Hypothesis Testing of Finite Element Models Using Load Uncertainty Probability Density Functions

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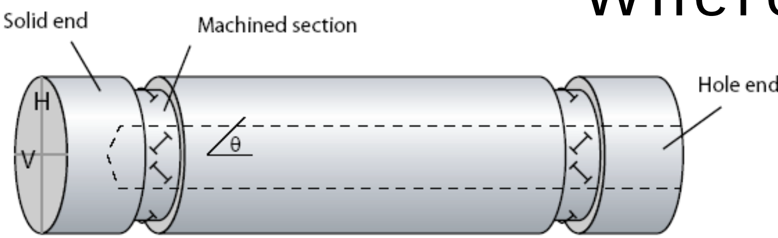
Project Motivation

- Good correlation between structural life predictions made from measured test data and finite element models requires confidence in the measured test loads and the finite element models.
- In the past twenty years, the size of FEM and the speed with which we can run them has increased over 100X! What kind of improvements have we made to the quality of the inputs to those models in that time?
- When FEA doesn't correlate to test data, it is sometimes difficult to tell whether model errors or load uncertainty is causing the correlation problem.
- Let's see if we can apply principles of hypothesis testing to FEM validation.
  - Express external load histories as probability density functions (PDF).
  - Propagate load uncertainty to calculated strain PDF.
  - Does the measured strain believably reside within that distribution?

Hypothesis Testing of Finite Element Models Using Load Uncertainty Probability Density Functions

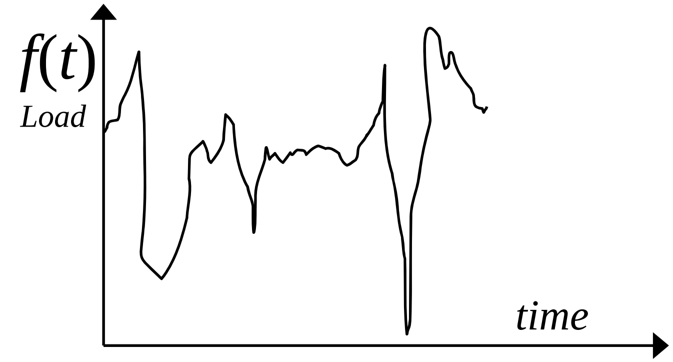


Instrumented Pin

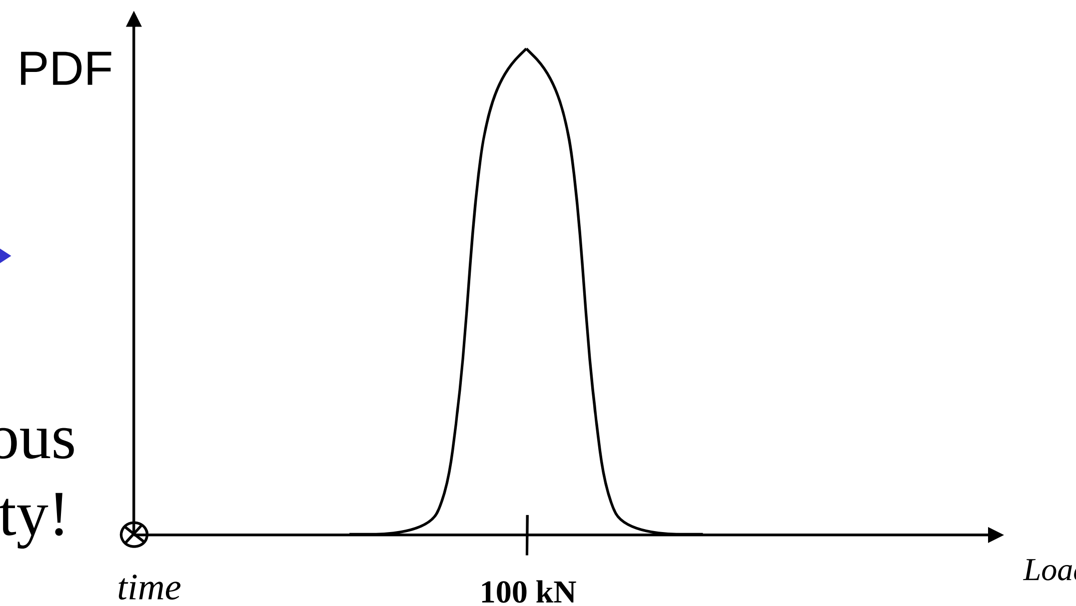
Where we are:

f=**A***s*

calibration



Where we want to be:



PDF addresses instantaneous **uncertainty**, not variability!

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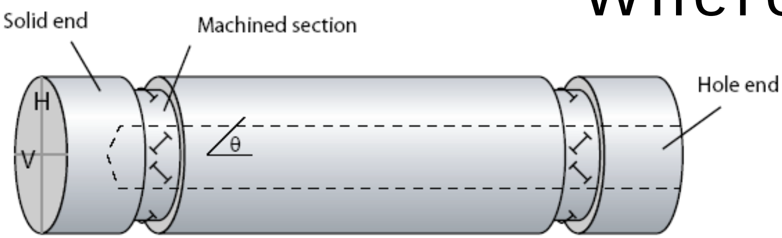

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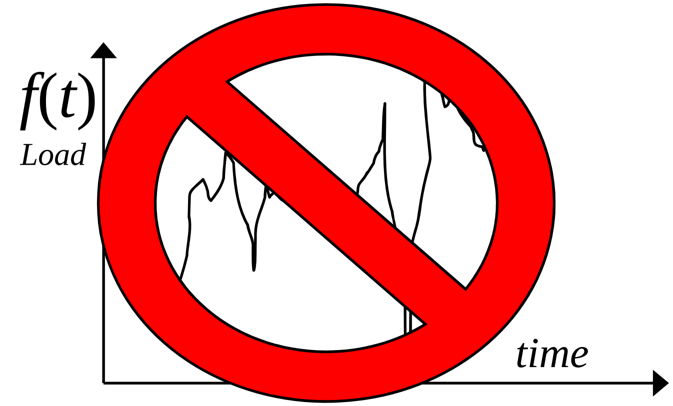

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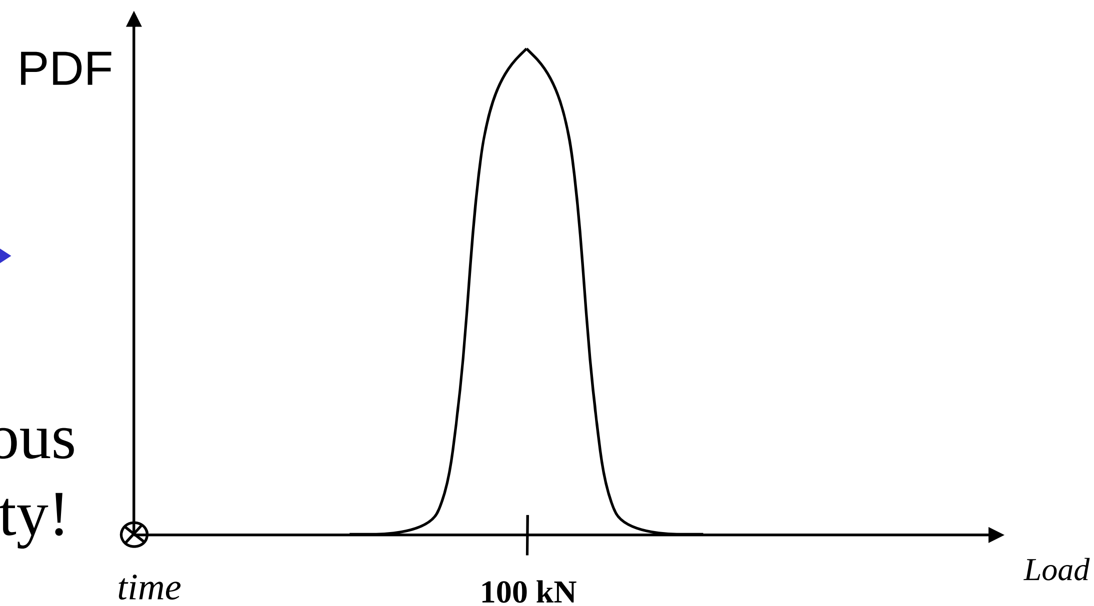

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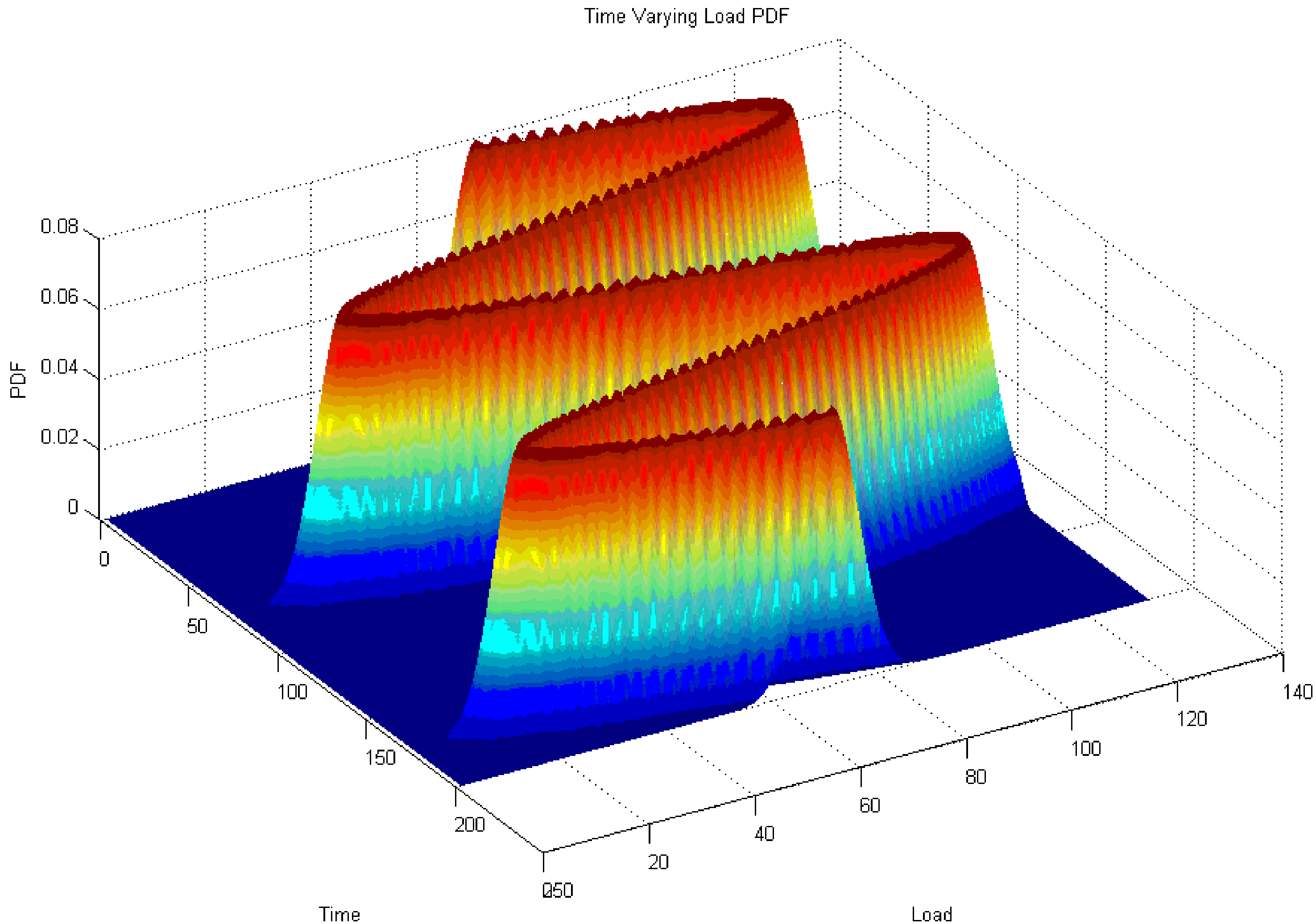

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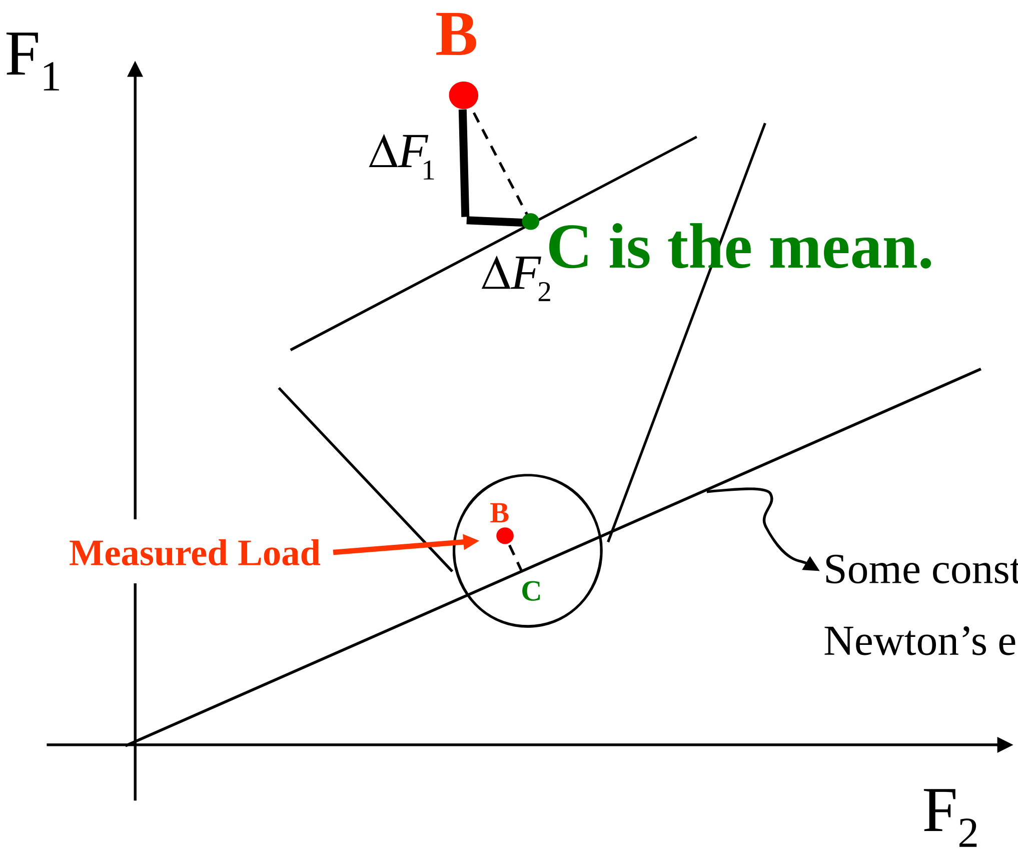
Basic Concepts Utilized in Creation of Load PDF

- Newtonian Constraint Surface Projection Method
- Linear Transformations of Continuous Random Variables with Normal Distributions

Newtonian Constraint Surface Projection Method



$$\Delta F_1 = F_{1_{measured}} - F_{1_{mean}}$$

$$\Delta F_2 = F_{2_{measured}} - F_{2_{mean}}$$

$$\sigma_i = \sqrt{\frac{1}{T}\int_0^T (\Delta F_i)^2 dt}$$

Some constraint surface in load space derived from Newton's equations.

Linear Transformations of Continuous Random Variables with Normal Distributions

If

$$X = N(\mu_x, \sigma_x^2)$$
$$Y = N(\mu_y, \sigma_y^2)$$

then $$W = aX + bY + c = N(a\mu_x + b\mu_y + c, a^2\sigma_x^2 + b^2\sigma_y^2)$$

X and *Y* must be *independent* random variables with normal distributions.

How do we get our Newtonian constraint equations?

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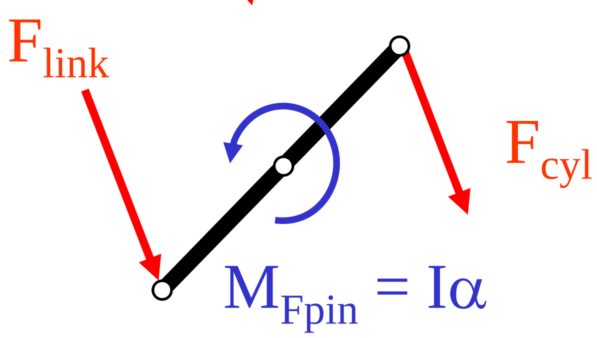
Example: Tilt Cylinder and Link

- Cylinder force measured (pressures & axial strain)
- Link force measured (axial strain)
- We know that the moment about the F-pin must obey Newton's rotational inertia equation.



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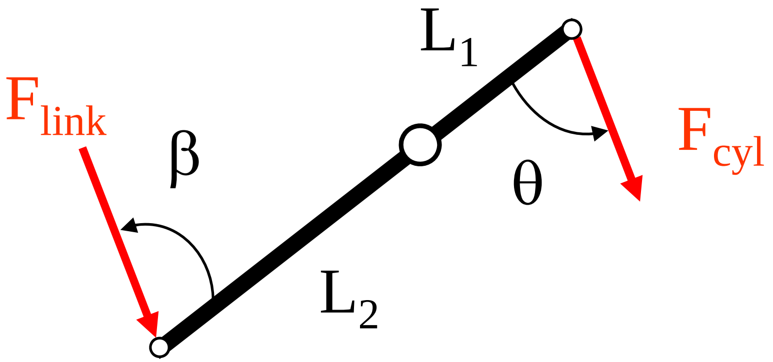
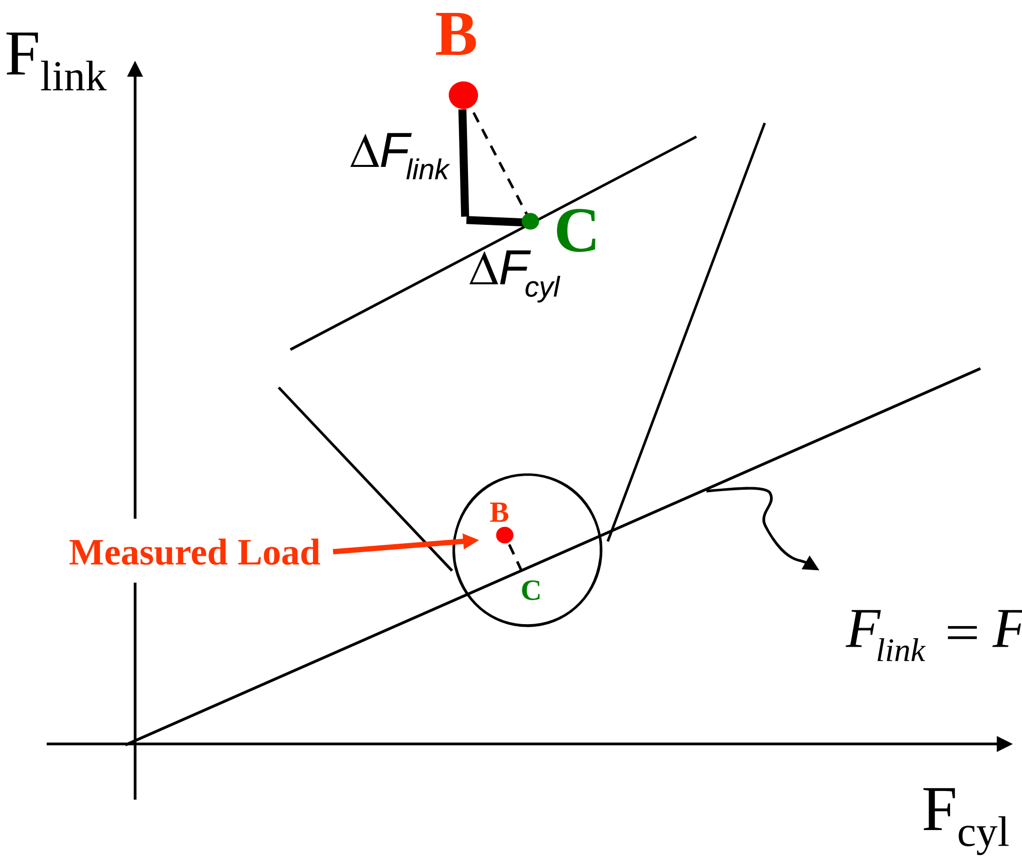


$$\Sigma M = I\alpha = -F_{cyl} \times L_1 \times \sin(\theta) + F_{link} \times L_2 \times \sin(\beta)$$

Constraint Equation:

$$F_{link} = F_{cyl} \frac{L_1 \sin(\theta)}{L_2 \sin(\beta)} + \frac{I\alpha}{L_2 \sin(\beta)}$$

How can we get the parameters of the PDF?



$$\Delta F_{link} = F_{link\ measured} - F_{link\ mean}$$

$$\Delta F_{cyl} = F_{cyl\ measured} - F_{cyl\ mean}$$

$$\sigma_i = \sqrt{\frac{1}{T}\int_0^T (\Delta F_i)^2 dt}$$

$$F_{link} = F_{cyl}\frac{L_1 \sin(\theta)}{L_2 \sin(\beta)} + \frac{I\alpha}{L_2 \sin(\beta)}$$

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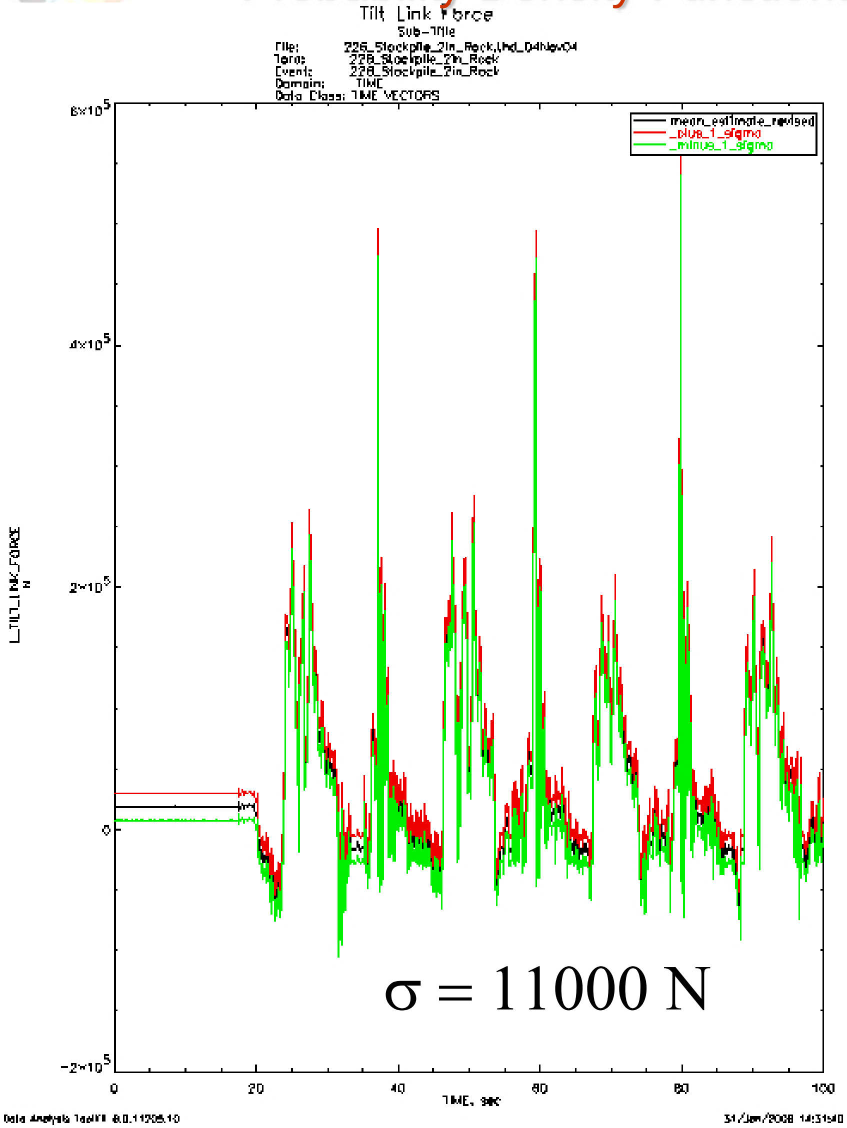

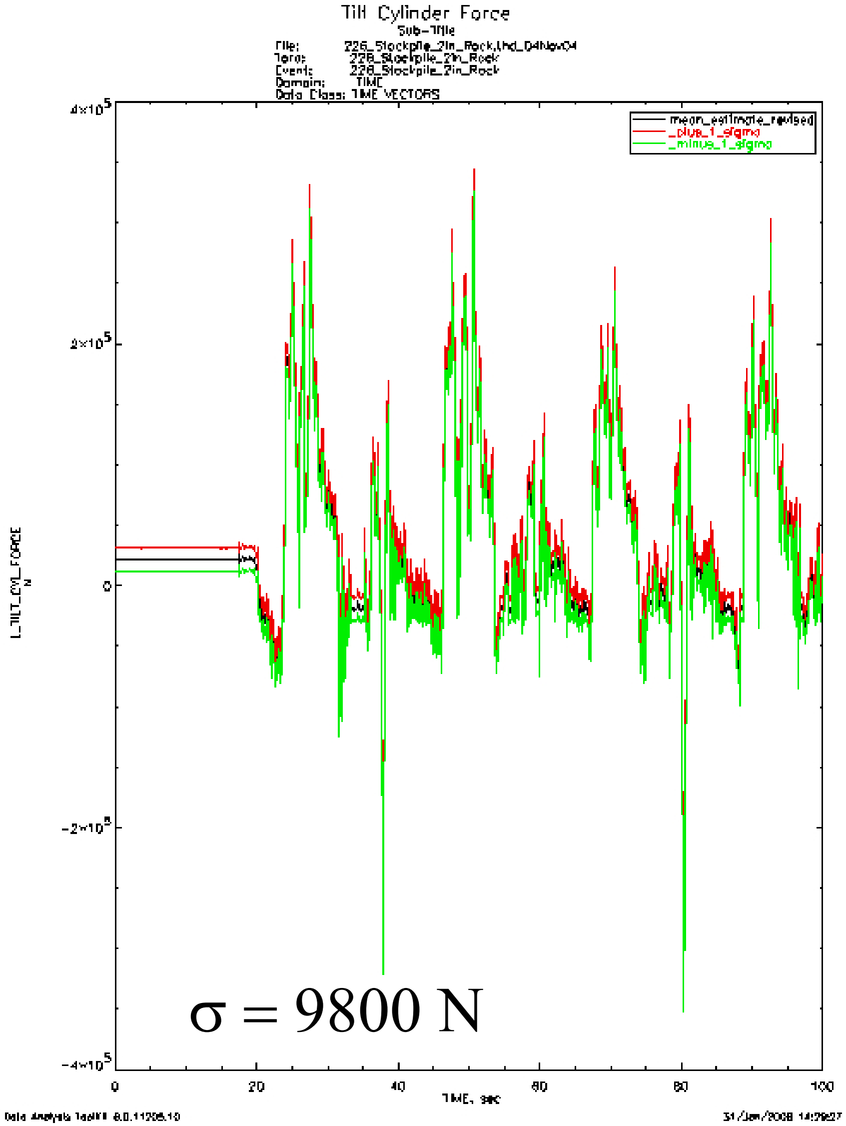

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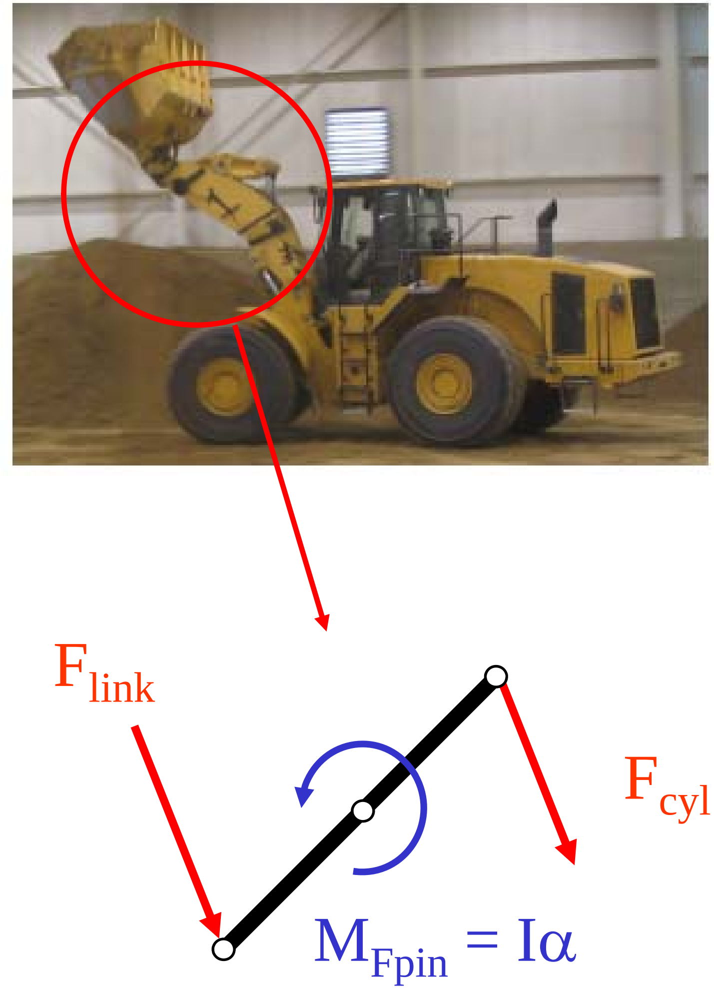
Hypothesis Testing of Finite Element Models Using Load Uncertainty Probability Density Functions

Now use the remaining in-plane Newtonian equations to calculate the F-pin loads (and the dump stop loads) as continuous random variables using the linear transformation laws for normal distributions. Continue with same concepts and work through the linkage.



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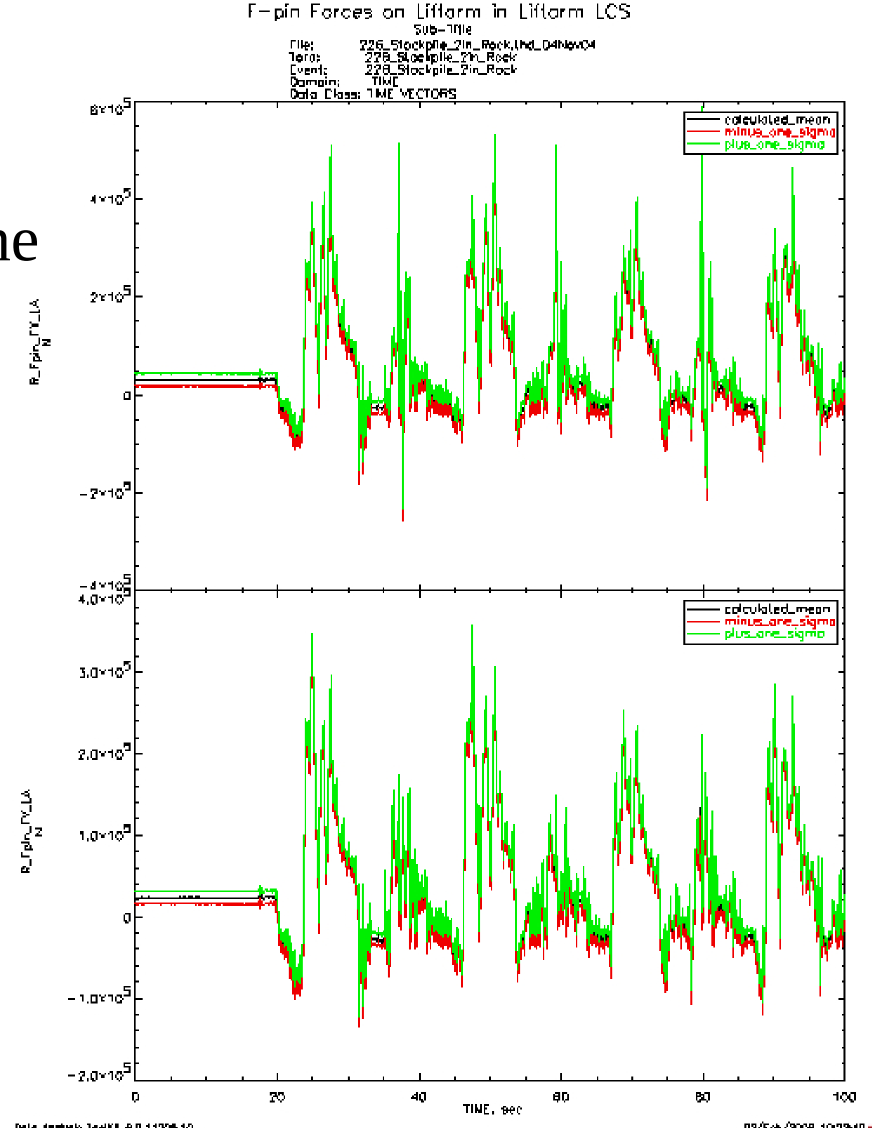

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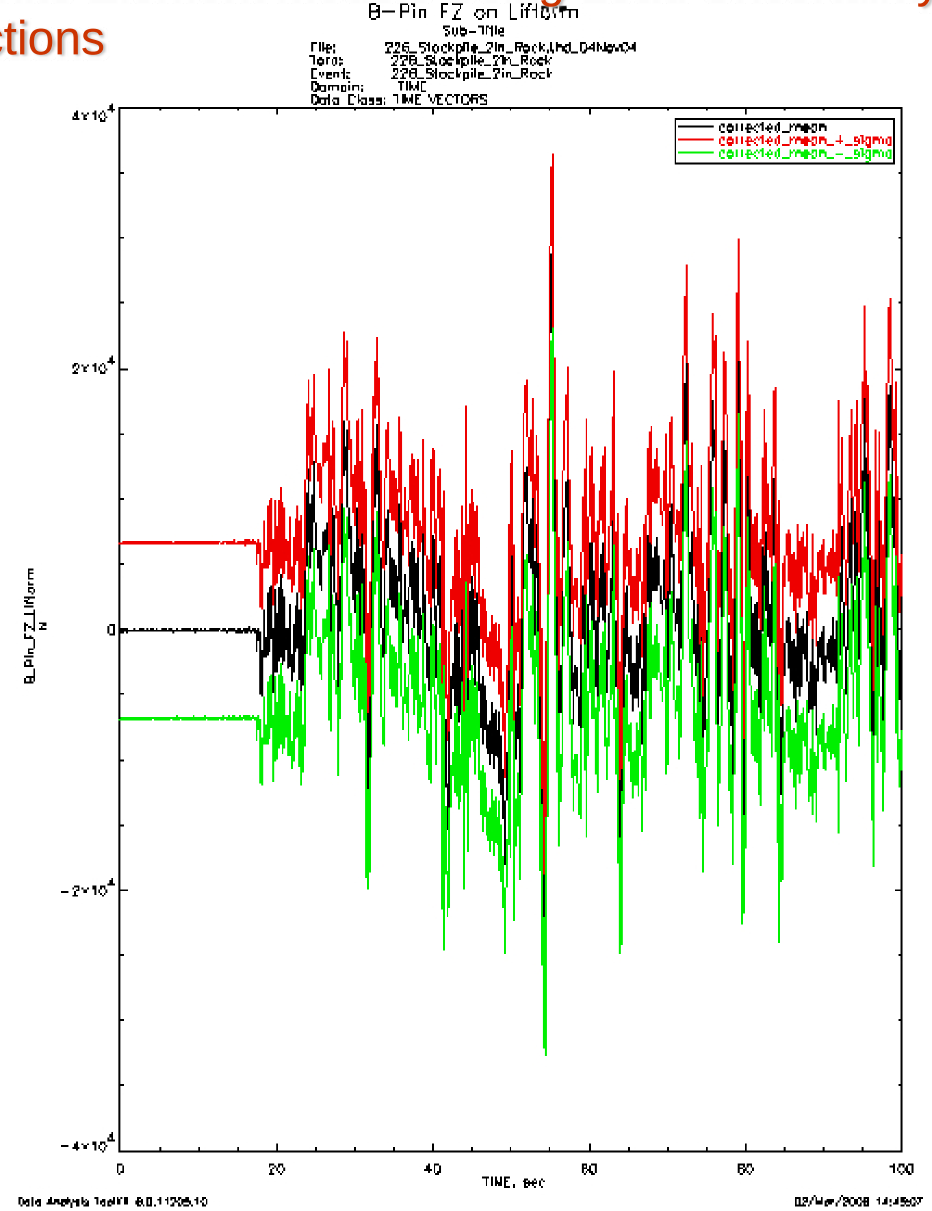

As we work through the linkage, we find that not all loads are known as accurately as some others.

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Now that we know how to get the loads in PDF format, what do we do with that?

s = Af (This is what a finite element model does.)

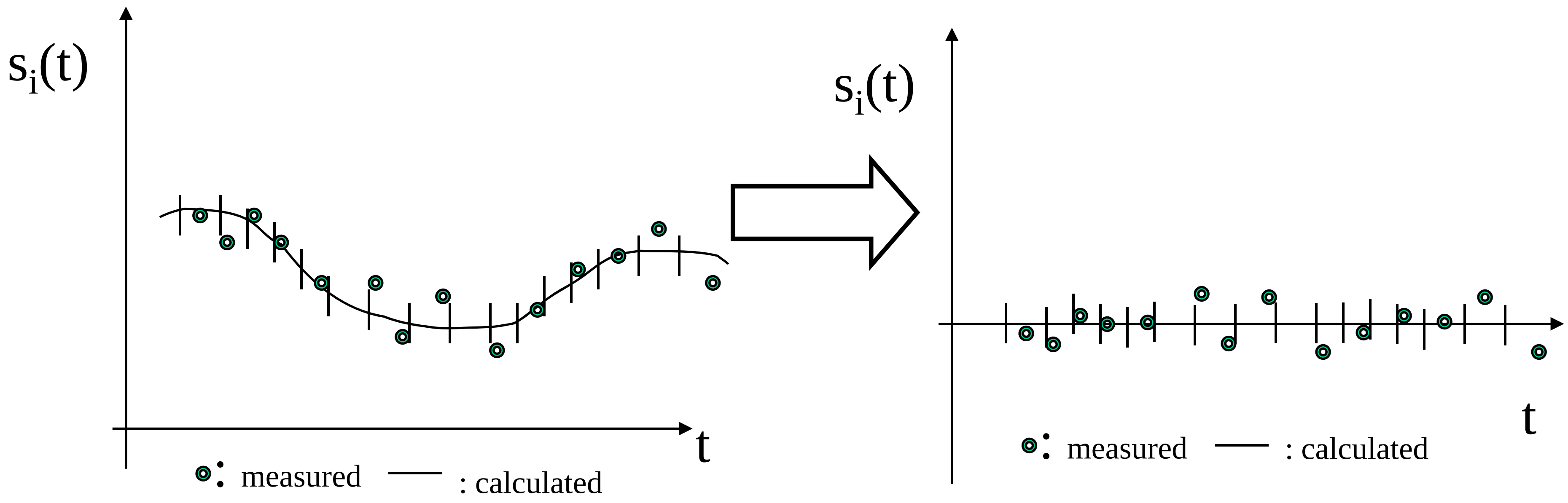
s & **f** are time-varying continuous random variables with (μ,σ^2)

Parameters of **s** come from rules of linear transformations on continuous random variables.

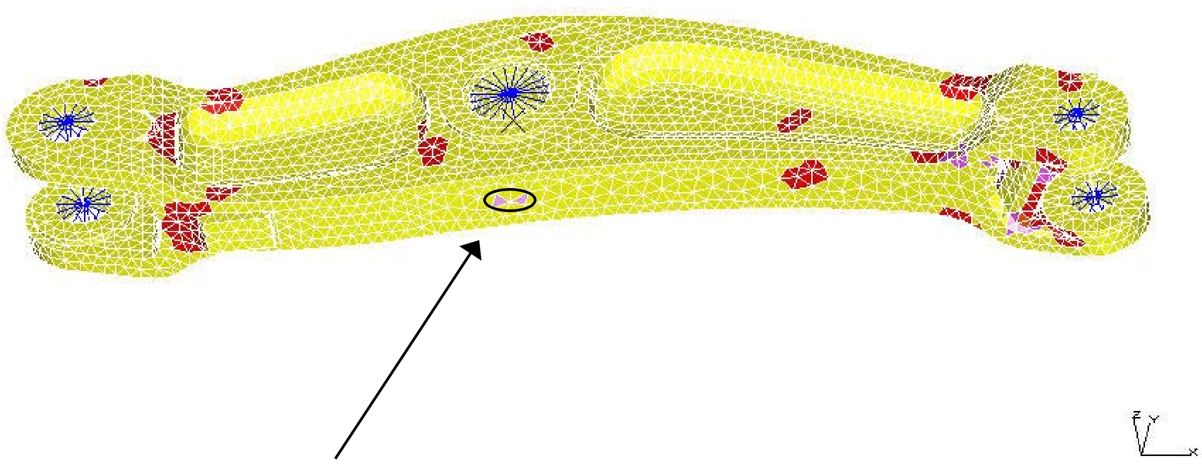
Null Hypothesis H_o: The **A** matrix is fine. The measured strains are simply random drawings from the distributions associated with calculated **s**.

How do we formulate the mathematical procedure to test the null hypothesis?

Specific procedure:
1) Subtract $\mu(t)$ from the hypothesized distribution and each sample point.
2) Also divide both by σ.



First Hypothesis Testing Example: Wheel Loader Tiltlever



Strain gage located here. Oriented pin-to-pin.

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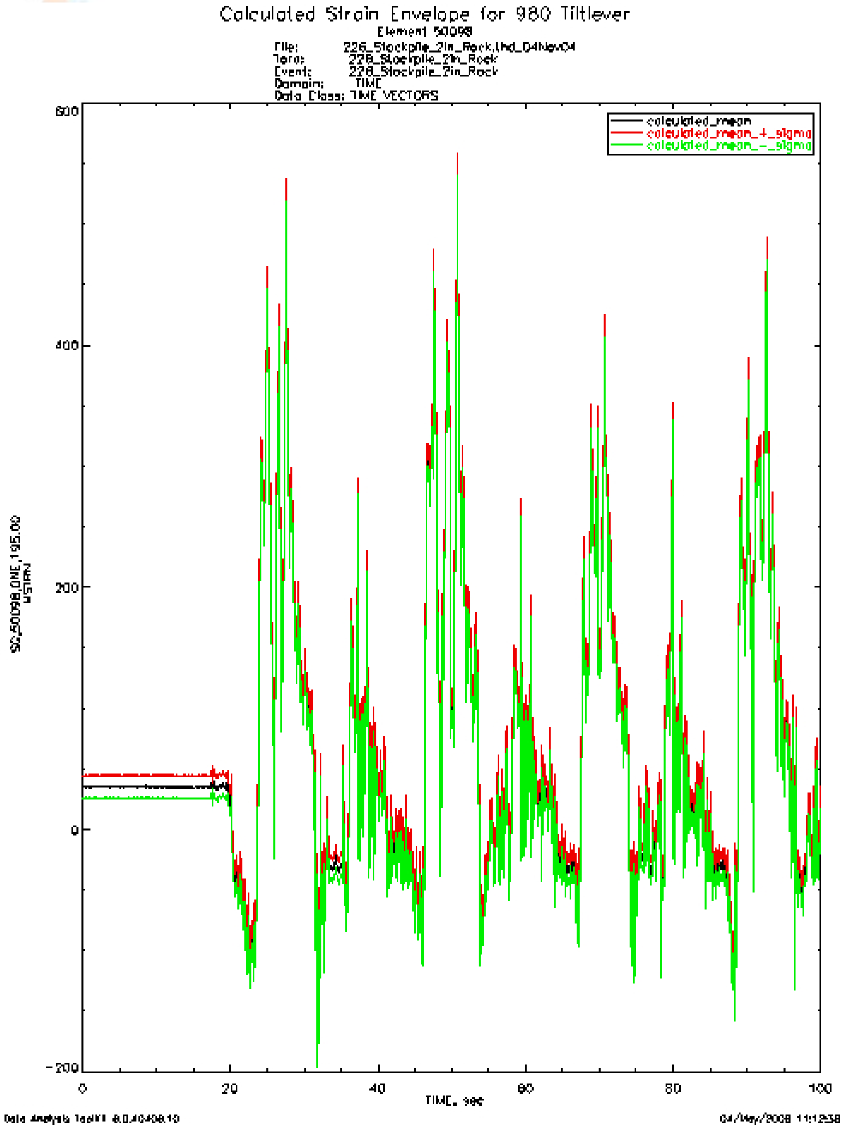

First, we must create the calculated strain distribution from **s=Af** and the transformation laws.

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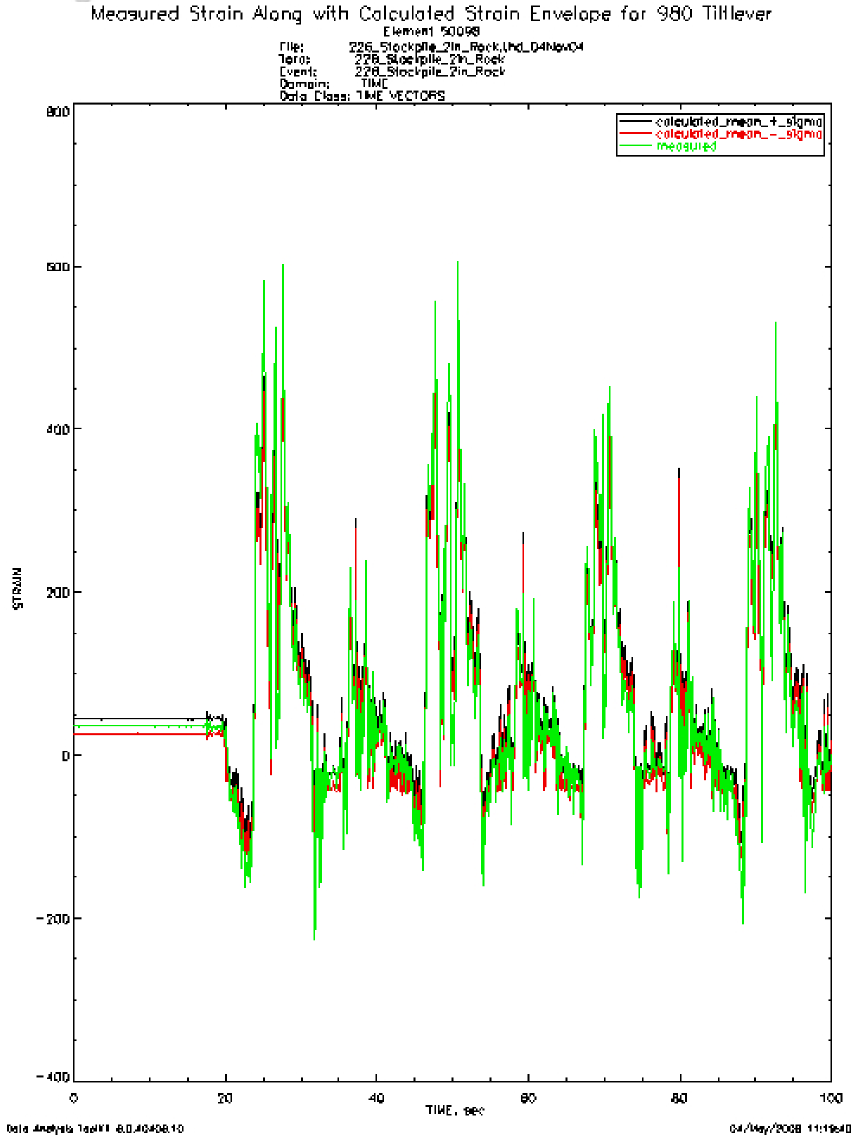

Here is the measured strain superimposed on that same envelope.

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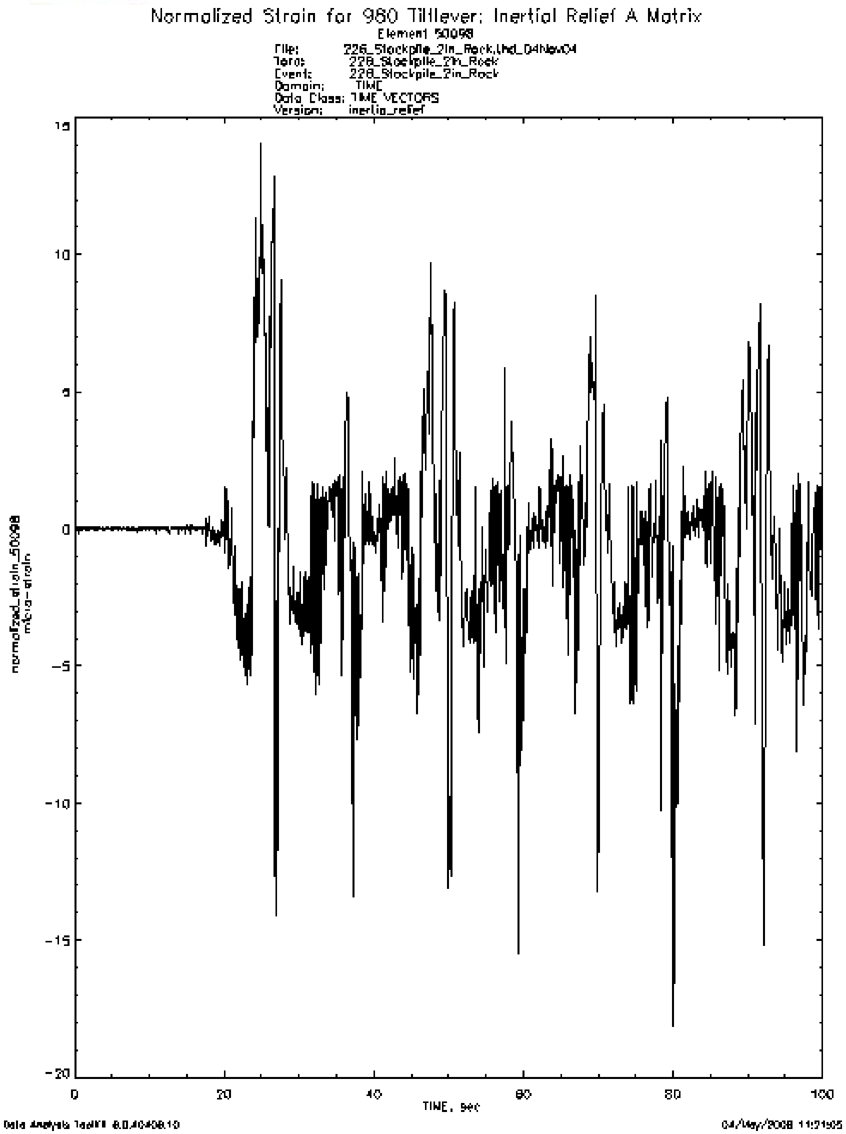


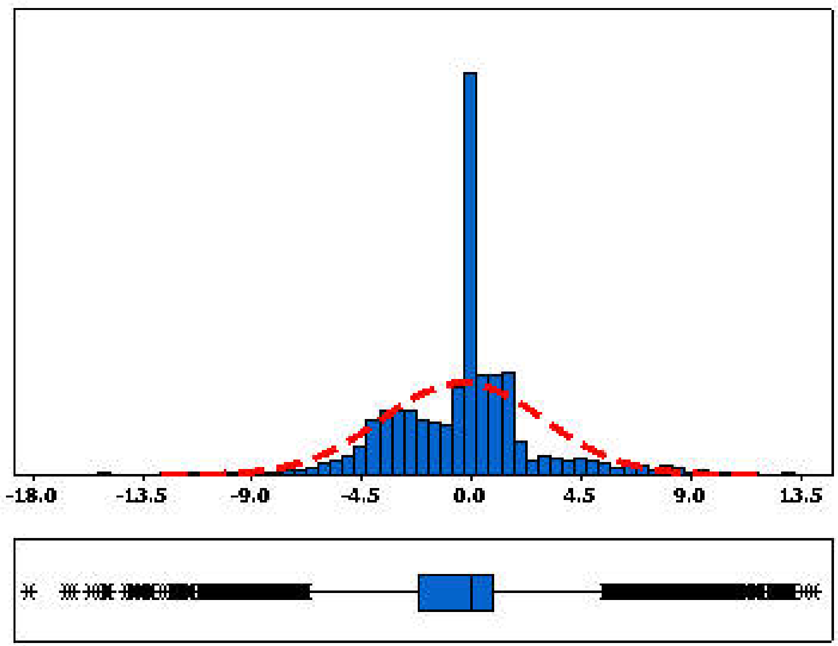
Now we have to normalize the measured strain.

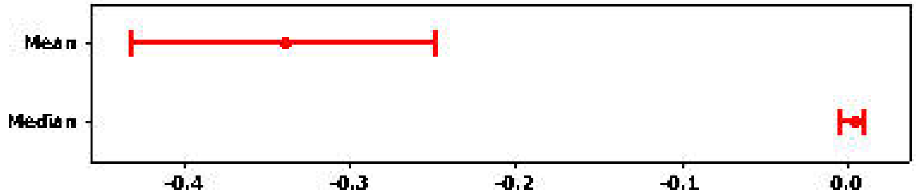
$$\tilde{s}_i(t) = [s_i(t) - \mu_i(t)] / \sigma_i(t)$$

measured ($s_i(t)$)

calculated ($\mu_i(t)$, $\sigma_i(t)$)

Summary for norm_strain_IR

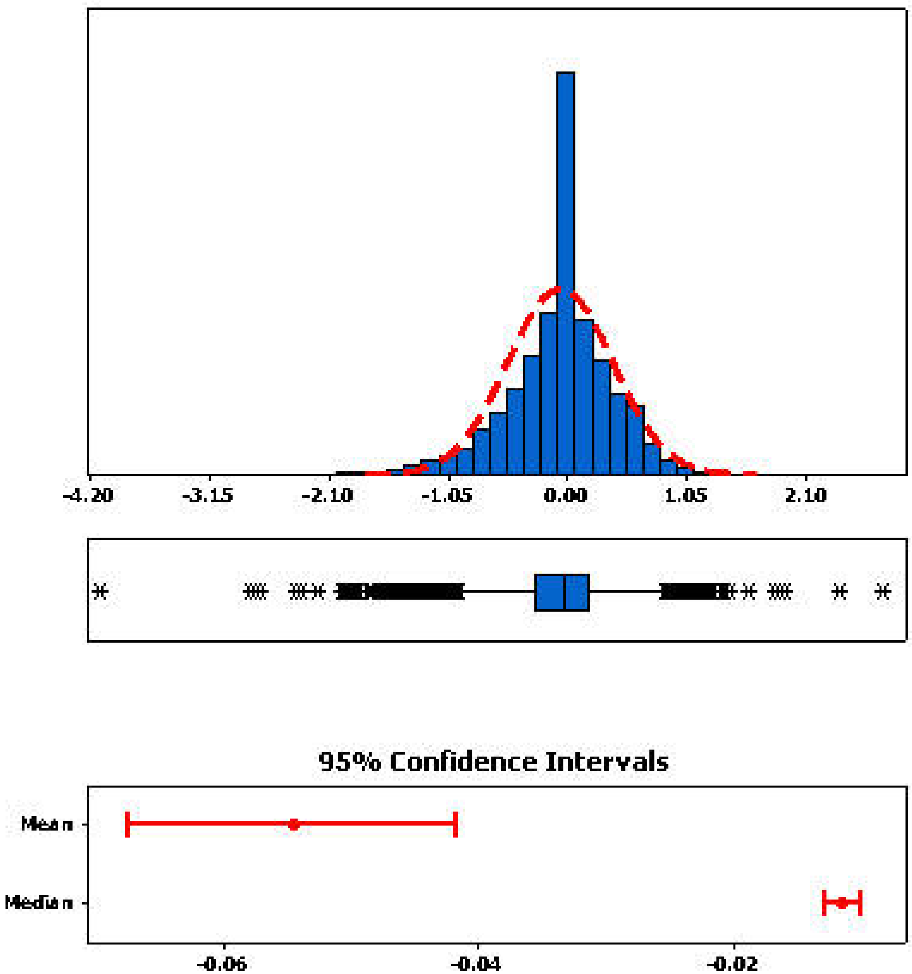




Anderson-Darling Normality Test	
A-Squared	138.59
P-Value <	0.005
Mean	-0.3394
StDev	3.3002
Variance	10.8915
Skewness	0.06525
Kurtosis	3.37319
N	5001
Minimum	-18.1488
1st Quartile	-2.1454
Median	0.0050
3rd Quartile	0.9217
Maximum	14.0780
95% Confidence Interval for Mean	
-0.4308	-0.2479
95% Confidence Interval for Median	
-0.0037	0.0109
95% Confidence Interval for StDev	
3.2368	3.3662

We reject the hypothesis.

Summary for norm_strain_s6_329811



Anderson-Darling Normality Test	
A-Squared	72.46
P-Value <	0.005
Mean	-0.05444
StDev	0.46144
Variance	0.21293
Skewness	-0.69352
Kurtosis	3.80225
N	5001
Minimum	-4.09769
1st Quartile	-0.26174
Median	-0.01147
3rd Quartile	0.19151
Maximum	2.77719
95% Confidence Interval for Mean	
-0.06724	-0.04165
95% Confidence Interval for Median	
-0.01282	-0.01005
95% Confidence Interval for StDev	
0.45257	0.47066

We do not reject the hypothesis for a one-sided test.

- Summary
  - Finite element model validation can consist of more than cross plots of measured and calculated strain with slopes and correlation coefficients.
  - Measured loads are not always the last word.
  - If you pass the hypothesis test, but your FEM correlation is not to your satisfaction, there isn't much point in trying to improve the FEM. Try instead to tighten your knowledge of the loads.
  - Perhaps this general methodology of hypothesis testing could be used more often for general model validation if some distribution can be associated with the inputs, and then mapped to calculated outputs.