

Introduction to Multi-physics Modelling

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Computational Modelling of multi-physics processes - What is it?

- **Computational Modelling is :**
 - “The application of numerical approximation methods and computers to the solution of problems in Engineering and Applied Sciences” - O. Zienkiewicz
- **Computational modelling can be used to predict a number of physical phenomena including:**
 - Fluid Flow
 - Heat Transfer
 - Solid mechanics
 - Electromagnetics
 - Etc
- **Multi-physics involves some or all of the above simultaneously**

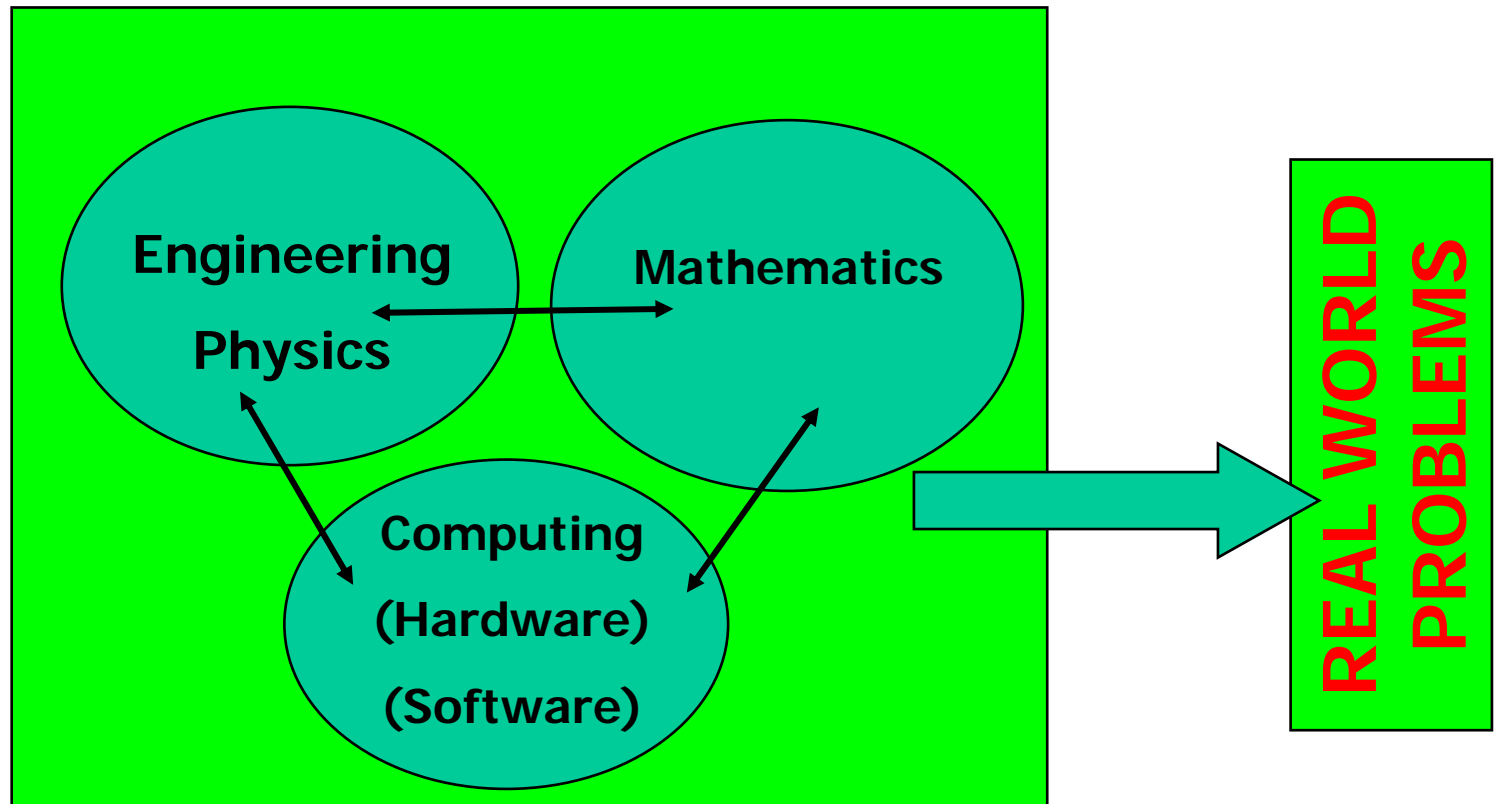


Computational Modelling – What is it?

- **Simulating real-world phenomena on a computer involves:**
 - understanding the governing physics.
 - formulating the problem in terms of mathematics.
 - writing computer software that solves the mathematical equations.
 - running the computer program
 - viewing (& analyzing) the results.
- **Generally most real-world problems require solution to thousands of equations.**
- **Many commercial software tools now available.**

Computational Modelling - What is it?

- **Computational Modelling Research - Interdisciplinary**



Why use Computational Modelling?

- **Cost of Quality**

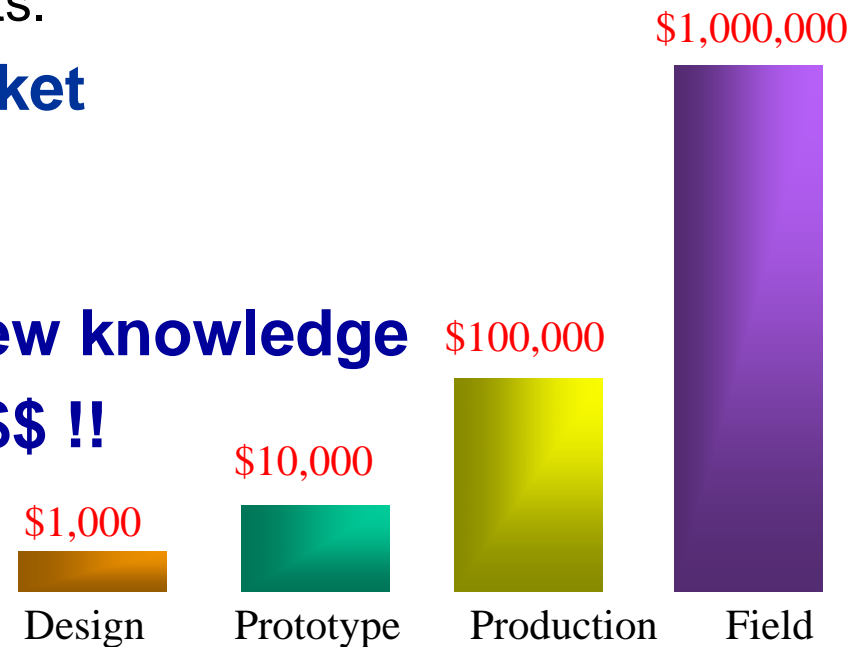
- warranty liabilities due to field failures;
- redesign; rework; and scrap costs.

- **Lateness of product to market**

- First two manufactures to market lock up 80% of business

- **Modelling helps generate new knowledge**

- **Modelling upfront saves \$\$\$\$!!**

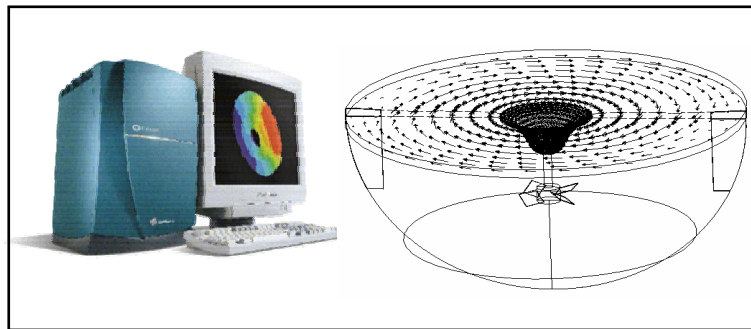


Modelling – real world interaction

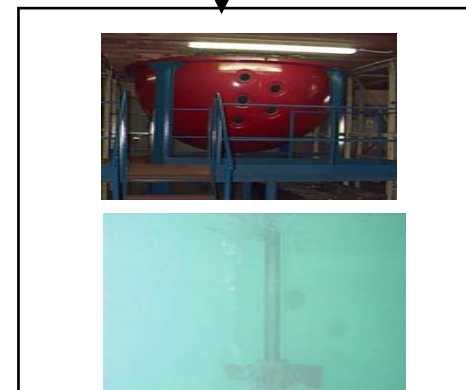
- Compliments Experimentation
- Validation should always be undertaken when possible



Refining Process



Computational
Modelling



Water Modelling

Governing Physics

- **Materials Processing**

- Fluid Flow + Free surface
- Chemical Reactions
- Electromagnetic Fields
- Heat Transfer
- Solidification
- Stress



Casting

- **Many processes governed by interactions of the above**

- Multi-physics

- **Multi-scale - 0.1μ - 1m**



Refining

Governing Equations

- The governing equations can be expressed in a standard form:

$$\frac{\partial}{\partial t}(\rho A \phi) + \nabla \cdot \underline{Q} = \nabla \cdot (\Gamma \nabla \phi) + S$$

	ϕ	A	Γ_{ϕ}	S	\underline{Q}
<i>Continuity</i>	1	1	0	S_{mass}	$\rho \underline{v}$
<i>Momentum</i>	\underline{v}	1	Γ_v	$(S + \underline{J} \times \underline{B} - \nabla P)$	$\rho \underline{v} \cdot \underline{v}$
<i>Heat transfer</i>	h	1	k/c	S_h	$\rho \underline{v} h$
<i>Electromagnetism</i>	\underline{B}	1	η	$(\underline{B} \nabla) \underline{v}$	$\underline{u} \cdot \underline{B}$
<i>Solid Mechanics</i>	\underline{u}	$\partial / \partial t$	μ	$\rho \underline{f}_b$	$\mu(\text{grad } \underline{u})^T + \lambda(\text{div } \underline{u} - (2\mu + 3\lambda)\alpha T)\underline{I}$

Steps In Computational Modelling

- CAD generates geometry
- CAD captures 'patches'
- CAD domain meshed
- Number of methods used to discretise equations
 - Finite Difference
 - Finite Element
 - Finite Volume
- Graphically based software tools for results analysis & visualisation.
- 'patches' – are points, surfaces or volumes

Build Geometry

Specify patches for bdy conditions & vol loads

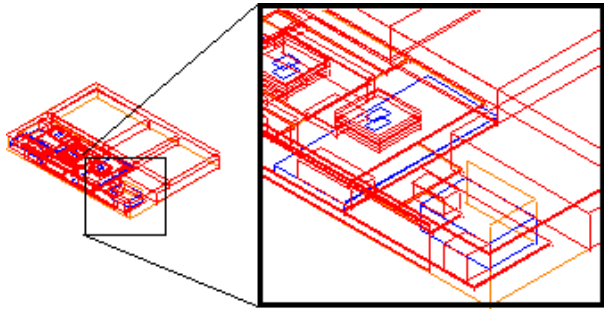
Mesh the Geometry

Specify & Solve governing equations

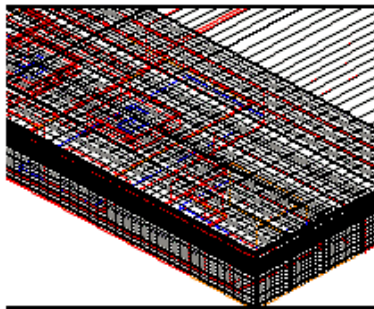
Visualise/analyse the results

Steps in Computational Simulation

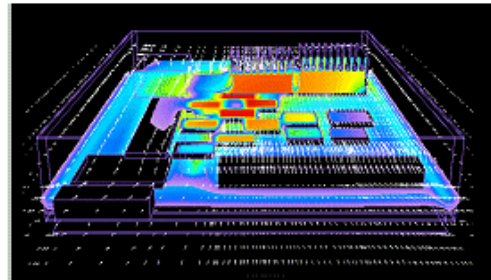
- CAD – MESHING – ANALYSIS - VISUALISATION



CAD



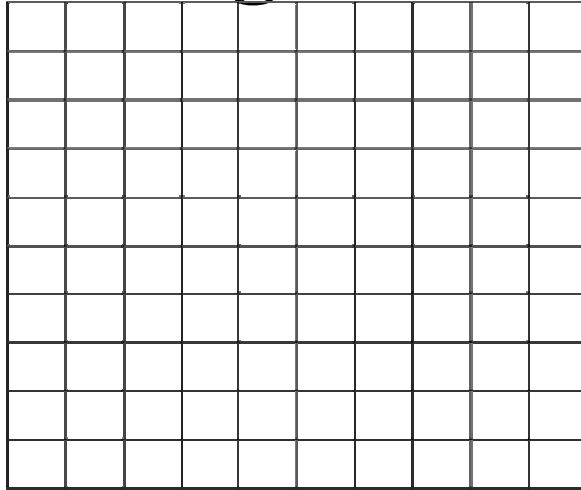
MESH



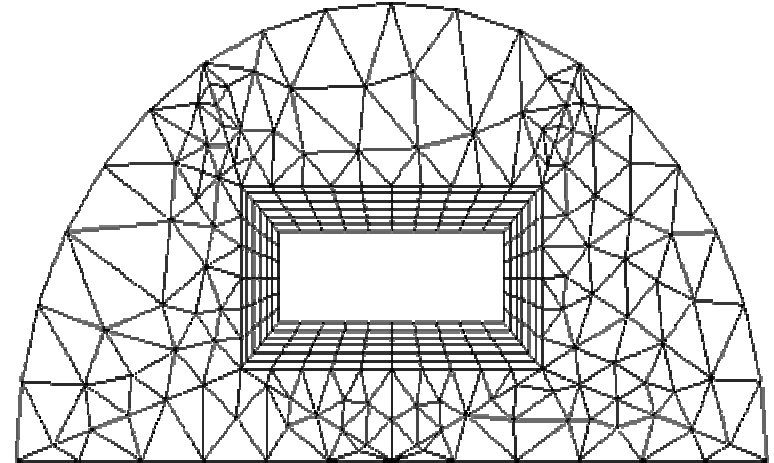
ANALYSIS VISUALISATION



Meshing – Structured and Unstructured



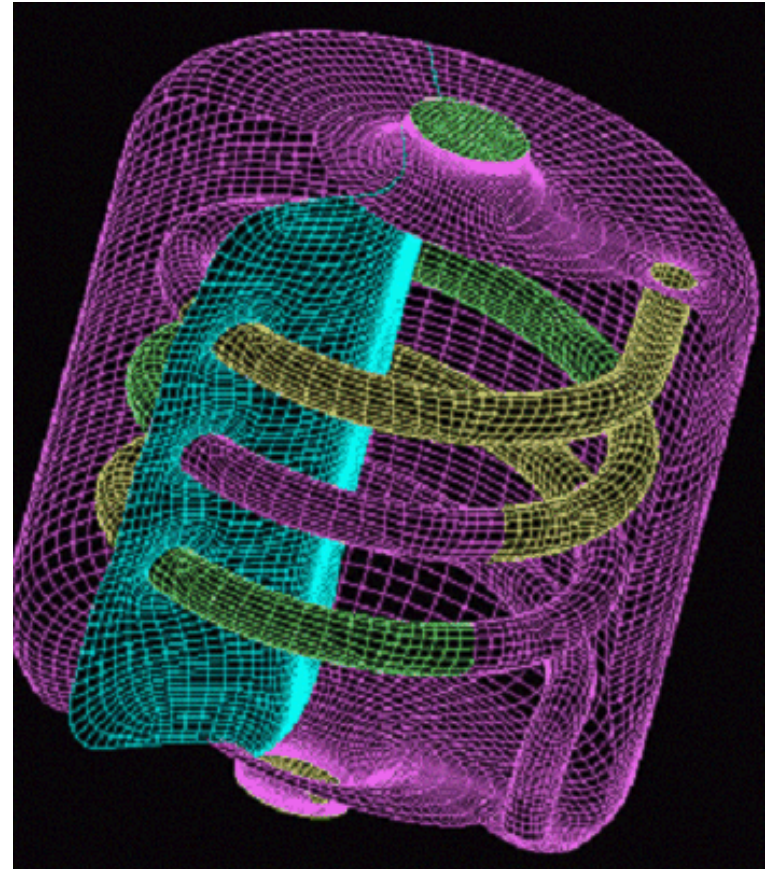
- **Data referenced using I,J grid lines**
- **Allows line-line solvers.**
 - Low in-core memory requirement
- **Poor on complex shapes.**
 - Use BFC or Block-Structured



- **Good for complex geometry**
- **Topology representation**
 - Points - Faces - Elements
- **Mix element types.**
- **Requires whole field solvers.**
 - Large memory needed

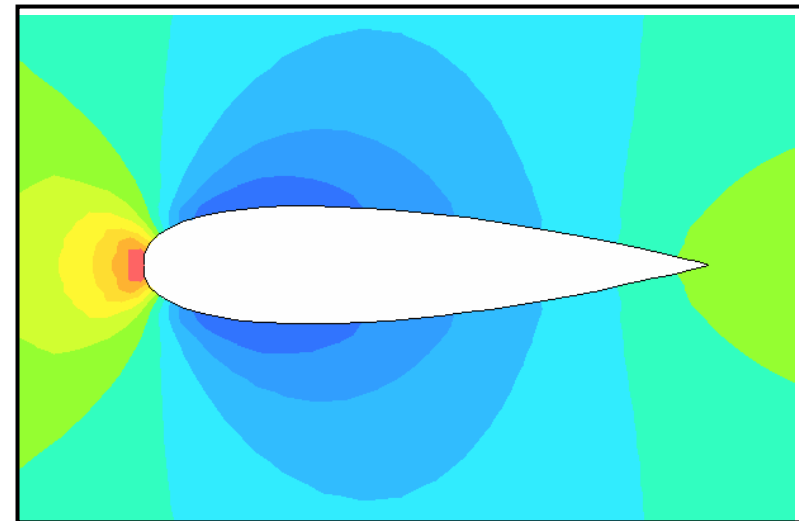
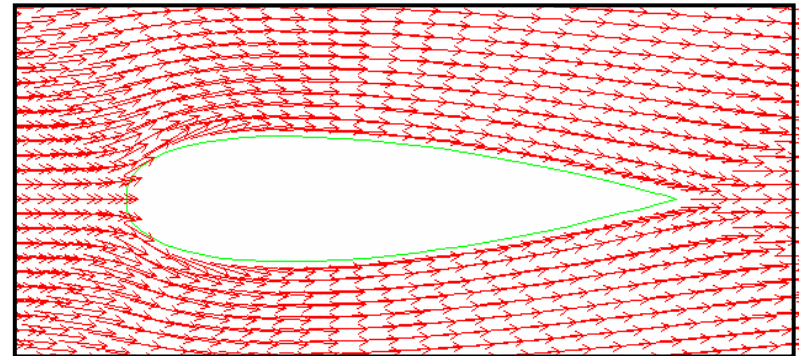
CAD + Meshing Software

- **PRO/ENGINEER**
 - <http://www.ptc.com/>
- **CATIA**
 - <http://www.catia.com/>
- **Mentor Graphics**
 - <http://www.mentor.com/>
- **FEMSYS**
 - <http://www.femsys.co.uk/>
- **GRIDPRO**
 - <http://www.gridpro.com/>



Computational Fluid Dynamics - CFD

- **Started in early late 60's early 70's**
- **Based on Finite Difference/Volume Methods**
- **Simulations now include**
 - Turbulence
 - Free surface Flows
 - Heat Transfer + Solidification
 - Chemical Reactions
- **Early work based on structured grids**
- **Unstructured Meshes now used**
- **FE Methods also used.**



Computational Fluid Dynamics

$$\frac{\partial \rho \phi}{\partial t} + \text{div}(\rho \underline{u} \phi - \Gamma_{\phi} \nabla \phi) = S_{\phi}$$

Typically

- **Used Finite Volume Method**
- **Eulerian Approach (Fluid moves through a static mesh)**
- **Highly non-linear**

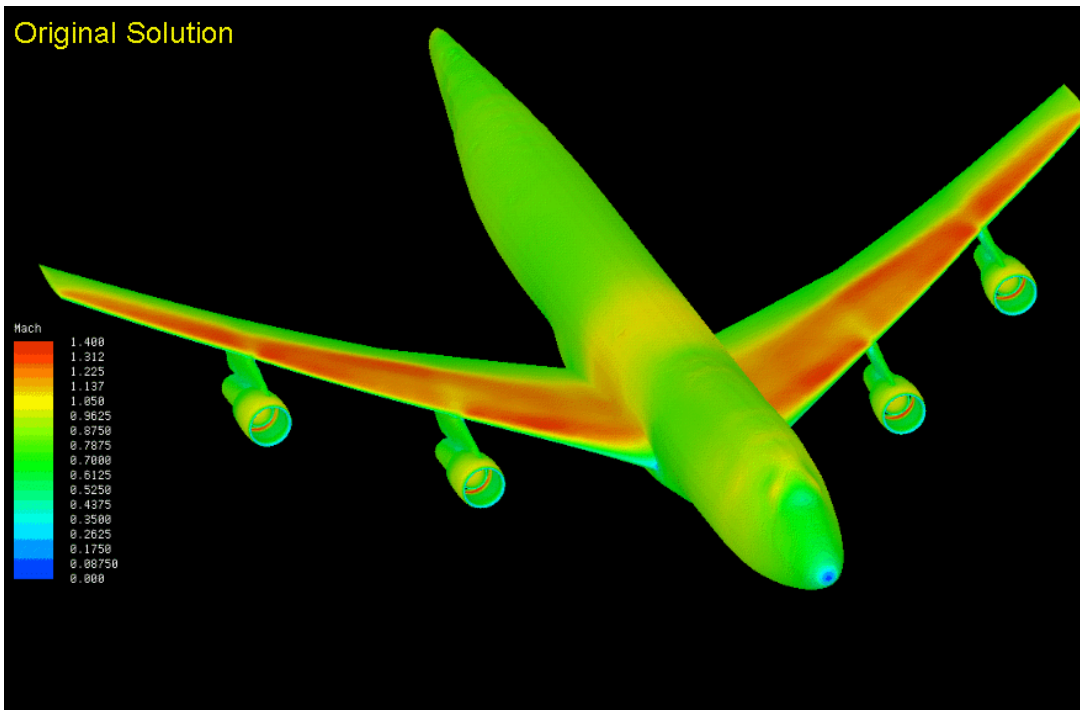
Fluid Flow

$$\frac{\partial \rho u_x}{\partial t} + \text{div}(\rho \underline{u} u_x - \mu \nabla u_x) = -\frac{\partial p}{\partial x} + \rho g$$

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \underline{u}) = S_m$$

- Navier-Stokes equation
- Newtonian fluid
- SIMPLE type algorithm resolve pressure
- Staggered – Multi-block - Cartesian or cylindrical polars

Fluid Flow



Turbulence

- **Solved variables contain a**
 - time averaged component
 - fluctuating component
- **Turbulence models the effects of the fluctuating component**
- **Effects other equations by affecting the diffusion coefficient**
- **Many turbulence models**
- **k-e most commonly used**

Turbulence

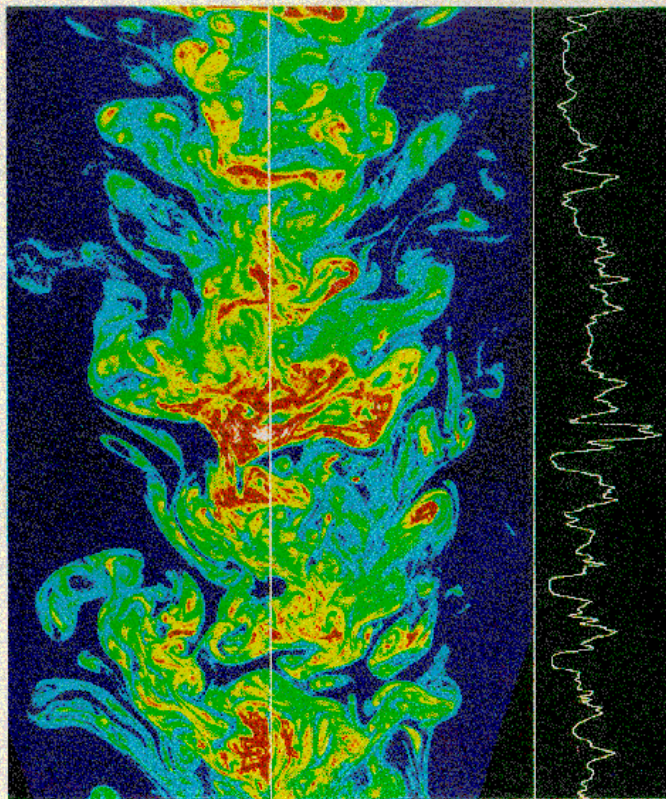
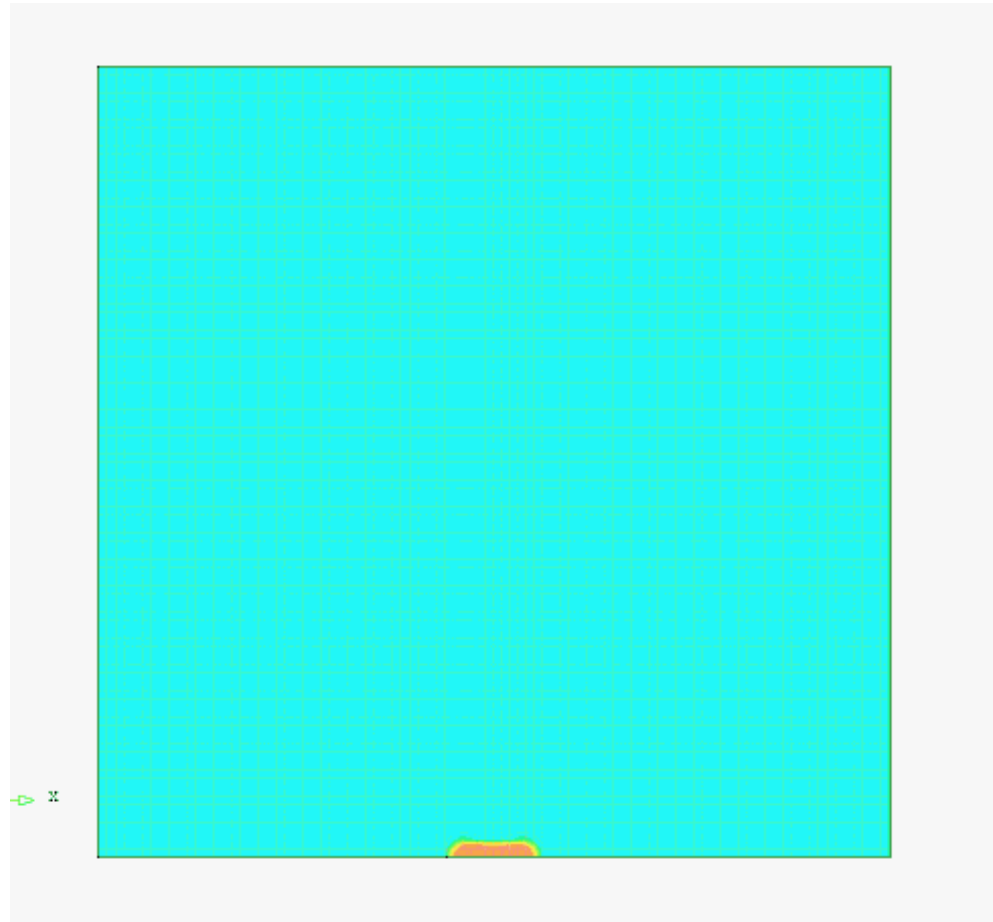
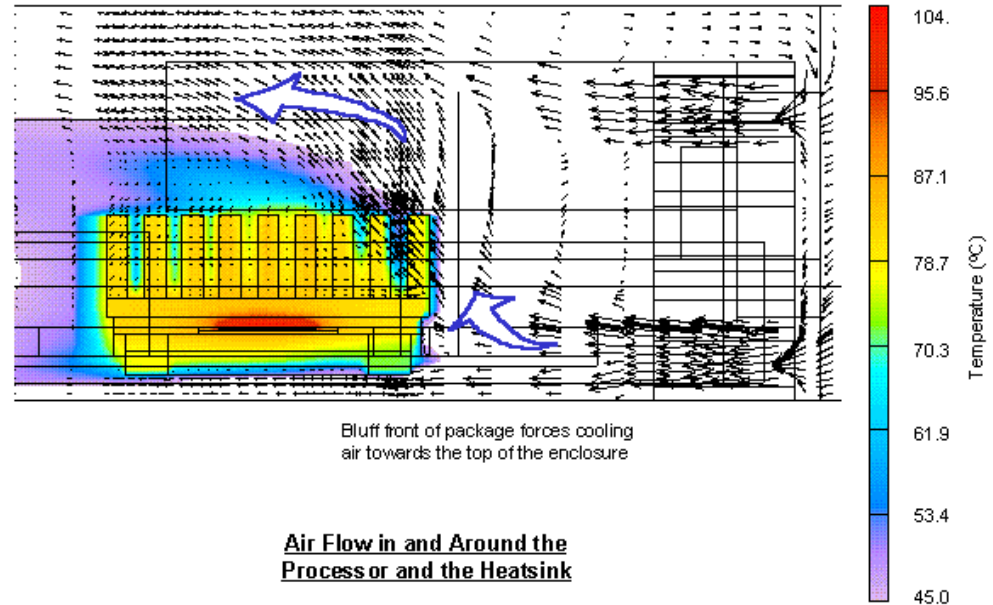


Figure 23 The result of applying a conformal mapping to the jet shown in Figure 3. The map, due to Everson et al (1990), corrects for the growth of the jet by covering a wedge into a slot. The downstream decay of the mean concentration has been normalized using similar considerations. The signal to the right corresponds to the vertical line cut shown in the jet.



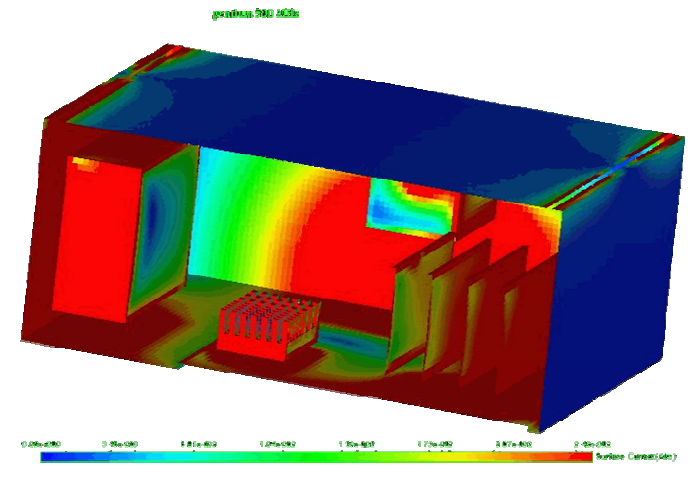
Commercial Software – CFD

- Resources at : <http://www.cfd-online.com/>
- **FLUENT**
 - <http://www.fluent.com/>
- **PHOENICS**
 - <http://www.cham.co.uk/>
- **STAR-CD**
 - <http://www.cdadapco.com>
- **PHYSICA**
 - <http://www.multi-physics.com>
- **CFX**
 - <http://www.anssys.com/cfx>



Computational Electromagnetics

- **Solution to Maxwells equations**
- **Low Frequency (10-100KHz)**
 - Time dependence importance
- **High Frequency (>1MHz)**
 - Frequency domain
- **Number of methods used**
 - Finite Difference
 - Transmission Line Methods
 - Finite Element Methods



Surface currents
at 482MHz

MHD Equations

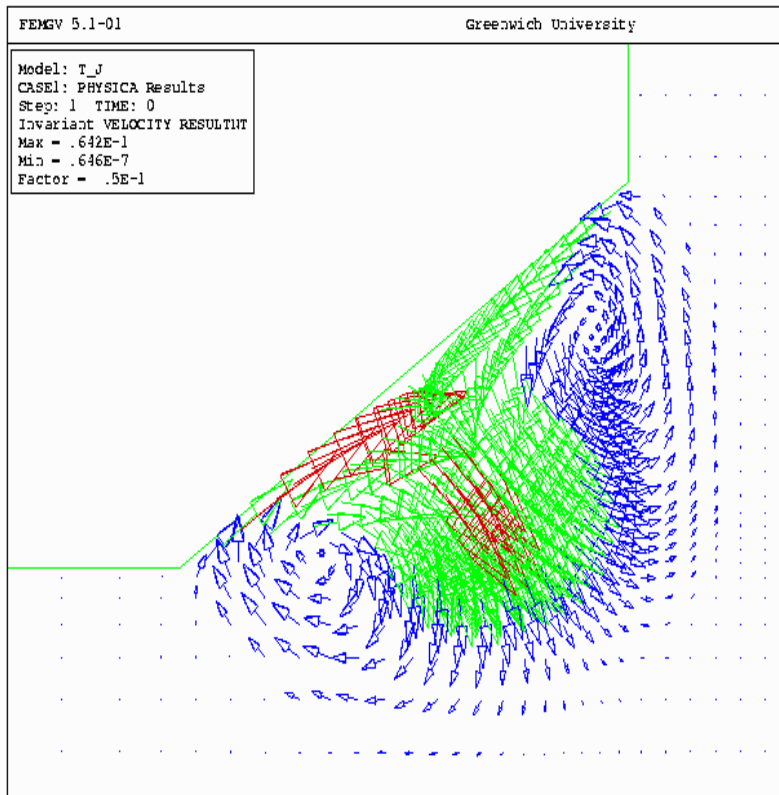
- Assume I have the NS equations, etc then ADD
- For example a simple derivative form of the Maxwell's Equations

$$\nabla \cdot (\sigma \nabla \phi) = \nabla \cdot (\mathbf{u} \times \mathbf{B}) + S_\phi$$

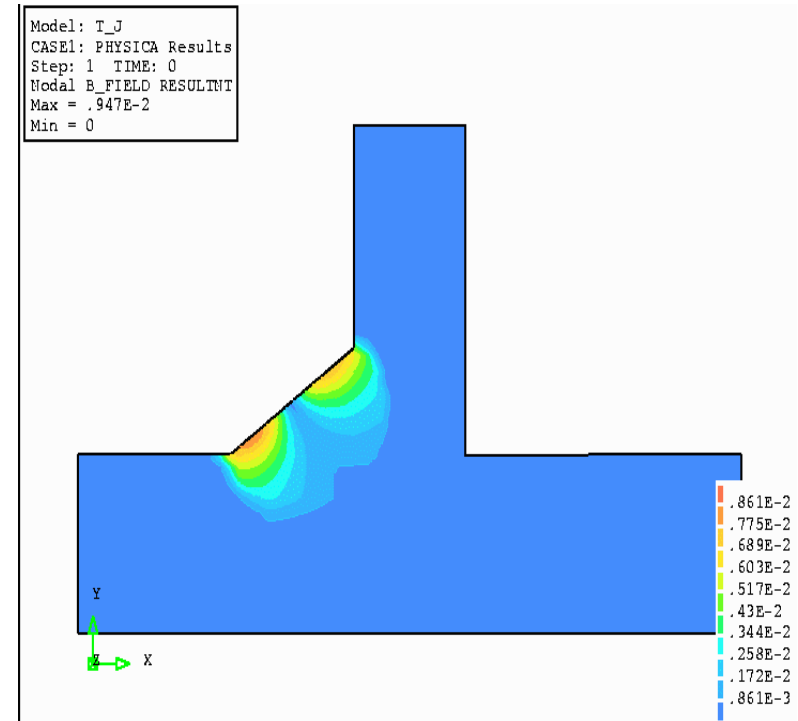
$$\mathbf{E} = -\nabla \phi \quad \text{and} \quad \mathbf{J} = \sigma_e \mathbf{E} .$$

- **B from the Biot – Savart Law**
 - e.g. $\mathbf{B} = (\mu_0 / 4\pi) \int d\mathbf{l} \times \mathbf{r} / |\mathbf{r}|^3$
- **Lorentz force: $\mathbf{J} \times \mathbf{B}$**
 - add to the NS momentum equations*
 - Extra source term in heat equation due to magnetic field $|\mathbf{J}|^2 / \sigma$
and include both in the CFD solution loop

Weld Pool Dynamics



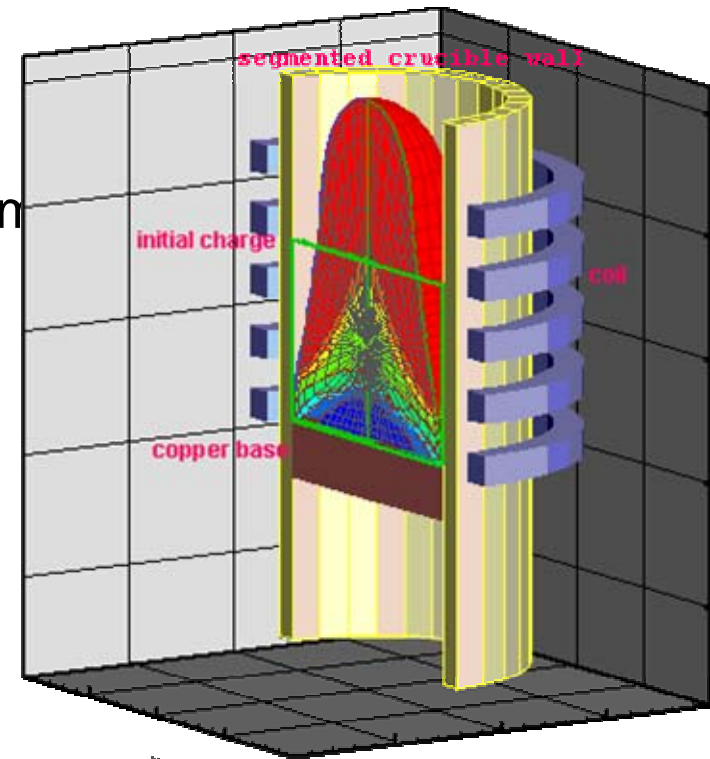
Velocity vectors in crosssection



Lorentz force distribution in the weld-pool

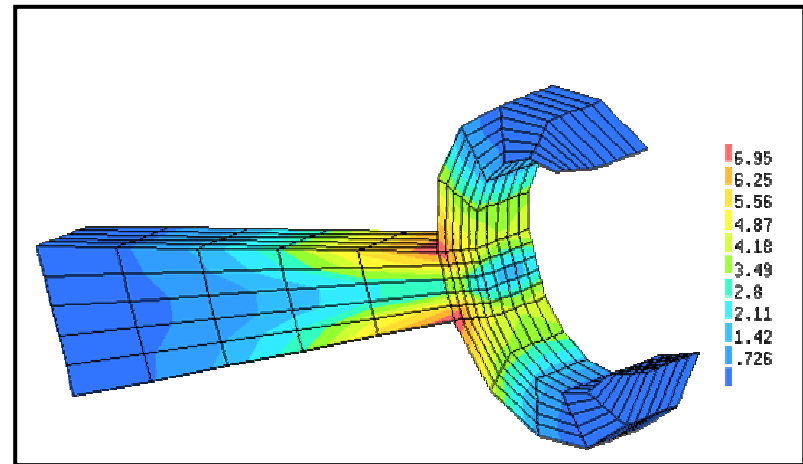
Commercial Software - Electromagnetics

- **General resource** : <http://emlib.jpl.nasa.gov/>
- **Vector Fields**
 - <http://www.vectorfields.com/>
- **ANSYS/EMAG**
 - <http://www.ansys.com/products/emag.htm>
- **Microstripes**
 - <http://www.flomerics.com/>
 - Close links to Flotherm
- **FLO/EMC**
 - For Electronic enclosures
 - <http://www.flomerics.com/>



Computational Solid Mechanics

- **Early work : Finite Difference**
 - Simple shapes.
- **Finite Elements introduced in late 50's**
 - Restricted to linear problems
 - Structural elements (beams, Trusses)
- **Continuum elements introduced mid 60's**
- **Plasticity included in early 70's**
- **Finite Volume Methods now also being used (early 90's)**



Solid Mechanics

$$\frac{\partial}{\partial t} \left(\rho \frac{\partial u_i}{\partial t} \right) = \frac{\partial \sigma_{ij}}{\partial x_j}$$

$$\sigma_{ij} = 2\mu \varepsilon_{ij}^{el} + \lambda \varepsilon_{kk}^{el} \delta_{ij}$$

$$\varepsilon_{ij}^{el} = \varepsilon_{ij}^{tot} - \varepsilon_{ij}^{th}$$

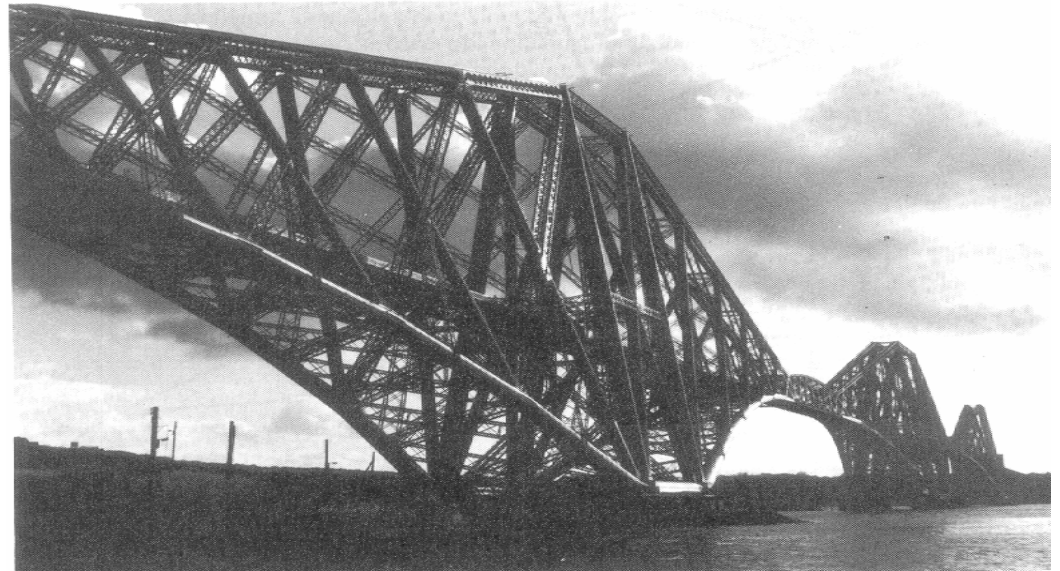
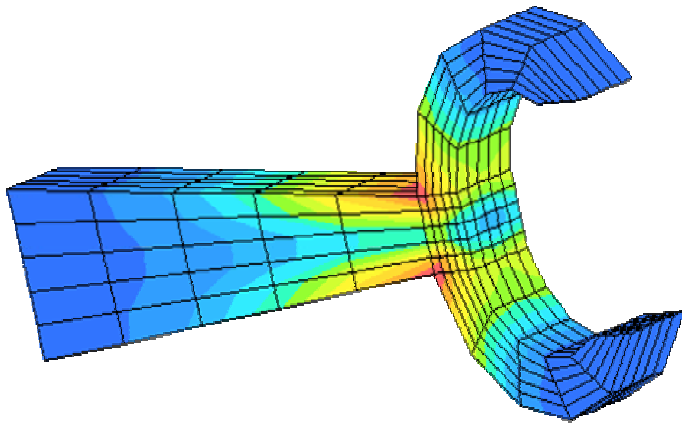
$$\varepsilon_{ij}^{tot} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\varepsilon_{ij}^{th} = \alpha \Delta T \delta_{ij}$$

Typically

- Uses the Finite Element Method
- Lagrangian Approach (Mesh moves with material)
- Elastic material => linear,
- Plasticity => some non-linearity

Stress - Displacement



Fatigue



Commercial Software – Stress Analysis

Resources at :

http://www.engr.usask.ca/~macphed/finite/fe_resources/fe_resources.html

- **ANSYS**

- <http://www.ansys.com/>

- **MARC**

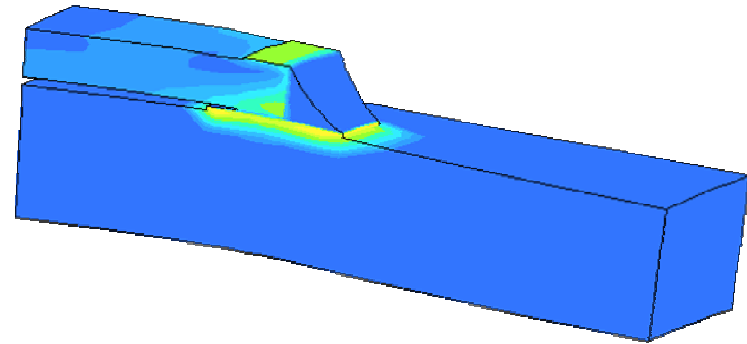
- <http://www.marc.com/>

- **NASTRAN**

- <http://www.mscsoftware.com/>

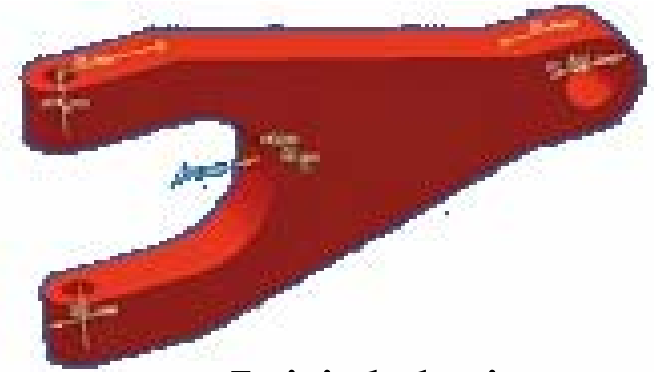
- **ABAQUS**

- <http://www.abaqus.com>

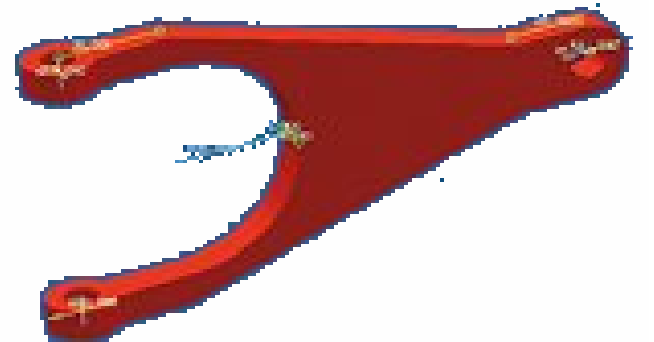


Optimisation

- **Optimisation Techniques**
 - Numerical
 - Design of Experiments (DOE)
- **Numerical**
 - Exact
 - Slow
- **DOE**
 - Approximate
 - Fast



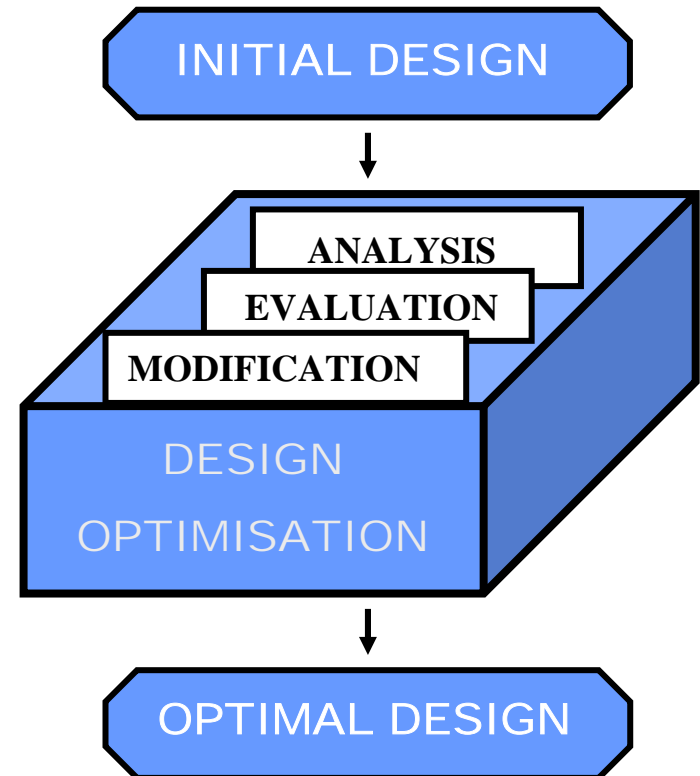
Initial design



Final design

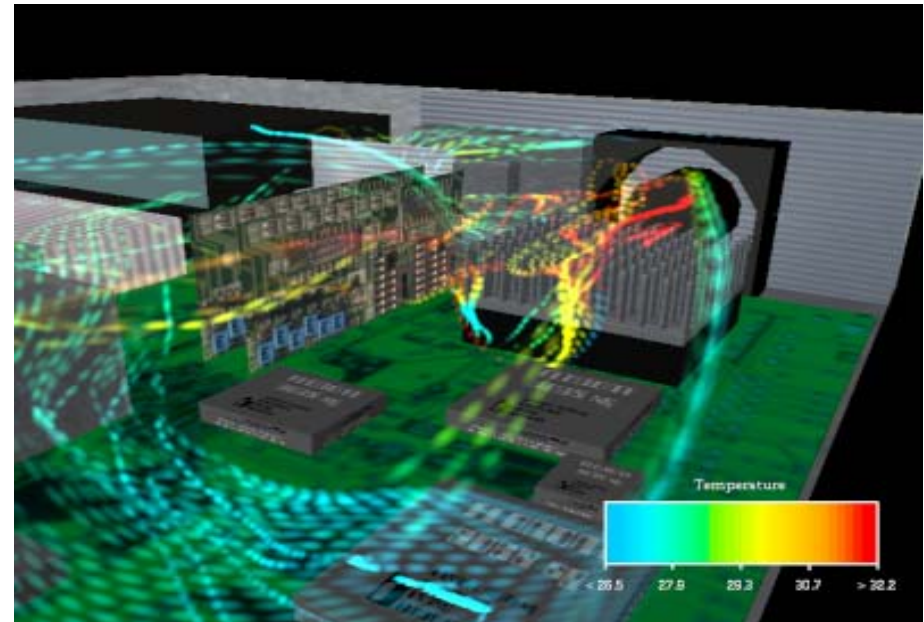
Optimisation Tools

- <http://optimal.hypermart.net/OPTM00.htm>
- **Optimisation Tool (DOT)**
 - VanderPlatts
 - VisualDoc (Graphical version)
 - Flexible software
 - Links to Analysis codes
 - <http://www.vrand.com/>
- **OptiStruct**
 - Altair Engineering
 - <http://www.altair.com/>
 - Structural Optimisation
- **Most major CAE tools have their own optimisation modules (e.g. ANSYS)**



Visualisation Software

- Resource <http://www.roe.ac.uk/~acd/vissys/>
- Note that all analysis codes will have some visualisation capabilities.
- Leading-edge visualisation codes:
 - **Wavefront**
 - <http://www.aliaswavefront.com/>
 - **AVS**
 - <http://www.avs.com/>
 - **Enight**
 - <http://www.ceintl.com/>

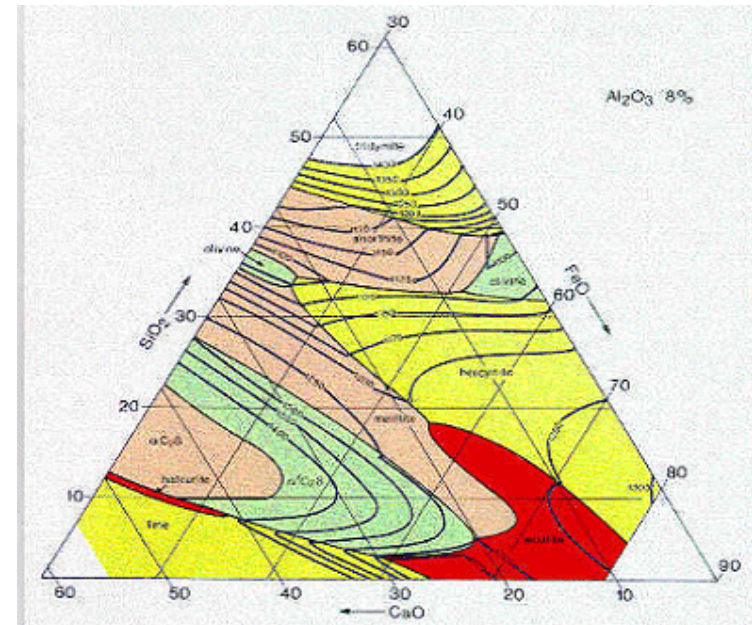


Thermodynamics Software

- Computational Thermodynamics + Databases
- Chemical and Phase equilibrium
- Function of composition and process conditions
- Can link to Macro-Models
 - i.e solidification

Software

- **MTDATA**
 - <http://www.npl.co.uk/npl/cmmt/mtdata/mtdata.html>
- **Thermo-Calc**
 - <http://www.thermocalc.se>



Multi-Physics and Multi-scale Modelling




Story So Far

- **Computational Fluid Dynamics (CFD)**
- **Finite Elements – Computational Solid Mechanics (CSM)**
- **Electro-Magnetics + Magneto-Hydro-Dynamics**
- **Each subject solves its own set of equations for the quantities of interest (e.g. velocity, stress, current).**
- **Some cross over in the quantities solved for (e.g. temperature)**


We asked some people this question..
"Having seen such a phenomenal growth in Finite Elements over the last 30 years, what do you foresee as the next giant leap in Engineering Simulation?"

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 The developments yet more new methods derived on the basis of FE, coupled with the dramatic expansion of computer power. <i>Paul Dextler</i> BAF System	 Multi-disciplinary applications - much more wide spread. No more single topics, its time to look at things as a whole. <i>Paul Dextler</i> BAF System
 Simulation of multiphysic & coupled phenomena. Application to health sciences and analysis and design of materials at the molecular level. <i>Roland Glowinski</i> University of Houston	 Real-time virtual reality simulations. <i>Denis Blackmore</i> New Jersey Institute of Technology
 Bridging dimensions from nano to macro physics. Quantitative (absolute) error controlled, adaptivity of approximations & in mathematic models in analysis, subdomains. Plus knowledge based tools for giving insight & advice for users for using complex software systems in complex virtual science. <i>Erwin Stein</i> University of Hannover	 Automated mesh generation in all fields of analysis (e.g. non-linear). <i>Cody Mayhew</i> Century Dynamics
 More powerful and cheaper software with more simulation. Less experiences people. <i>Gerhard Krause</i> Krause Software GmbH	 Real-time FE simulations for large scale manufacturing solids and particulates. <i>Roger Owen</i> University College of Swansea
I think the future lies in fluid structure interaction (Hydrodynamic impact) <i>Bernard Peseux</i> E.C. Navire	 Multi-scale modelling and computing. Multi-physics and coupled phenomena, as well as uncertainty. <i>H. Matthie</i> University of Braunschweig
 Automated Project (eg. vehicle) simulation (multi-level, multi-physics, multi-optimization, etc.). Putting these all together. <i>Raymond Löhner</i> George Mason University	 Fluid structure and teaching the theory while not forgetting the practical side (the usage). Plus the joy of and desire to combine the two. <i>G. Sandberg</i> Lund University
 In aeronautics, customised design of air vehicles (e.g. missiles) will become feasible within the next five years. <i>Doyle Knight</i> Rutgers University	 Fluid structure interaction. <i>Mike Chrisfield</i> Imperial College
 Adaptive Finite Element methods for fluid dynamics <i>Anne Burbanck</i> ONERA France	 Many small steps towards more computer problems in optimisation. <i>Tomaz Rodic</i> University of Ljubljana
 In the future, uncertainties for analysis optimization will be very important.	 Multi-disciplinary applications - much more wide spread. No more single topics, its time to look at things as a whole. <i>Paul Dextler</i> BAF System




Simulation of multiphysic & coupled phenomena. Application to health sciences and analysis and design of materials at the molecular level
Roland Glowinski
 University of Houston



Multi-scale modelling and computing. Multi-physics and coupled phenomena. as well as uncertainty.
H. Matthie
 University of Braunschweig



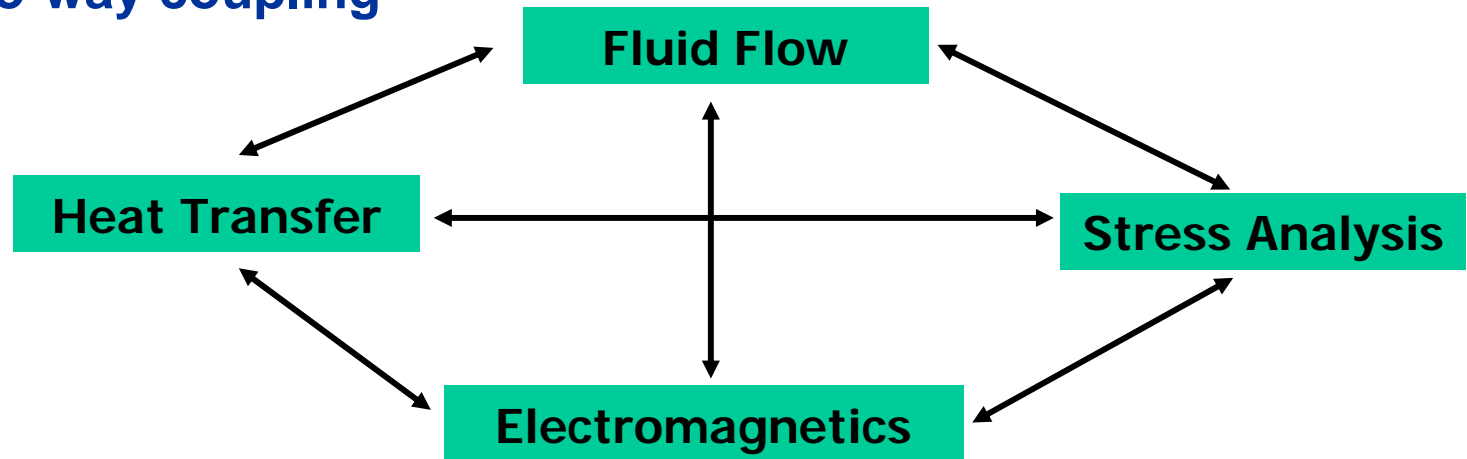
Fluid structure interaction.
Mike Chrisfield
 Imperial College



Multi-disciplinary applications - much more wide spread. No more single topics, its time to look at things as a whole.
Paul Dextler
 BAF System

Why Multi-physics Modelling ?

- Large number of real world problems require multiphysics simulation tools.
- Examples
 - Solidification problems – Solder Joints
 - Fluid-Structure interaction – Flutter in aircraft wings
- Need to solve for integrated physics
- Ensure two-way coupling



Multi-physics Modelling

Current modelling technology mainly focused on distinct physics

- CFD (Fluid Flow)
- CSM (Stress)
- CEM (Electromagnetics)

Multi-physics modelling

- Interface codes together: CFX - ANSYS.
- Single modelling frameworks evolving.

Discretisation + Solution Procedure ?

CAE analysis tools market history

FEA started mid 60's with

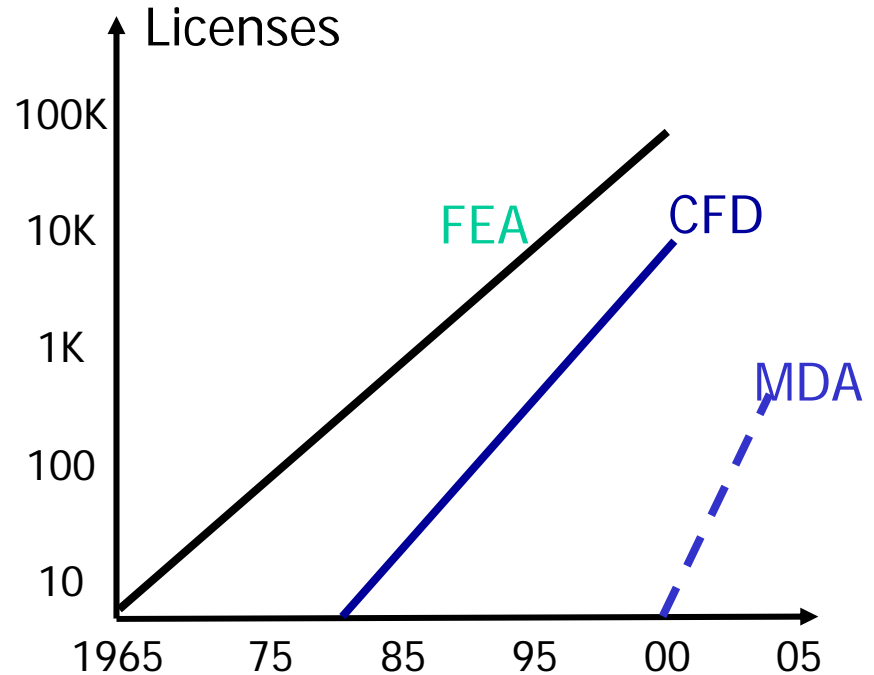
- NASTRAN, Abaqus, ANSYS, etc as major players

CFD started 1980 with

- FLUENT, CFX, PHOENICS & STAR-CD major players

MDA started mid 1990's

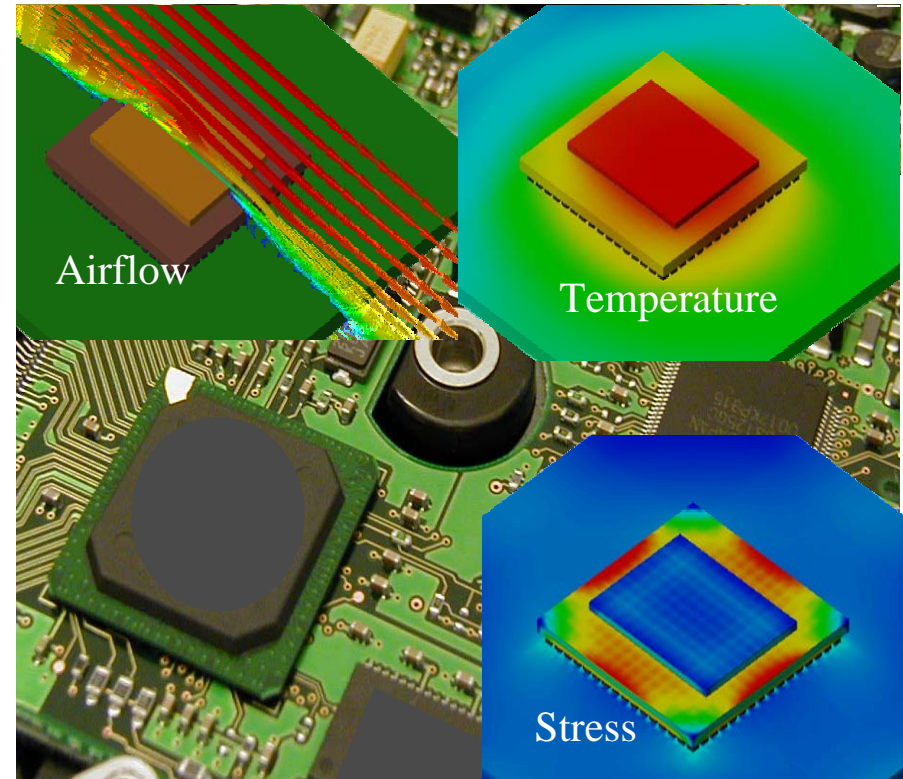
- Coupling codes
- MDICE, mpCCI, Spectrum, PHYSICA



Commercial Software – Multi-physics

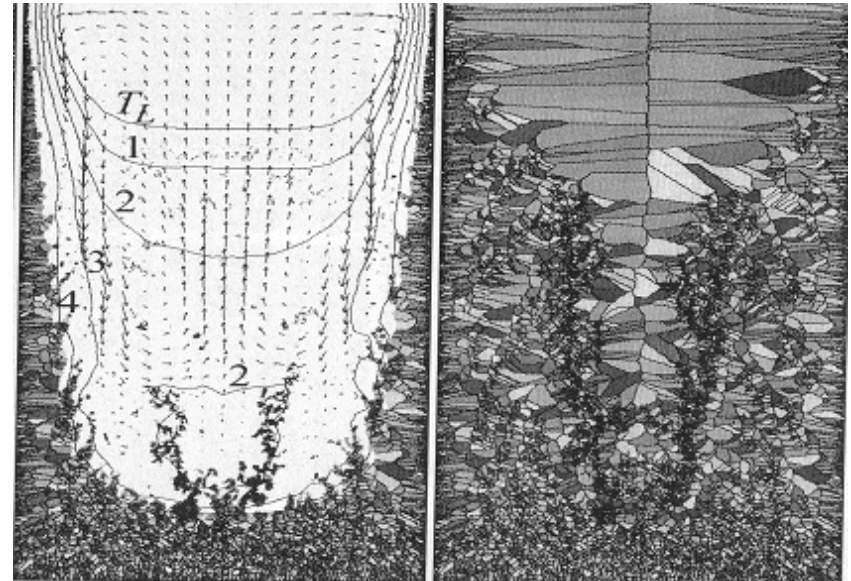
Number of products claiming to be multi-physics:

- **ANSYS/Multi-physics**
 - <http://www.ansys.com/>
- **PHYSICA**
 - <http://www.multi-physics.com/>
- **FEMLAB**
 - <http://www.femlab.com/>
- **ADINA**
 - <http://www.adina.com/>
- **AUTODYN, etc**
 - <http://www.centdyn.com/>
- **RADIOSS**
 - <http://www.radioss.com/>
- **DYTRAN**
 - <http://www.mscsoftware.com/>
- **Algor**
 - <http://www.algor.com/>



Why Multi-scale Modelling?

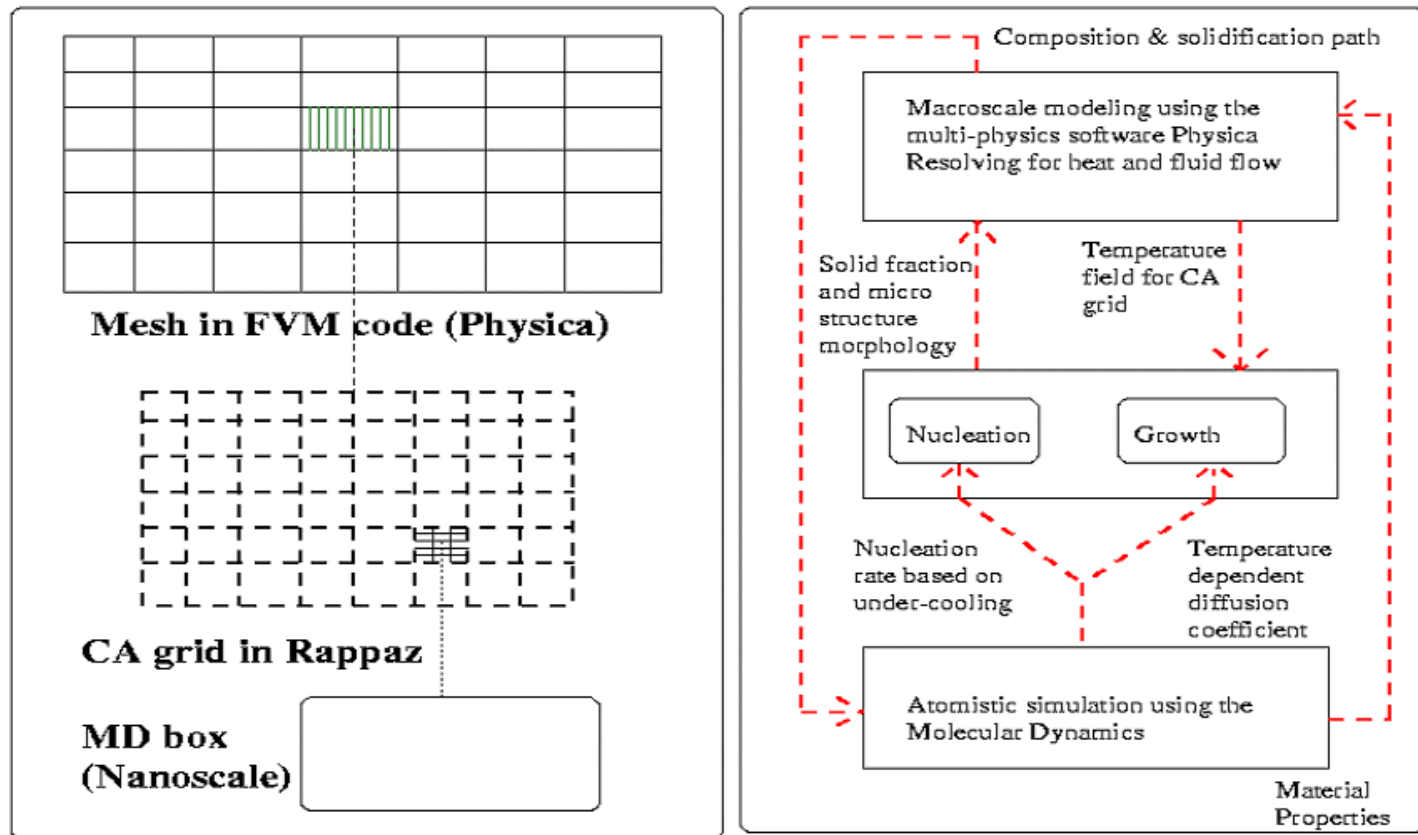
- **Material performance will be governed by its microstructure**
- **Nucleation and grain growth dependent on process conditions (Temperature, etc)**
- **Need to couple Macro – Micro codes**



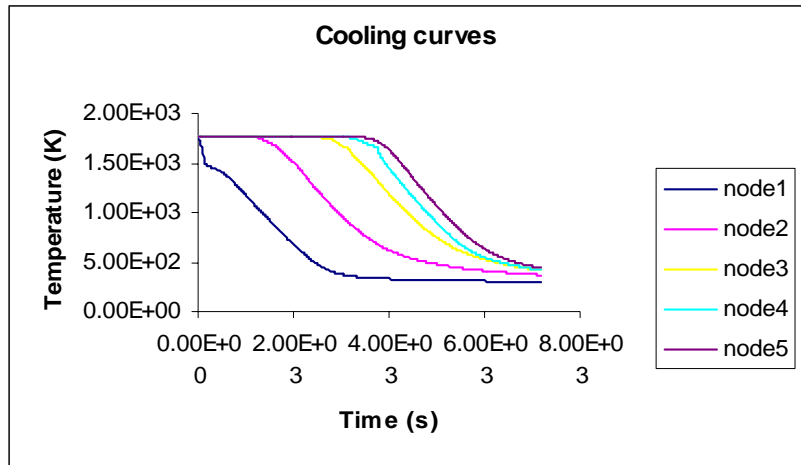
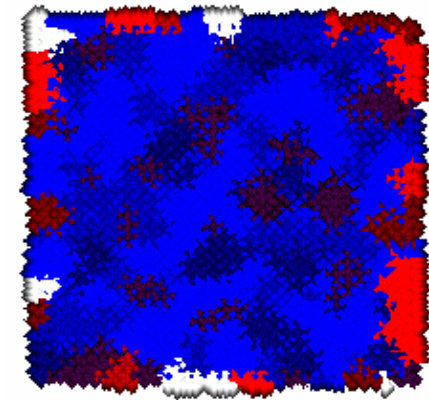
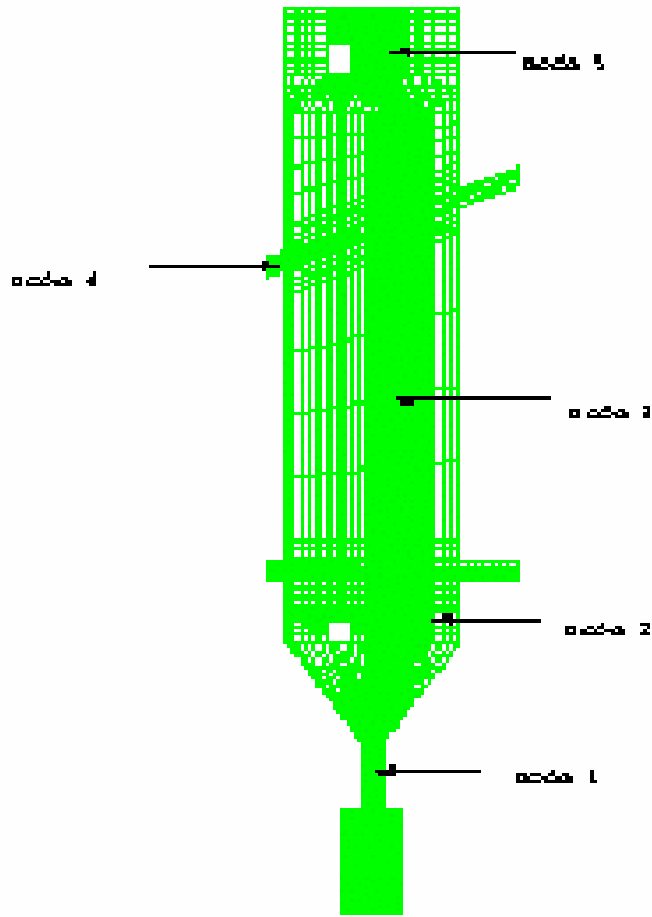
Micro-Macro Simulation

(From Ch.-A Gandin et-al, MCWASP VIII, Pub TMS (1998))

Macro – Structure including Nano Modelling



Macro – Structure including Nano Modelling



Definition of Multi-physics Modelling

- **Ask any software vendor to define what a multi-physics code is able to do:**
- **Ask any software vendor to define what a multi-physics code is able to do:**

And most will describe their code

Multi-physics modelling – a definition

- **Definition of a multi phenomena problem:**
 - *A problem that requires the solution of two or more variables whose evolution is described by different classes of equations.*
- **For the problem to require *multi-physics* modelling there must be two way coupling between the constituent physics.**
- **If there is only one way coupling then the problem is *multi-disciplinary*.**

Solution of Multi-physics Problems

- **Two alternative approaches for a coupled problem, e.g. fluids and structures:**
- **Two codes**
 - e.g. Ansys (CSM) and CFX (CFD)
 - linked through mpCCI

Versus

- **Single code**
 - e.g. PHYSICA

Issues in Multi-physics Modelling

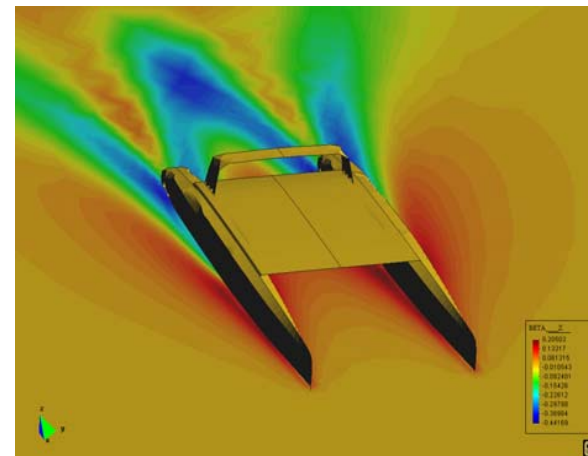
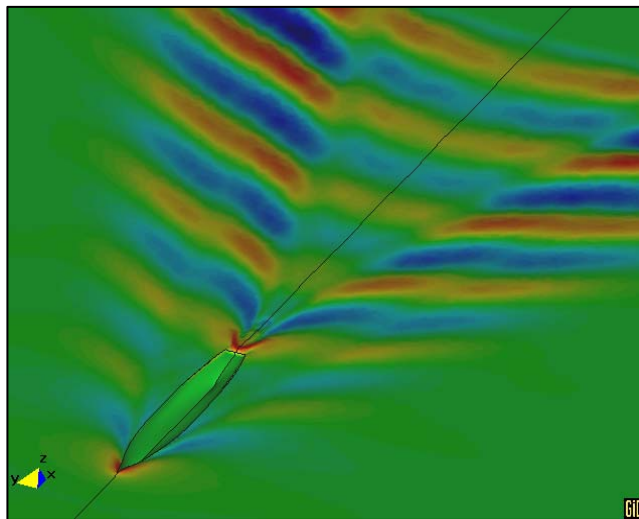
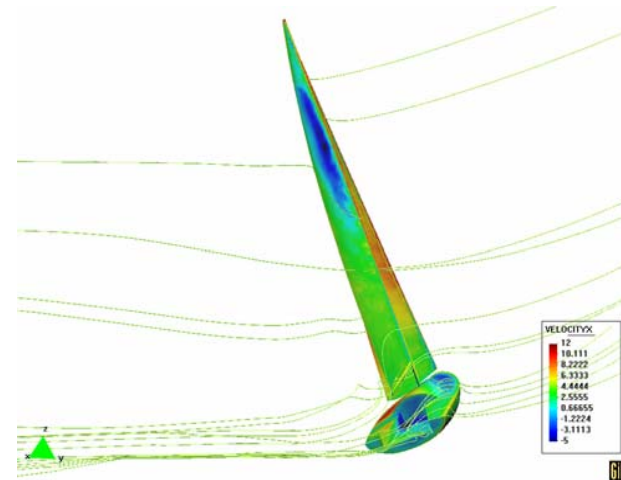
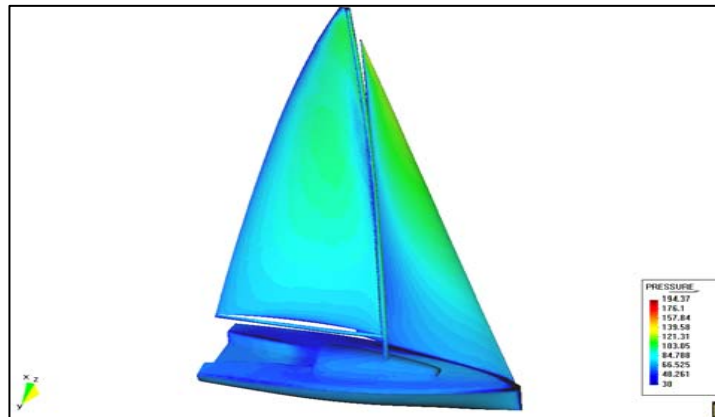
- **Levels of coupling between constituent physics**
 - How do variables influence each other.
 - How often do variables have to be updated
- **Interpolation of values between solvers**
 - Accuracy
 - Consistency

Tacoma Narrows Bridge

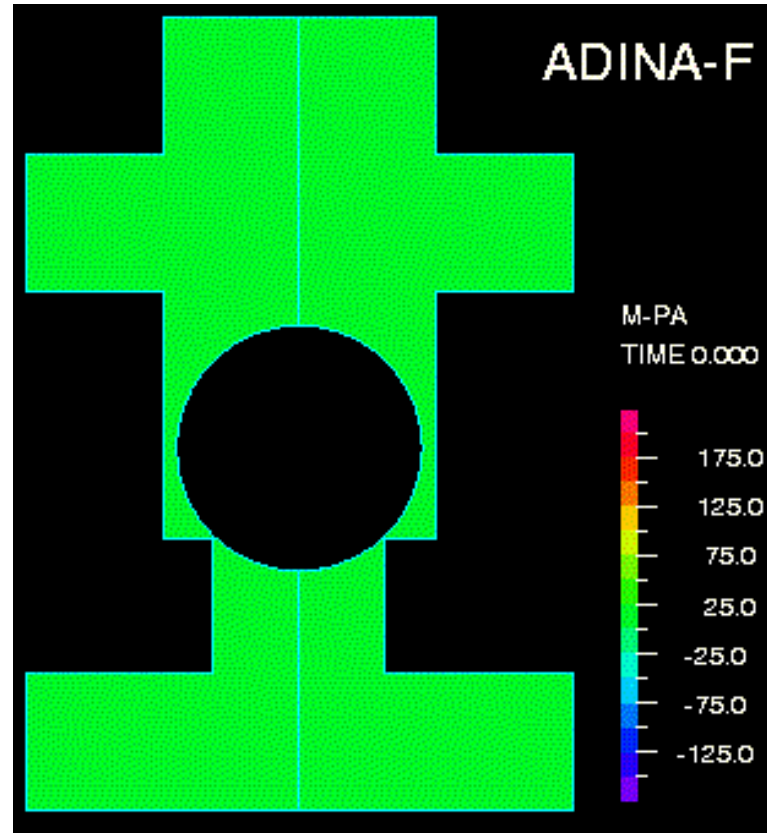
- serious dynamic fluid structure interaction



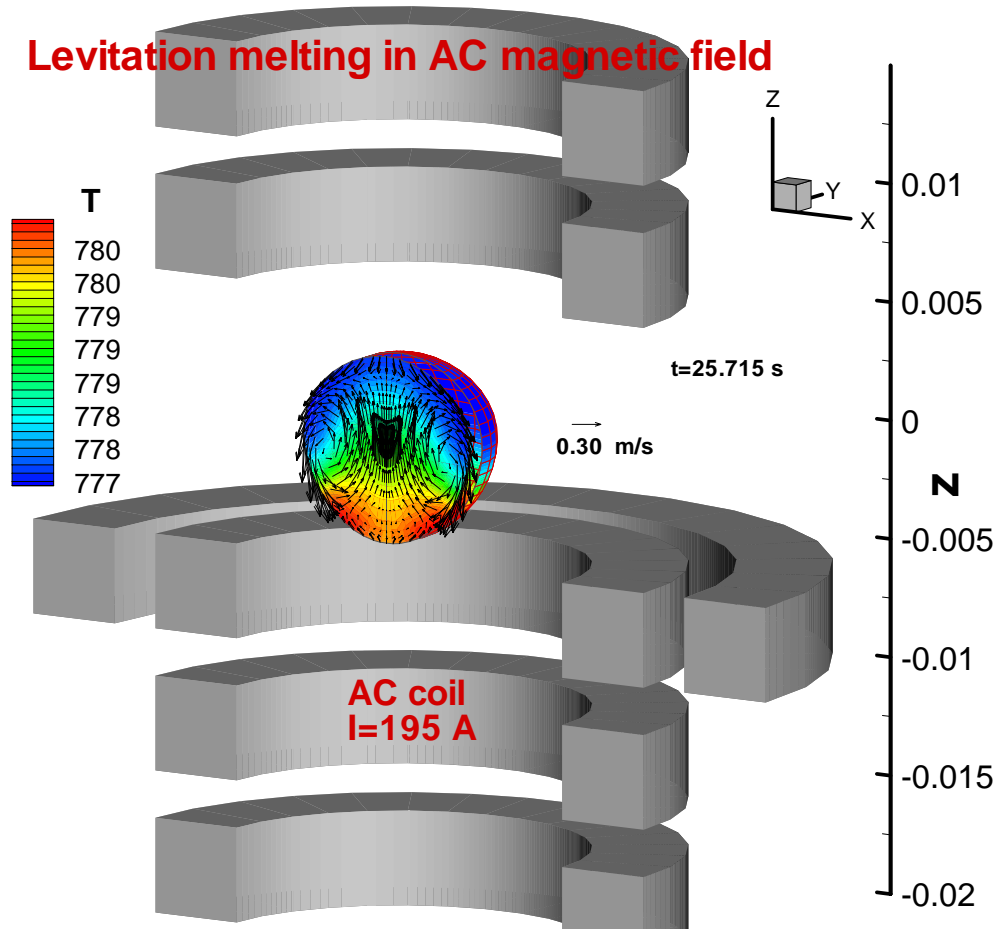
Sail Boat – more DFSI



Anti-Locking Brakes



Magnetic Levitation



Parallel Multi-Physics Modelling



Multi-physics simulations : compute issues

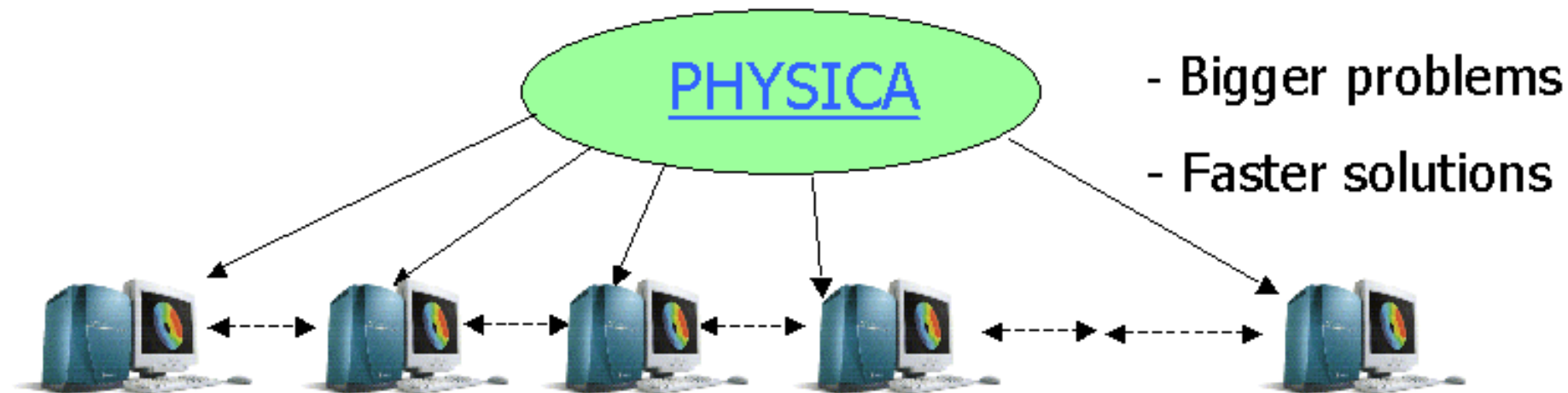
- **Genuine multi-physics has many key interactions**
- **Macro-defects predicted as a function of these interactions**
- **Simulation time on Compaq Alpha can be horrendous- measured in weeks, for a problem with only 75K nodes - not practical!!!**
- **Conclusion - can do multi-physics casting, etc simulation, but compute demand is extensive**

Multi-physics compute demands:

- **Unstructured Mesh analysis = 3* Structured mesh analysis**
- **Performance on a Compaq alpha 466Mhz**
 - Seconds per node or element per time step per problem class
 - Heat Transfer (HT) + Solidification (Sol) = $2 \cdot 10^{-3}$
 - Fluid Flow (FF) + HT + Sol = $6 \cdot 10^{-3}$
 - HT + Sol + Stress = .09/ .55
 - FF + HT + Sol + Stress = .14
- **Casting simulation with 100K nodes, and 100 time steps is in excess of 300hours!**
- **Need simulation times that are 10-20x faster**
- **PARALLEL – WITH CHANGING PHYSICS**

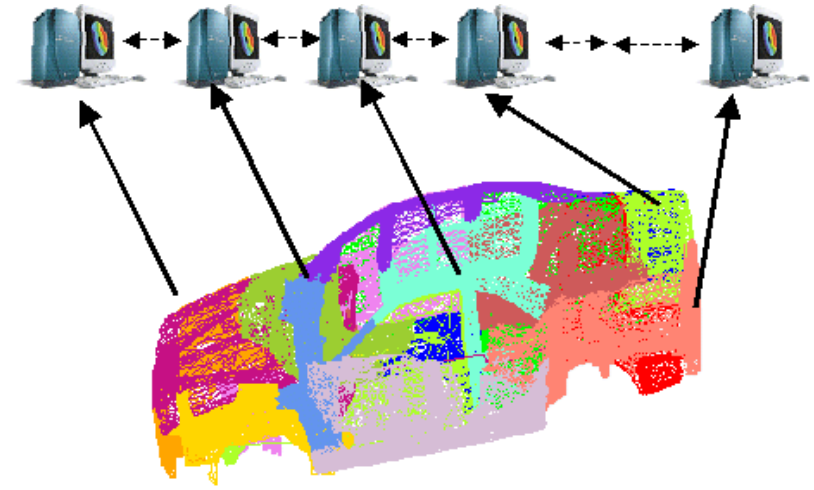
Parallel Strategy

- **Single Program Multiple Data (SPMD)**
- **Program resident on each processor**
- **Mesh Partitioned across processors.**
- **Minimise communication times.**



Mesh Partitioning - JOSTLE

- Partition of 3D unstructured mesh by JOSTLE assuming a non-homogeneous load balance across the mesh:
- SPMD Strategy
 - load balanced (even distribution per processor)
 - attempts to minimise sub-domain interface elements
 - sub-domain connectivity matches processor topology of the parallel system

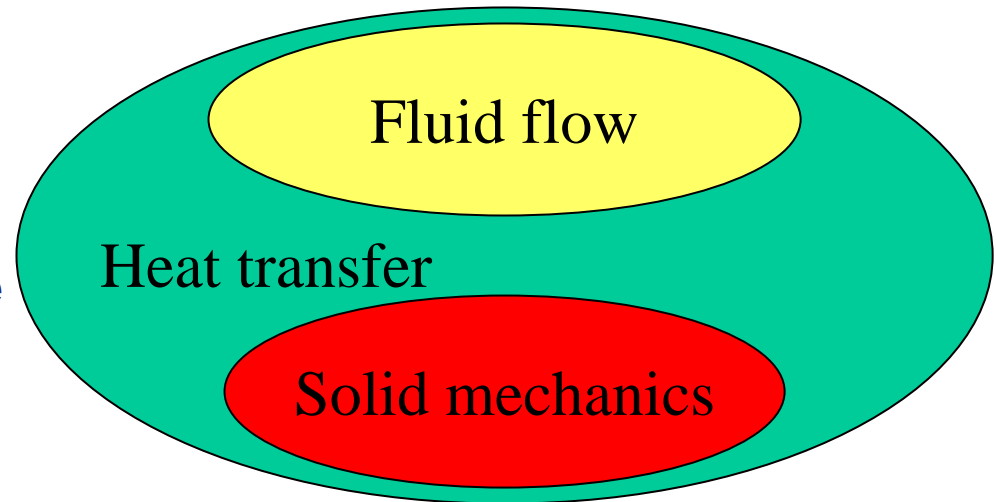


<http://www.gre.ac.uk/~jostle/>

Multi-physics Simulation parallel issues

- **Sub-domains have specific physics so partition must reflect this:**
 - non-uniform load/node
- **Distinct physics uses distinct discretisation procedures:**
 - secondary partitions
- **Also, sub-domains may change as problem develops:**
 - dynamic load balance

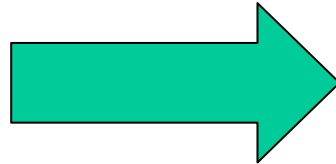
- **Strategy needs to address all the above issues**



Multi-physics Modelling

Physics Requirements

- Fluid Flow
- Solidification
- Stress
- Electromagnetics



MULTI-PHYSICS

Geometry

- Complex



UNSTRUCTURED
MESHING

Large simulations



PARALLEL

Single Software Framework

- **Key route to closely coupled multi-disciplinary (multi-physics) simulation**
- **Basic requirements of a SSF:**
 - consistency of mesh for all phenomena
 - compatibility in the solution approaches to each of the phenomena
 - single database & memory map so that no data transfer & efficient memory use between programs
 - facility to enable accurate exchange of boundary or volume sources (e.g. body force)

PHYSICA – Multi-physics Framework

- **Begun in 1988 at University of Greenwich**
- **Used FV methods on unstructured mesh (FV-UM) using either cell centred or vertex based discretisation approaches**
- **Phenomena addressed:**
 - Fluid Flow – turbulent, free surface, multi-component
 - Electro-magnetics
 - Heat transfer with phase change & chemical reactions
 - Solid mechanics, linear, non-linear and dynamic (also in FE!)
- **Prototypes moved from 2D to 3D and from scalar to parallel**
- **Key issue was to ensure FLOW worked well in all contexts**
- **Solidification processes was originally a key target**

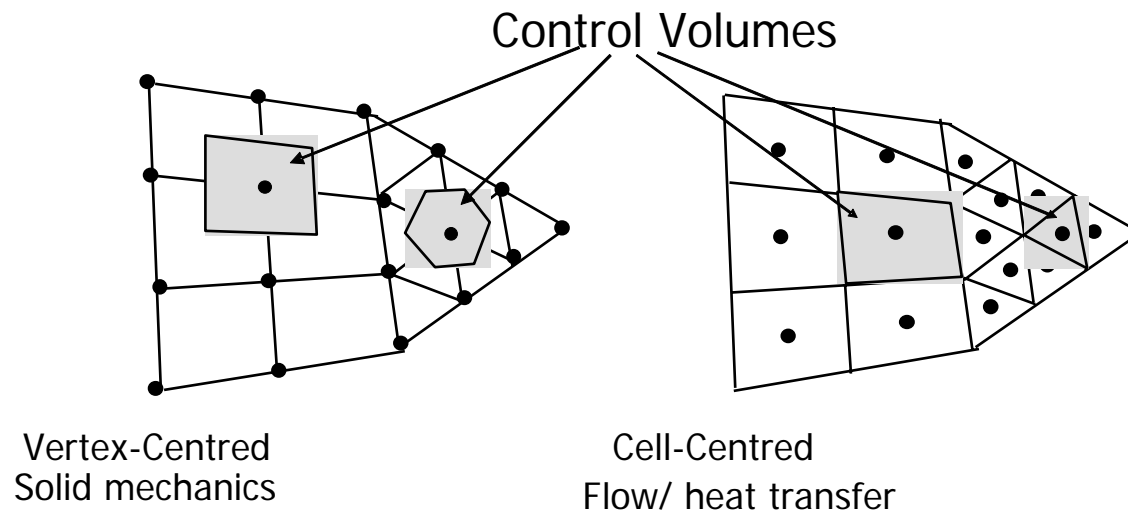
PHYSICA :

Design of a FV-UM multi-physics modelling software tool

- **Framework for the solution of any coupled set of PDEs up to second order**
- **Design concept - object oriented with reusable software modules in FORTRAN77**
- **Multi-level toolkit :**
 - focus at high level for model implementation
 - maximise control over numerical issues
 - essentially **open source**
- **Conceived and implemented in parallel**

PHYSICA – a SSF for multi-physics simulation

- Uses finite volume or element procedures on unstructured mesh



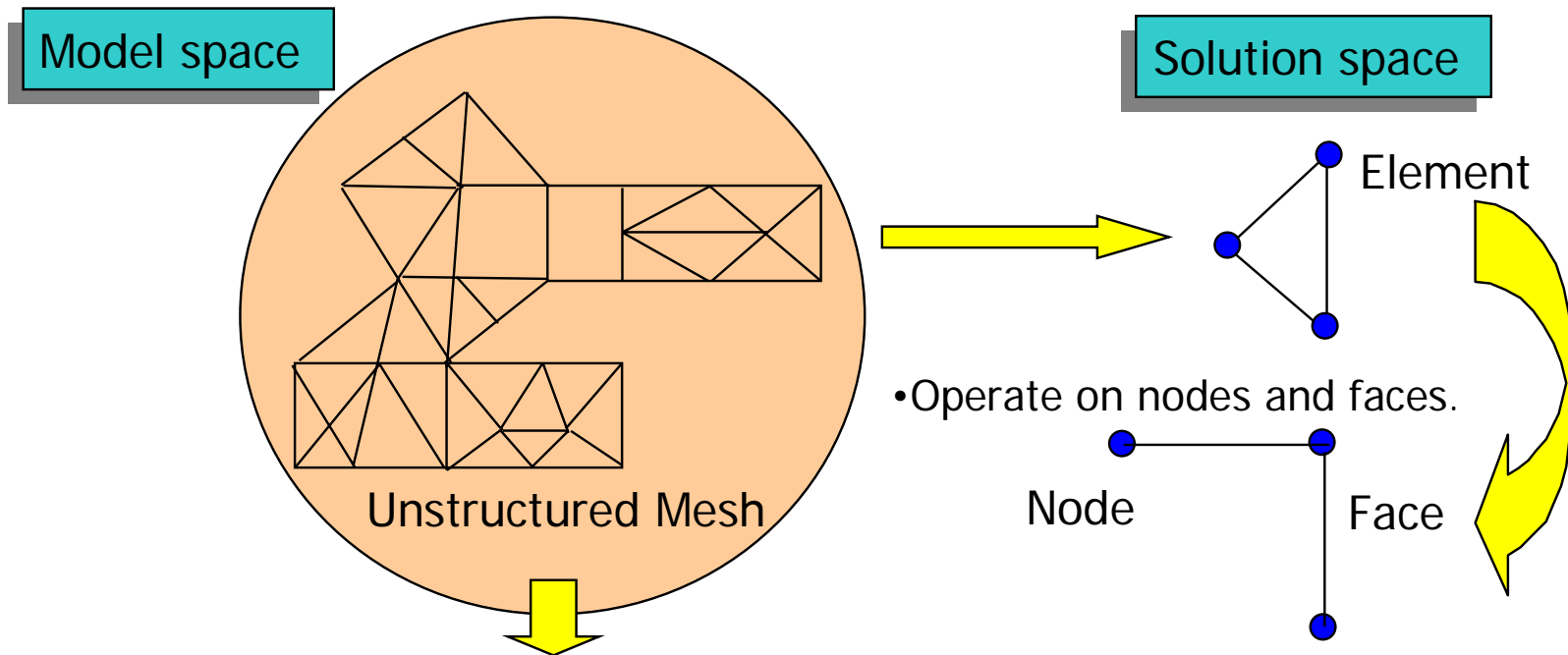
Implemented in 3D in PHYSICA

PhD's on FV-UM techniques and tools in PHYSICA

- **Chow (93)** FV-UM CFD procedures in 2D
- **Fryer (93)** FV-UM CSM procedures in 2D
- **Chan (94)** FV SEA procedure in 3D - free surfaces
- **Hughes (94)** FV-UM for MHD in 2/3D using PHOENICS
- **Taylor (96)** FV-UM material nonlinear CSM procedures in 3D
- **McManus (96)** Parallel multi-physics algorithms in 2D & 3D
- **Croft (98)** FV-UM CFD procedures in 3D turbulence/particle tracking/reactions
- **Wheeler (00)** FV-UM 3D free surfaces with surface tension and using level set
- **Slone(00)** FV-UM dynamic fluid-structure interaction in 3D
- **Chirazi (00)** FV-UM multi-scale ala Rappaz
- **Fallah (01)** FV-UM CSM - large strain elasticity & cell centred approximations
- **Edussriya (03)** FV-UM non-Newtonian free surface 3D fluids
- **McBride (03)** FV-UM Vertex based and hybrid CFD procedures in 3D
- **Stoyanov (04)** FV-UM multi-physics with optimisation

Object based design

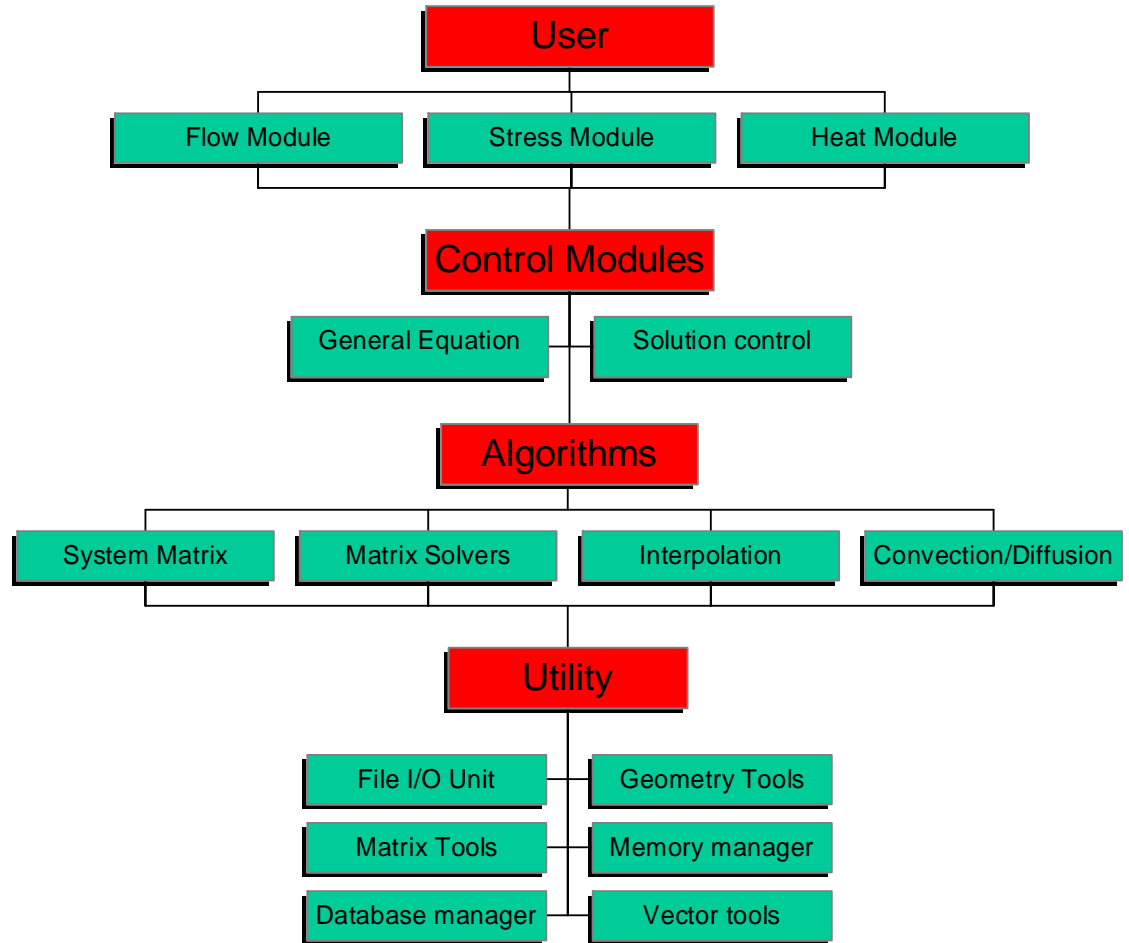
- Algorithms are built on methods for “Objects”.



- Traditional approach is to use routines that perform a large number of operations on an element.

Levels of code abstraction

User can access user, control and algorithmic levels.

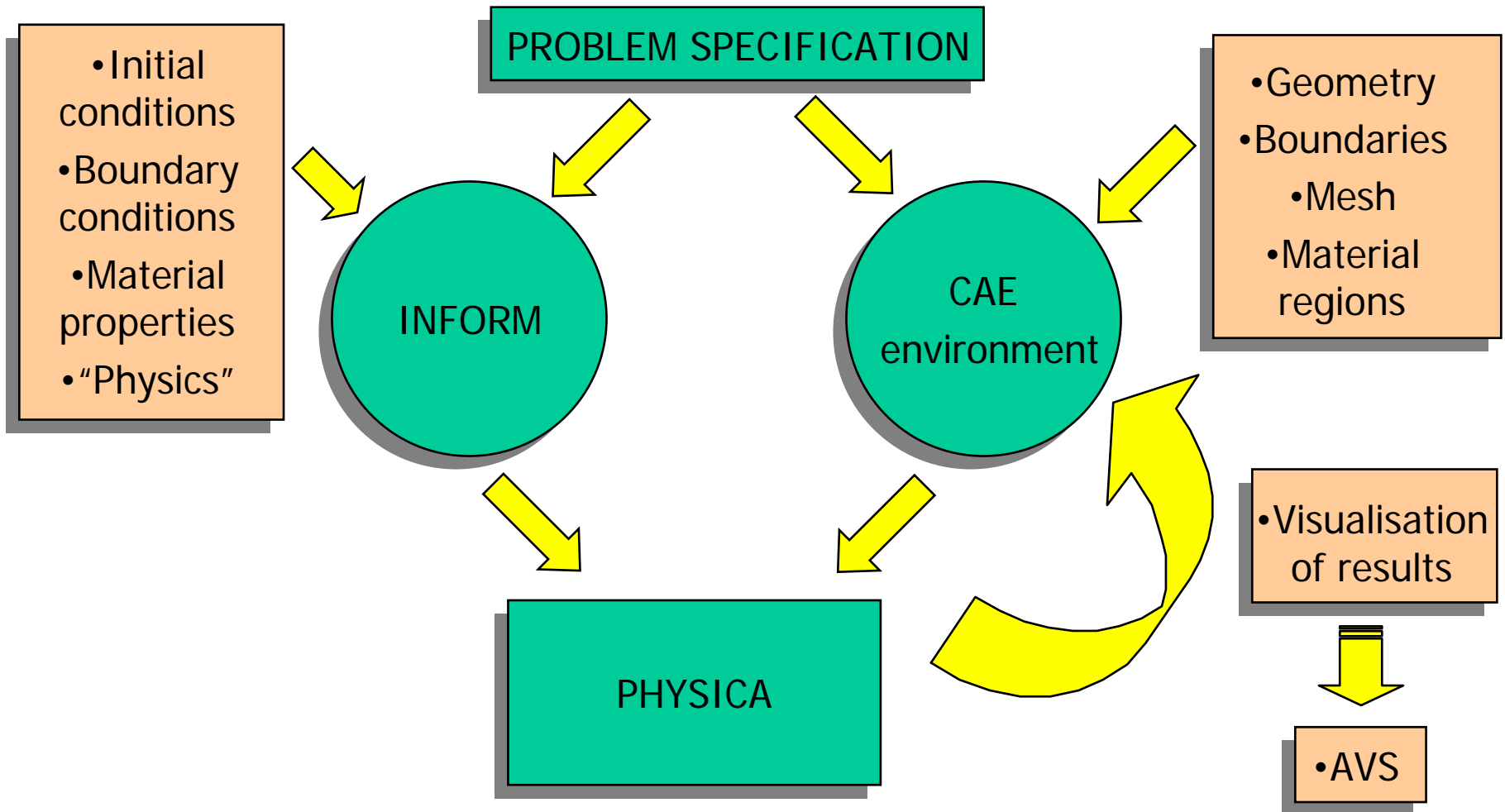


Governing Equations

- The governing equations can be expressed in a standard form:

$$\frac{\partial}{\partial t}(\rho A \phi) + \nabla \cdot \underline{Q} = \nabla \cdot (\Gamma \nabla \phi) + S$$

	ϕ	A	Γ_{ϕ}	S	Q
<i>Continuity</i>	1	1	0	S_{mass}	$\rho \underline{v}$
<i>Momentum</i>	\underline{v}	1	Γ_v	$(S + \underline{J} \times \underline{B} - \nabla P)$	$\rho \underline{v} \cdot \underline{v}$
<i>Heat transfer</i>	h	1	k/c	S_h	$\rho \underline{v} h$
<i>Electromagnetism</i>	\underline{B}	1	η	$(\underline{B} \nabla) \underline{v}$	$\underline{u} \cdot \underline{B}$
<i>Solid Mechanics</i>	\underline{u}	$\partial / \partial t$	μ	$\rho \underline{f}_b$	$\mu(\text{grad } \underline{u})^T + \lambda(\text{div } \underline{u} - (2\mu + 3\lambda)\alpha T)\underline{I}$



PHYSICA+

- **PHYSICA+ = PHYSICA + Femgv**
- **Femgv is a pre and post processor**
- **PHYSICA is a modelling framework that can solve CFD, CSM and coupled problems.**
- **PHYSICA developed at Greenwich specifically to solve classes of multi-physics problems.**

Course Presenters

Dr Avril Slone

- BSc (Maths) PhD (Multi-Physics)
- Senior Research Fellow, University of Greenwich
- Core development team for PHYSICA
- In another life mother of two grown up children & a Chartered Statistician.

Prof Mark Cross

- BSc (Maths) PhD (Math Physics) DSc (Comp Engg)
- Professor and Director of Centre for Numerical Modelling and Process Analysis, University of Greenwich
- Worked on Multi-Physics since inception of subject
- In another life father of 3 grown up kids and Pro Vice Chancellor for Research at Greenwich