CONTACT BETWEEN FLEXIBLE BODIES IN NONLINEAR ANALYSIS, USING LAGRANGE MULTIPLIERS

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Abstract

This paper deals with contact in nonlinear analysis. Only implicit schemes are looked at. In order to solve the contact problem, an augmented lagrangien procedure is used. The resulting system of equations is solved simultaneously for the displacements and Lagrange multipliers. Contact between two flexible bodies and contact between a flexible body and a rigid surface are treated. A special care is taken when the flexible body is modelised by second order element in 3d analysis.

1 Introduction

In nonlinear analysis, there are different ways to solve a contact problem. Penalty method or Lagrangien method can be used. The contact problem can be solved at the same time as the other nonlinearities or it can be solved in an uncoupled way. In this case, either for each structural iteration, the contact problem is solved or the structural iterations are performed with frozen contact conditions. In this paper, we solve the contact at the same time as the other nonlinearities and we use an augmented Lagrangien method [1,2]. The resulting system of equations is solved simultaneously for the displacements and Lagrange multipliers.

Contact between flexible and rigid bodies and contact between two flexible bodies are looked at. In the first case, we analyze the contact between one slave node and a master rigid surface, in the second case, we analyze the contact between one node and a master face. When second order elements are used in 3d analysis, we add a node linked to the face. That means we have brick element with 21 nodes and tetrahedral elements with 11 nodes.

2 Contact equations

The first step is a geometrical step. We search the projection of the node on the master surface, either a rigid surface or a flexible face. From this projection, we can compute the normal distance d_n between the node and the surface and the variation of tangential displacements (Δu_1 Δu_2) between the current iteration and the beginning of the time step.

We can compute the variation and increase of these values in function of the nodal unknowns q. The nodal unknowns are the displacements of the slave node, and either the displacements of the node to witch the rigid surface is linked or the displacements of the nodes of the face. We write :

$$\delta d_n = \underline{n}^T B \delta \underline{q}$$

$$\delta \Delta u_1 = \underline{t}_1^T B \delta \underline{q}$$

$$\delta \Delta u_2 = \underline{t}_2^T B \delta \underline{q}$$

$$d\, d_n = \underline{n}^T B d\underline{q}$$

$$d \Delta u_1 = \underline{t}_1^T B d\underline{q}$$

$$d \Delta u_2 = \underline{t}_2^T B d\underline{q}$$

where n is the normal and t_1 t_2 the tangents to the surface.

The second step is to define the contact condition. For this purpose, 3 Lagrange multipliers are introduced, one for the contact and two for the friction. We introduce the 3 quantities:

$$\sigma_n = k\lambda_n + p d_n$$

$$\sigma_{t1} = k\lambda_1 + p\Delta u_1$$

$$\sigma_{t2} = k\lambda_2 + p\Delta u_2$$

where k is scaling factor and p a regularization parameter. In order to decide if there is contact or not, we look at σ_n. The contact criteria is a mixed between forces and displacements. If we look at Figure 1, the solution at the convergence is the bolt line, either the normal distance or the Lagrange multiplier should be equal to 0. During the iterations, we are not on the solution, the dotted line divides the space in two zones, one where the node is said in contact and the other one where the node is said not in contact. The slope of this dotted line depends on the regularization parameter p. At the convergence, the solution will not depend on p but the convergence property depends on p. When the value of p changes, the criteria to say if a point is in contact or not will change.

We do not discuss on the value to be given to p. In the numerical application, we take it as a function of the stiffness of the structural elements.

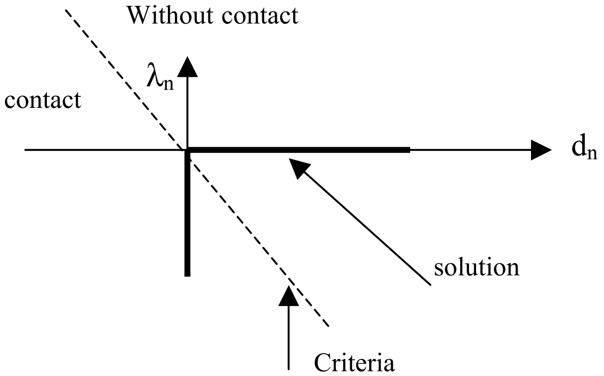

Figure 1 – Contact condition

2.1 No contact

If σ_n is larger than 0, there is no contact and we impose that the Lagrange multipliers linked to contact and friction is equal to 0. The nodal forces at the element level are computed from:

$$\delta \underline{q}^T \underline{F} = -\left(\delta\lambda_n k \lambda_n + \delta\lambda_{t1} k \lambda_{t1} + \delta\lambda_{t2} k \lambda_{t2}\right)$$

2.2 Contact

If σ_n is smaller than 0, there is contact. The nodal forces at the element level are computed from:

$$\begin{aligned}\delta \underline{q}^T \underline{F} &= \delta d_n\left(p d_n + k\lambda_n\right) + \delta\lambda_n k d_n \\ &= \delta \underline{q}^T B^T \underline{n}\left(p d_n + k\lambda_n\right) + \delta\lambda_n k d_n\end{aligned}$$

At the equilibrium, the normal distance is equal to 0 (equation linked to the Lagrange multiplier) and it can be seen that the regularization parameter has no influence on the solution. It can also be seen that the Lagrange multiplier is the contact force divided by the scaling parameter *k*. From this equation, we can compute an iteration matrix which is symmetrical. If the derivative of *B* is neglected, This iteration matrix is computed from:

$$\left\langle \delta \underline{q}^T \;\; \delta\lambda_n \right\rangle \begin{vmatrix} p B^T \underline{n}\,\underline{n}^T B & k B^T \underline{n} \\ k \underline{n}^T B & 0 \end{vmatrix} \begin{vmatrix} d\underline{q} \\ d\lambda_n \end{vmatrix}$$

We see that *k* is introduced in order to have the same order of magnitude for the equation linked to the Lagrange multiplier and the equations linked to the displacements. It is taken equal to the stiffness of the structural elements.

2.3 Friction

For the friction behavior, there is three status. The first one is the simplest one, there is no friction coefficient. In this case, we just impose that the Lagrange multipliers linked to the friction are equal to 0. We take as element forces:

$$\delta \underline{q}^T \underline{F} = -\left(\delta\lambda_{t1} k \lambda_{t1} + \delta\lambda_{t2} k \lambda_{t2}\right)$$

If there is friction, it remains two status. Either the node is stick to the surface or the node is sliding. In order to see what is the status, we first compute the following quantity:

$$\sigma_t = \sqrt{\sigma_{t1}^2 + \sigma_{t2}^2}$$

Stick

We compare σ_t to the normal force. The status is stick if:

$$\sigma_t - \mu\left|\sigma_n\right| \leq 0$$

In this case, we impose that the variation of tangential displacements is equal to 0. The element forces are computed from:

$$\begin{aligned}\delta \underline{q}^T \underline{F} = {} & \delta\Delta u_1\left(p\Delta u_1 + k\lambda_{t1}\right) + \delta\Delta u_2\left(p\Delta u_2 + k\lambda_{t2}\right) \\ & + \delta\lambda_{t1} k \Delta u_1 + \delta\lambda_{t2} k \Delta u_2\end{aligned}$$

As for the contact, the regularization parameter has no influence on the solution. We can build an iteration matrix which is symmetrical.

Slip

The status is slip if:

$$\sigma_t - \mu\left|\sigma_n\right| \geq 0$$

We first define the direction of sliding :

$$\begin{aligned} v_1 &= \sigma_{t1}/\sigma_t \\ v_2 &= \sigma_{t2}/\sigma_t \end{aligned}$$

The element forces are computed from:

$$\delta \underline{q}^T \underline{F} = \left\langle \delta\Delta u_1 \delta\Delta u_2 \delta\lambda_1 \delta\lambda_2 \right\rangle \begin{vmatrix} \mu\left|F_n\right| v_1 \\ \mu\left|F_n\right| v_2 \\ \left(\mu\left|F_n\right| v_1 - k\lambda_1\right) k/p \\ \left(\mu\left|F_n\right| v_2 - k\lambda_2\right) k/p \end{vmatrix}$$

We can check that the friction Lagrange multipliers are parallel to the variation of sliding displacements. At the equilibrium, the two last equations are proportional to *p*. But as they are local equations, *p* has no influence on the results.

If we introduce:

$$\begin{vmatrix}\delta v_1\\ \delta v_2\end{vmatrix}=\frac{1}{\sigma_t}\begin{vmatrix}1-v_1^2 & -v_1v_2\\ -v_1v_2 & 1-v_2^2\end{vmatrix}\begin{vmatrix}p\delta\Delta u_1+k\delta\lambda_1\\ p\delta\Delta u_2+k\delta\lambda_2\end{vmatrix}=\frac{1}{\sigma_t}T\begin{vmatrix}p\delta\Delta u_1+k\delta\lambda_1\\ p\delta\Delta u_2+k\delta\lambda_2\end{vmatrix}$$

The iteration matrix is equal to:

$$\delta\underline{q}^T\underline{dF}=\langle\delta\Delta u_1\,\delta\Delta u_2\,\delta\lambda_1\,\delta\lambda_2\rangle\begin{vmatrix}\mu v_1\\ \mu v_2\\ \mu v_1 k/p\\ \mu v_2 k/p\end{vmatrix}\left(pdd_n+kd\lambda_n\right)+$$

$$\langle\delta\Delta u_1\,\delta\Delta u_2\,\delta\lambda_1\,\delta\lambda_2\rangle\begin{vmatrix}p\frac{\mu|\sigma_n|}{\sigma_t}T & k\frac{\mu|\sigma_n|}{\sigma_t}T\\ k\frac{\mu|\sigma_n|}{\sigma_t}T & \frac{k^2}{p}\left(\frac{\mu|\sigma_n|}{\sigma_t}T-I\right)\end{vmatrix}\begin{vmatrix}d\Delta u_1\\ d\Delta u_2\\ d\lambda_1\\ d\lambda_2\end{vmatrix}$$

In this case, it is not symmetrical.

3 Projection of the slave nodes

Local Newton iterations are performed in order to find the projection of the node on the surface, except if the face is a triangle at degree 1. We will not discuss this point here. Just recall that when we compute the *B* matrix in the rigid surface case, there is a master node to which is linked the surface and with whom the surface is moving.

For flexible to flexible contact, the faces seen by a node change during the computation. We update these faces at each iteration.

4 Second order element in 3d analysis

We consider two brick elements with 20 nodes (Figure 2). The first one is fixed along z at the bottom face. The rigid body modes in the xy plane of the two elements are fixed. Contact is introduced between the two elements. A constant pressure is applied at the top of the structure.

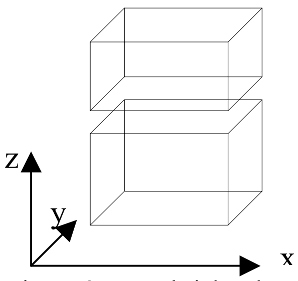

Figure 2 – two bricks elements

If the area of the upper face and the pressure are equal to 1, the nodal forces conjugated to the pressure are equal to:

$$F^T=\langle -1/12\ 1/3\ -1/12\ 1/3\ -1/12\ 1/3\ -1/12\ 1/3\rangle$$

Some are positive, some are negative. For a constant pressure, we should have the same distribution on the contact zone. With a contact strategy node to face, it will not be the case. The forces will go only through the mid-side nodes and nothing on the corner nodes. So, with standard 20 nodes element, it is not possible to have the exact solution for this small problem.

The solution is to add a node at the middle of the face. Its shape function is equal to:

$$N_{face}=\frac{1}{2}\left(1-\xi^2\right)\left(1-\eta^2\right)\left(1+\zeta\right)$$

if the contact face is the upper one. We have also to change the shape function of the nodes belonging to the face. For a corner node or a mid-side node, we have simple relationship between the new shape function and the classical one:

$$N_{new,corner}=N_{classical,corner}+\frac{1}{4}N_{face}$$

$$N_{new,midside}=N_{classical,midside}-\frac{1}{2}N_{face}$$

Once the shape functions are defined, it is a classical isoparametric element with 21 nodes. The nodal forces conjugated to a constant pressure are now equal to:

$$F^T=\langle 1/36\ 1/9\ 1/36\ 1/9\ 1/36\ 1/9\ 1/36\ 1/9\ 4/9\rangle$$

They have the same sign. For the small problem, we now have the exact solution. For the triangular faces, we adopt the same strategy and get tetrahedral element with 11 nodes.

5 Numerical application

The method has been introduced in the Samcef Mecano software [3,4].

5.1 Buckling of a cylinder

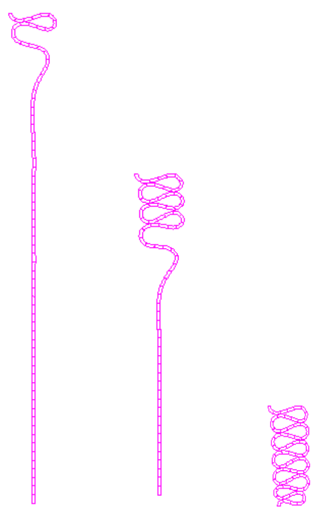

Figure 3 – Buckling of a cylinder

As first example, we study the buckling of a cylinder. The radius is equal to 100, the height to 200 and the thickness 1. The Young modulus is equal to 210000, the Poisson coefficient to 0.3 and the initial yield stress 1500. An axial force is introduced. First a linear analysis is performed followed by a buckling analysis. We introduce as imperfection for the nonlinear analysis the 5 first buckling modes. We impose the axial displacement of the upper face. In this case, we have flexible to flexible contact and we also introduce a flexible to rigid contact with rigid plane at the top and bottom of the cylinder. We only look at the axisymmetrical behavior. A static analysis is performed. Figure 3 show the deformed shape for different load level. The axial force in function of the axial displacement is shown on Figure 4.

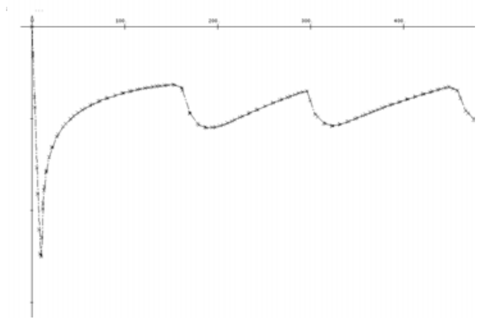

Figure 4 – buckling of a cylinder – load displacement curve

5.2 Hertz contact problem

The second example is a classical Hertz contact problem. We study the contact between a flexible cylinder (radius 1m, Young modulus 3. 10^6Pa, Poisson coefficient 0.4) and a rigid plane. We use plane strain hypothesis and second degree elements. By symmetry, we study a half model with 500 element. In the contact area, we use a 8 by 8 regular mesh, elsewhere we use a free mesh. The applied load is equal to 2224N.
Figure 5 shows the distribution of vonmises stresses in the cylinder

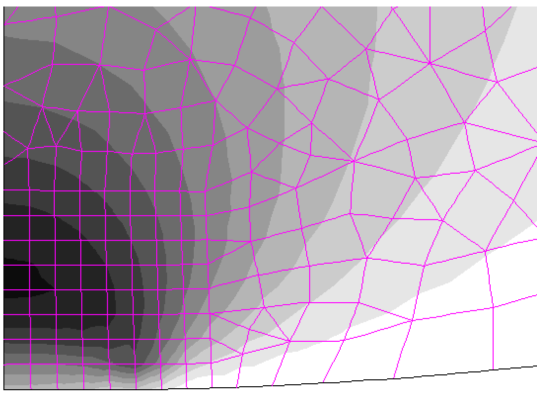

Figure 5 - vonmises stress distribution

The solution is close to the theoretical solution. The load is applied in 5 steps, good convergence is observed. A maximum of 6 iterations per step is needed. At the maximum load, 12 nodes are in contact. On Figure 6, we can see the distribution of the contact pressure in the contact area.

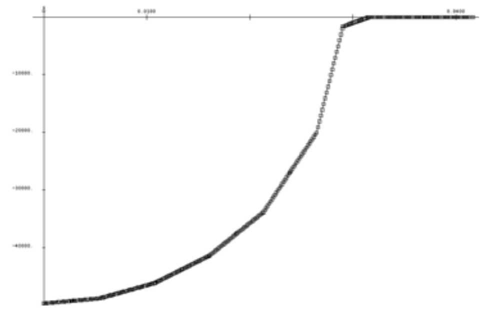

Figure 6 – contact pressure

5.3 Key problem

The next example is a pin insertion, it was proposed by S. Adeff [5]. Both part are flexible.

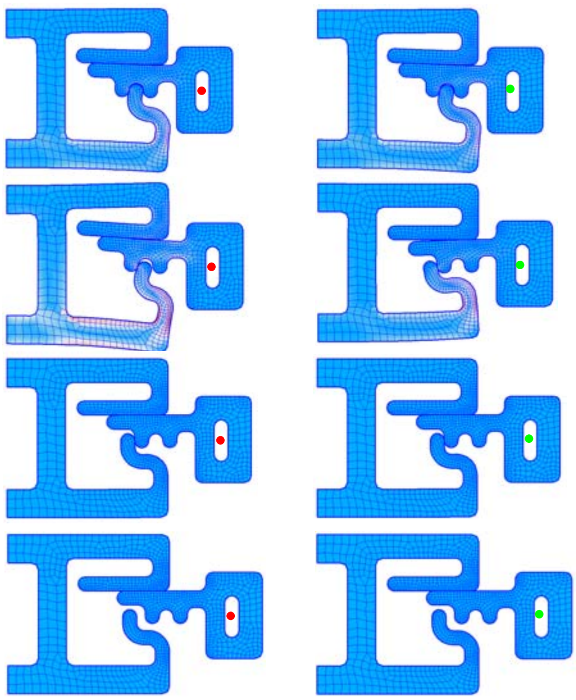

Figure 7 – key problem

Figure 7 shows the different positions of the pin. It should be looked starting from the bottom at the right and turning in a trigonometric way. The key is pushed (right side from bottom to top) and after that is pulled back (left side, from top to bottom).

In this case, the faces seen by the slave nodes change during the analysis. We have to update the topology at each time step.

5.4 Pre-stressing of a cylinder

The purpose of this example is to introduce a cylinder in a hole the radius of which is smaller than the radius of the cylinder. The hole belong to a structure which has a cyclic symmetry, so only 1/6th of the structure is modelised. 3d second degree elements are used, cyclic conditions are introduced at the boundary.

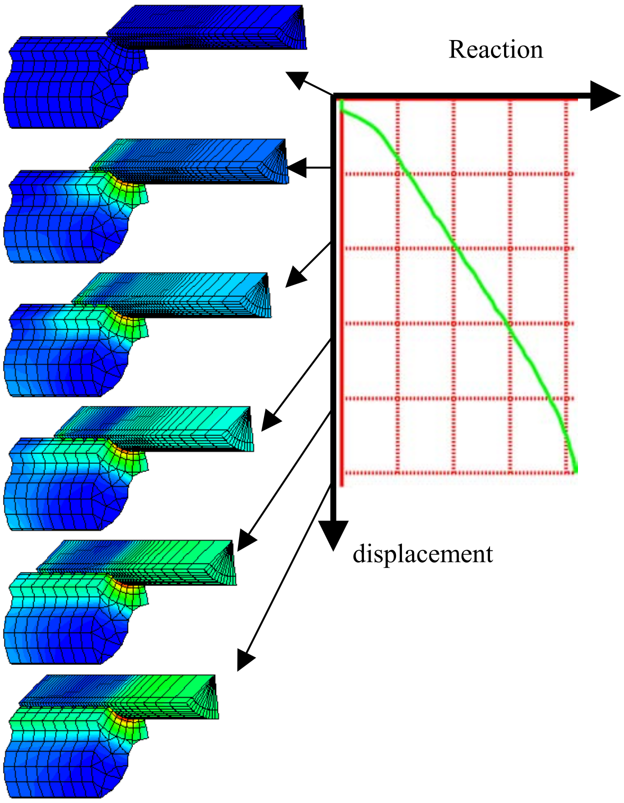

Figure 8 – prestressing of a cylinder

Figure 8 shows different positions of the inner cylinder and the reaction force corresponding to different displacements.

5.5 STENTOR tank, stability with contact

The STENTOR (Satellite de Télécommunication et Etudes de Nouvelles Technologies en Orbite) program has as objective to put in orbit a satellite with new technology in order to test it. In this program, Aerospatiale Matra develops a new high pressure tank.

The tank is made of a cylindrical part and two hemispherical parts (Figure 9). The external structure is made of composite material and the internal part is a metallic liner. The liner is not glued on the composite, there is only contact with friction between both materials. A pressure is introduced in the tank. The composite material has a linear behavior, plasticity appears in the liner. When the pressure is removed, residual stresses are present in the structure. In this way, under current load, the behavior of the tank will be linear.

There is some compression in the liner, buckling can occur.

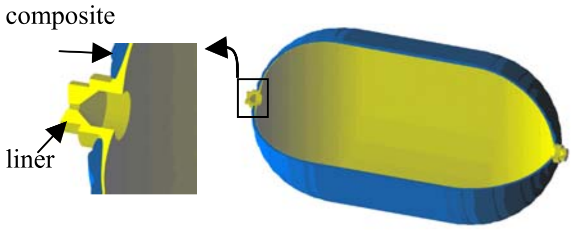

Figure 9 – STENTOR tank

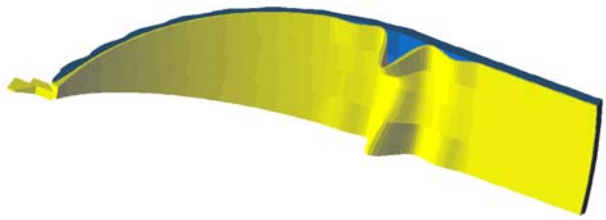

Figure 10 – buckling mode

The purpose of the study is to see for which size of imperfection buckling will occur. For an imperfection equal to the thickness of the liner, we have an instability. The displacement at the end of the analysis are shown on figure 10.

6 Conclusion

A general method has been presented in order to handle contact in nonlinear analysis. A special care has been given to second degree element in 3d analysis. Only static examples are performed in an implicit way.

7 Reference

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