Introductory Dynamic FE Analysis Webinar

April 2nd, 2009
Agenda
Introductory Dynamic FE Analysis Webinar
April 2nd, 2009
11am EDT (New York) / 4pm GMT (London)

Welcome & Introduction (Overview of NAFEMS Activities)
Matthew Ladzinski, NAFEMS North America

Dynamic FE Analysis
Tony Abbey, FETraining

Q&A Session
Panel

Closing
An Overview of NAFEMS Activities

Matthew Ladzinski
NAFEMS
North America
Planned Activities

Webinars

- New topic each month!
  - Dynamic FE Analysis

- Recent webinars:
  - Modal Analysis in Virtual Prototyping and Product Validation
  - Practical Advice for Finite Element Analysis of Your Design
  - Pathways to Future CAE Technologies and their Role in Ambient Intelligent Environments
  - Computational Structural Acoustics: Technology, Trends and Challenges
  - FAM: Advances in Research and Industrial Application of Experimental Mechanics
  - Practical CFD Analysis
  - Complexity Management
  - CCOPPS: Creep Loading of Pressurized Components – Phenomena and Evaluation
  - Multiphysics Simulation using Implicit Sequential Coupling
  - CCOPPS: Fatigue of Welded Pressure Vessels
  - Applied Element Method as a Practical Tool for Progressive Collapse Analysis of Structures
  - AUTOSIM: The Future of Simulation in the Automotive Industry
  - A Common Sense Approach to Stress Analysis and Finite Element Modeling
  - The Interfacing of FEA with Pressure Vessel Design Codes (CCOPPS Project)
  - Multiphysics Simulation using Directly Coupled-Field Element Technology
  - Methods and Technology for the Analysis of Composite Materials
  - Simulation-supported Decision Making (Stochastics)

To register for upcoming webinars, or to view a past webinar, please visit: [www.nafems.org/events/webinars](http://www.nafems.org/events/webinars)
Established in 2009

Proposed initial course offerings:
  - Dynamic FE Analysis
  - Stochastics
  - Composites
  - Verification & Validation

Next course:
  - Topic: Dynamic FE Analysis
  - Start: April 21st, 2009 (six-week course)

For more information, visit: www.nafems.org/e-learning
When: June 16\textsuperscript{th} – 19\textsuperscript{th}, 2009

Where: Crete, Greece

Updates:

- Over 200 presentations
- Six Keynote Presentations
- Additional Workshops and Activities:
  - Mini-symposium: Analysis and Simulation of Composite Structures Including Damage and Failure Prediction
  - Engineering Analysis Quality, Verification & Validation
NWC09 Keynotes

➢ Erich Schelkle - Porsche AG and Automotive Simulation Center
  Stuttgart, Germany

➢ Tsuyoshi Yasuki - Toyota Motor Corporation, Japan

➢ Martin Wiedemann - DLR German Aerospace Center, Germany

➢ Jacek Marczyk - Ontonix, Italy

➢ Louis Komzsik - Siemens PLM Software, USA

➢ François Besnier - Principia RD, France
For more information about the NWC09, please visit: www.nafems.org/congress.

Sponsorship and Exhibition Opportunities Still Available!

For more information, please visit: www.nafems.org/congress/sponsor.
Welcome and Agenda

Introduction to FETraining

Overview of the e-Learning Course

Introductory Dynamic FE Analysis

Q and A
Introduction to FETraining

Tony Abbey

BSc Aero. Eng. University of Hertfordshire, UK
MSc Struct. Eng. Imperial College, London

• Started at BAC Warton, UK in 1976

• Worked in UK Defence Industry for 20 years;
  Hunting Engineering, BAe Systems, RRA

• Joined MSC.Software as UK support and Training
  Manager in 1996

• Transferred to MSC.Software US in 2000

• Joined Noran Engineering 2003

• Formed FE Training in 2007

Primary Skill Set:
NASTRAN
PATRAN
FEMAP
Intro to FETraining: Consultancy Solutions

Statics
Dynamics
Composites
Non-Linear
Fatigue
Fracture Mechanics
Thermal
Aero Elasticity

Details
Email: tony@fetraining.com
www.fetraining.com
Intro to FETraining: Training Solutions

Interactive DVD

Live Training On-Site or Public Courses

E-Learning – multiple or one-on-one

Details

Email: tony@fetraining.com

www.fetraining.com
April 21st - June 2nd, 2009

Six-Week Training Course

Members Price: £243 | €264 | $350
Non-Members Price: £382 | €415 | $550
Order Ref: el-001
Event Type: Course
Location: E-Learning, Online
Date: April 21, 2009

www.nafems.org/events/nafems/2009/e-dynamicfea/
Overview of Dynamics e-Learning Class

Dynamics Analysis

Many problems facing designers and engineers are dynamic in nature. The response of a structure cannot be simply assessed using static assumptions. The nature of the problem may be to understand the resonant frequencies of your design, so that key driving frequencies such as equipment rotational speed, acoustic or external pressure frequencies, ground motion frequency content or vehicle passing frequency.

Your design may face external driving forces from adjacent components; cams, push rods, pistons or from vehicle input sources such as a bumpy road, wave loading, air pressure or inertial forces.

Whatever the nature of the challenge, this objective of this course is to break down the dynamic problem into clearly defined steps, give an overview of the physics involved and show how to successfully implement practical solutions using Finite Element Analysis.
Overview of Dynamics e-Learning Class

Why an e-learning class?

In the current climate travel and training budgets are tight. To help you still meet your training needs the following e-learning course has been developed to complement the live class.

The e-learning course runs over a six week period with a single two hour session per week.

Bulletin Boards and Email are used to keep in contact between sessions, mentoring homework and allowing interchange between students.

E-learning classes are ideal for companies with a group of engineers requiring training. E-learning classes can be provided to suit your needs and timescale. Contact us to discuss your requirements.

We hope that small companies or individuals can now take part in the training experience.
Introductory Dynamic FE Analysis Webinar

Agenda

What are Natural Frequencies, Normal Modes
- and why are they important?

Why can’t I get displacements out of Normal Modes Analysis?
- are these ‘real’ values?

Importance of Mode Identification
- don’t just quote a frequency!

What are Rigid Body Modes?
- my structure is elastic, why do I see them?
Introductory Dynamic FE Analysis Webinar

Agenda (continued)

Damping
*What level of damping should I use?*

Transient Analysis background
– *that looks simple, stepping through time?*

Frequency Response Analysis
– *how is that different from transient analysis?*

Quick peek at Shock Spectra and Random analysis
What are Natural Frequencies, Normal Modes

- and why are they important?

Analysis of the Normal Modes or Natural Frequencies of a structure is a search for its resonant frequencies.

- A diving board vibrating after the diver makes his leap
- A car cup holder rattling after we go over a bump

We tend to use both terms to mean the resonant frequency of oscillation, but more accurately:

**Natural Frequency** - the actual measure of frequency in cycles per second (Hz) or similar units

**Normal Mode** - the characteristic deflected shape of the structure as it resonates – imagine using a strobe to ‘freeze’ motion

There are many resonant frequencies in a typical structure
What are Natural Frequencies, Normal Modes

Here are the first 4 modes of the board:

1. First Bending 1.3 Hz
2. Second Bending 3.6 Hz
3. Side Shear 14.5 Hz
4. First Torsion 19.1 Hz

- We expect mode 1 to dominate intuitively, but notice we have a twisting mode—maybe a very heavy ‘athlete’ could just catch a corner of the board and put some twist in?
- How could mode 2 get excited?
- Is mode 3 of any practical interest?
What are Natural Frequencies, Normal Modes
What are Natural Frequencies, Normal Modes

First 5 modes of the cup holder

- The first mode is a cantilever nodding mode, at 15.7 Hz - almost certainly a dominant mode
- Then two twisting modes, at 34.6 Hz and 45.8 Hz
- Then two hogging modes at 124.7 Hz and 144.7 Hz.

Intuitively we would expect all these to be important modes, but as yet we can’t prove that.

The next range of modes are 160 Hz and above and are very complex shapes.

Important question; what is the range of input or driving frequencies? That will dictate which modes are critical.
**What are Natural Frequencies, Normal Modes**

Assume we are a subcontractor to the auto manufacturer. If we are very lucky he will tell us the range of frequencies we can expect to see at the attachment region on the dashboard, based on his analysis and test results.

We can use that as the basis of the range of interest. We can’t use the range directly as that ignores the very complex interaction we may see between harmonics of the system and other factors. So we typically take **an upper bound of 1.5 or 2 times** the upper frequency.

The lower bound should go right down to the lowest frequency we find, as it is difficult to provide a sensible cut off here.

So that gives us a set of modes of interest to investigate. We know a lot about the dynamic characteristics of the cup now. The basic **building block** approach.

Next stage is to look at how the cup is excited by the dashboard – the **response** analysis.

It could be an actual time history, frequency response, shock spectra or random response at the interface.
What are Natural Frequencies, Normal Modes

Frequency Ranges

Theoretically there are an infinite number of natural frequencies in any structure.

- In FE analysis this reduces to the number of degrees of freedom (DOF) in the model.

- We are normally only interested in the dominant frequencies of a system, luckily for us these are *usually* the first few natural frequencies

- Energy required to get a significant response increases as frequency increases

  *Bridge – say .5 Hz to 5 Hz important. Above 20 Hz effectively ignores excitation*

However – excitation frequency may be very high and drives the structural response

  *Crankshaft in a F1 racing car - say 60,000 RPM – means 1000 Hz is critical*
What are Natural Frequencies, Normal Modes

Theoretical Normal Modes analysis is a rather strange process in some ways.

- In practice a structure cannot reveal what natural frequencies it has until we jolt it or hit it or excite it in some way.

- As usual in physics we have to apply an input to the system to get a response out.

- **Physical testing** for normal modes takes this approach

  ![Image of shake and thump]

For **Theoretical analysis** we take a different route:
What are Natural Frequencies, Normal Modes

In the theoretical approach we look for all the frequencies of a system that show a perfect balance between:

- the internal stored energy
- kinetic energy due to motion

At these frequencies the interchange between the two forms of energy is very easily triggered by any external input which has the same driving frequency.

These ‘unique’ or ‘special’ frequencies were labeled Eigen Frequencies in the early 20th Century, from the German word. Mode shapes are correspondingly called Eigen Vectors.

In the theoretical solution we don’t need an external excitation!

The energy balance is calculated by considering the inertial and stiffness terms in isolation. We can find all the structural frequencies and mode shapes this way.
What are Natural Frequencies, Normal Modes

- $m =$ mass (inertia)
- $b =$ damping (energy dissipation)
- $k =$ stiffness (restoring force)
- $p =$ applied force
- $u =$ displacement of mass
- $\dot{u} =$ velocity of mass
- $\ddot{u} =$ acceleration of mass

responses $\dot{u}$ and $\ddot{u}$ vary in time
load input $p$ can vary in time
$m$, $k$ and $b$ are constant with time in linear analysis
What are Natural Frequencies, Normal Modes

\[ m\ddot{u}(t) + b\dot{u}(t) + k\dot{u}(t) = p(t) \]

- Inertia Force
- Damping Force
- Stiffness Force
- Applied Force
What are Natural Frequencies, Normal Modes

\[ m\ddot{u}(t) + b\dot{u}(t) + ku(t) = p(t) \]

Inertia Force  Damping Force  Stiffness Force  Applied Force

undamped free vibration analysis
What are Natural Frequencies, Normal Modes

In the SDOF equation of motion reduces to:

$$m \ddot{u}(t) + k u(t) = 0$$

Assume a solution of the form:

$$u(t) = A \sin \omega_n t + B \cos \omega_n t$$

This form defines the response as being **HARMONIC**, combinations of sine and cosine shape responses with a resonant frequency of:

$$\omega_n$$
What are Natural Frequencies, Normal Modes

For a SDOF system the resonant, or natural frequency, is given by:

$$\omega_n = \sqrt{\frac{k}{m}}$$

We can solve for the constants A and B:

When \( t = 0 \), \( \sin(\omega_n t) = 0 \) thus \( B = u(t = 0) \)

Differentiating solution:

\( \ddot{u}(t) = A \omega_n \cos \omega_n t - B \omega_n \sin \omega_n t \)

When \( t = 0 \), \( B \omega_n \sin(\omega_n t) = 0 \) thus

\[
A = \frac{\ddot{u}(t = 0)}{\omega_n} \\
\]

\[
u(t) = \frac{\ddot{u}(0)}{\omega_n} \sin \omega_n t + u(0) \cos \omega_n t
\]
What are Natural Frequencies, Normal Modes

For a SDOF we know the Natural Frequency or Eigen Value:

\[ \lambda_n = \frac{k}{m} = \omega_n^2 \]

The displacement response is indeterminate as we don’t know any initial conditions

\[ u(t) = \frac{\ddot{u}(0)}{\omega_n} \sin \omega_n t + u(0) \cos \omega_n t \]

The Eigen Vector \( \phi_n \) in this SDOF is literally any arbitrary number
What are Natural Frequencies, Normal Modes

- Now consider the system in terms of a Matrix Solution

![Diagram of a system with DOFs 1, 2, 3, and 4 connected by springs and masses M and 2M]

- The individual ‘element’ stiffness matrices $[K_1], [K_2],$ and $[K_3]$ are:

$$[K_1] = [K_2] = [K_3] = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
What are Natural Frequencies, Normal Modes

- To derive the model stiffness matrix \([K]\), we assemble the individual ‘element’ stiffness matrices \([K_1],[K_2],\) and \([K_3]\):

\[
\begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 + 1 & -1 & 0 \\
0 & -1 & 1 + 1 & -1 \\
0 & 0 & -1 & 1
\end{bmatrix}
\]

- If we eliminate grounded DOF at 1 and 4:

\[
[K] = k \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}
\] \quad \text{and} \quad
\[
[M] = m \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}
\]

We ‘lump’ masses at DOF.
What are Natural Frequencies, Normal Modes

- The equation of motion in matrix form is:
  \[ [M]\ddot{x} + [K]x = 0 \]
- If we substitute in
  \[ \{x\} = \{\phi\}e^{i\omega t} \]
  And \[ \ddot{x} = -\omega^2 \{\phi\}e^{i\omega t} \]
- Then \[ -\omega^2 [M]\{\phi\} + [K]\{\phi\} = 0 \]
- So \[ ([K] - \omega^2[M])\{\phi\} = 0 \]

This means we have a mode shape, \{\phi\}, which varies sinusoidally with a frequency \(\omega\).

This means we can find a mode shape, \{\phi\}, and frequency \(\omega\) where the inertia terms and elastic terms balance.

The Eigenvalue problem
What are Natural Frequencies, Normal Modes

- If we have a set of \( n \) physical degrees of freedom (2 in our case)

  - Then we have \( n \) sets of unique Eigenvalues \( \omega_i^2 \) and eigenvectors \( \{ \phi_i \} \)
    - where \( i = 1 \) to \( n \)

  - For each of these sets, the inertia terms balance the elastic terms and this is the definition of resonance
What are Natural Frequencies, Normal Modes

- So at $\omega_1 = \sqrt{\frac{0.634 k}{m}}$, the motion is defined by: $\{\phi\} = \begin{cases} 0.731 \\ 1.000 \end{cases}$

- And at $\omega_2 = \sqrt{\frac{2.366 k}{m}}$, the motion is defined by: $\{\phi\} = \begin{cases} -2.731 \\ 1.000 \end{cases}$
What are Natural Frequencies, Normal Modes

- Let us add some values in and check out the numbers:
  - Let $k = 1000$ units of force / length
  - Let $m = 20$ units of mass

- Then

$$\omega_1 = \sqrt{\frac{0.634}{m} \frac{k}{m}} = 5.629 \frac{\text{rad/s}}{\text{s}} = 0.896 \text{Hz}$$

$$\omega_2 = \sqrt{2.366 \frac{k}{m}} = 10.875 \frac{\text{rad/s}}{\text{s}} = 1.731 \text{Hz}$$

- Notice the conversion of Frequency from Radians/s to Cycles/s (Hertz)

$$\omega = \frac{f}{2\pi}$$
What are Natural Frequencies, Normal Modes

Class Topics taking this further

• Eigenvalue Extraction Methods
• Error Checks in Normal Modes
• Reduction techniques
• Residual Vectors
• Modal Effective Mass
• Case Studies
Introductory Dynamic FE Analysis Webinar

Agenda

What are Natural Frequencies, Normal Modes
- and why are they important?

Why can’t I get displacements out of Normal Modes Analysis?
- are these ‘real’ values?

Importance of Mode Identification
- don’t just quote a frequency!

What are Rigid Body Modes?
- my structure is elastic, why do I see them?
Displacements out of Normal Modes Analysis

- *are these ‘real’ values?*

In life, sadly we can never get something for nothing.

• The side effect of the Eigen Value method is that we do not know the actual amplitude of the shapes that we calculate.
• This confuses many users who are new to modal analysis.

The 2 DOF spring example gave us

\[
\{\phi_1\} = \begin{bmatrix} 0.731 \\ 1.000 \end{bmatrix} \quad \{\phi_2\} = \begin{bmatrix} -2.731 \\ 1.000 \end{bmatrix}
\]

We can equally say:

\[
\{\phi_1\} = \begin{bmatrix} 0.731 \\ 1.000 \end{bmatrix} \quad \{\phi_2\} = \begin{bmatrix} 1.000 \\ -0.366 \end{bmatrix}
\]
Displacements out of Normal Modes Analysis

*How can we predict a shape without knowing its magnitude?*

The simple answer is because we haven’t provided any excitation input or initial conditions.

- We don’t know the weight of the diver or how high his spring was.
- We don’t know the speed of the car or the height of the bump.

All we know is what the range of natural frequencies of the board and the holder will be and what their respective deflected shapes will look like.

*But I see screen animations, they must have a value for deformation and they look big?*

The answer here is that we scale the magnitude of the shapes to one of several types of arbitrary definitions for convenience and for shape comparison.

A post processor will further scale deformations so you can see each mode shape clearly – typically 10% of the maximum viewable dimension, again quite arbitrary.

The second part of the question is very important – we are only dealing with linear, small displacement theory. So in practice vibration amplitudes would have to be small relative to the size of the structure.
Displacements out of Normal Modes Analysis

Class Topics taking this further

- Pseudo static assumptions
- Modal Effective mass in base motion
Introductory Dynamic FE Analysis Webinar

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Importance of Mode Identification

– don’t just quote a frequency!

2144 Hz !!!

Very close son, 2133 Hz is better

0.5% error here?
### Importance of Mode Identification

<table>
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<th>Mode</th>
<th>( f )</th>
<th>Mesh 1b (2 x 1) Description</th>
<th>Mode</th>
<th>( f )</th>
<th>Mesh 1a (10 x 4) Description</th>
<th>Mode</th>
<th>( f )</th>
<th>Mesh 1c (50 x 20) Description</th>
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<td>1st Order Bending</td>
<td>1</td>
<td>133.1</td>
<td>1st Order Bending</td>
<td>1</td>
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<td>10</td>
<td>6433</td>
<td>4th Order Torsion</td>
</tr>
</tbody>
</table>
Importance of Mode Identification

Correlation between models and model and test is important

Corresponding points on test and analysis model are located and deflections measured, for each mode shape found
Importance of Mode Identification

Mass Normalised Eigenvectors within one system have the characteristic:

\[
\begin{align*}
\{\Phi\}_i^T [M] \{\Phi\}_i &= 1.0 \\
\{\Phi\}_i^T [M] \{\Phi\}_j &= 0.0
\end{align*}
\]

Consider Mass Normalised Eigenvectors across two systems

\[
\begin{align*}
\{\Phi\}_A \\
\{\Phi\}_T
\end{align*}
\]

Refers to the Analysis Results

Refers to the Test or External Model Results
Importance of Mode Identification

Mass Orthogonality Check is

\[
\begin{align*}
\text{Diagonal} & : \left\{ \Phi \right\}_A^T \left[ M \right] \left\{ \Phi \right\}_T = 1.0 \\
\text{Off-Diagonal} & : \left\{ \Phi \right\}_A^T \left[ M \right] \left\{ \Phi \right\}_T = 0.0
\end{align*}
\]

A perfectly correlated Analysis and Test result will be a Unit Diagonal Matrix

Closeness to 1.0 or 0.0 represents degree of orthogonality

It is the ratio between the diagonal and off diagonal terms which is important
Importance of Mode Identification
Importance of Mode Identification

Modal Assurance Criteria (MAC) has a similar approach

\[
\frac{\left[\{\Phi\}_A^T\{\Phi\}_T\right]^2}{\{\Phi\}_A^T\{\Phi\}_T \times \{\Phi\}_T^T\{\Phi\}_A}
\]

A perfectly correlated Analysis and Test result will be a Unit Diagonal Matrix as before

Various other variations on the basic idea are available, some include frequency shift as well as mode shape comparison.
Importance of Mode Identification

[3D bar chart showing MAC values for different input data sets]
Importance of Mode Identification
Importance of Mode Identification

Class Topics taking this further

- Project examples
- Describing and assessing modes
- Correlation Case studies
Introductory Dynamic FE Analysis Webinar

Agenda

What are Natural Frequencies, Normal Modes
- and why are they important?

Why can’t I get displacements out of Normal Modes Analysis?
- are these ‘real’ values?

Importance of Mode Identification
- don’t just quote a frequency!

What are Rigid Body Modes?
- my structure is elastic, why do I see them?
What are Rigid Body Modes?

– *my structure is elastic, why do I see them?*

For every DOF in which a structure is not totally constrained, it allows a Rigid Body Mode (stress-free mode) or a mechanism.

The natural frequency of each Rigid Body Mode should be close to zero.
What are Rigid Body Modes?

- Classic mechanism due to element DOF mismatch
What are Rigid Body Modes?

Class Topics taking this further

• Typical causes of mechanisms
• Low order elastic eigenvectors
• Grounding and other checks
Introductory Dynamic FE Analysis Webinar

Agenda (continued)

Damping
*What level of damping should I use?*

Transient Analysis background
--- *that looks simple, stepping through time?*

Frequency Response Analysis
--- *how is that different from transient analysis?*

Quick peek at Shock Spectra and Random analysis
Damping

What level of damping should I use?

The only honest answer is  - it depends!

Let’s review what damping is and come back to the question …
Damping

- If viscous damping is assumed, the equation of motion becomes:
  \[ m\ddot{u}(t) + b\dot{u}(t) + ku(t) = 0 \]

- There are 3 types of solution to this, defined as:
  - Critically Damped
  - Overdamped
  - Underdamped

- A swing door with a dashpot closing mechanism is a good analogy:
  - If the door oscillates through the closed position it is **underdamped**
  - If it creeps slowly to the closed position it is **overdamped**
  - If it closes in the minimum possible time, with no overswing, it is **critically damped**
Damping

DAMPED FREE VIBRATION SDOF (Cont.)

- For the **critically damped** case, there is no oscillation, just a decay from the initial condition:

\[ u(t) = (A + Bt)e^{-bt/2m} \]

- The damping in this case is defined as:

\[ b = b_{cr} = 2\sqrt{km} = 2m\omega_n \]

- A system is **overdamped** when \( b > b_{cr} \)

- We are generally only interested in the final case - **underdamped**
Damping

DAMPED FREE VIBRATION SDOF (Cont.)

- For the underdamped case $b < b_{cr}$ and the solution is the form:

$$u(t) = e^{-bt/2m} (A \sin \omega_d t + B \cos \omega_d t)$$

- $\omega_d$ represents the Damped natural frequency of the system:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

- $\zeta$ is called the Critical damping ratio and is defined by:

$$\zeta = \frac{b}{b_{cr}}$$

- In most analyses $\zeta$ is less than .1 (10%) so $\omega_d \approx \omega_n$
Damping

DAMPED FREE VIBRATION SDOF (Cont.)

- The graph is from a transient analysis of the previous spring mass system with damping applied.

![Graph showing damping effects]

- Frequency and period as before.
- Amplitude is a function of damping.

- 2% Damping
- 5% Damping
Damping

I have discussed viscous damping – but this is just one *simulation* method.

Other types of damping simulations exist which may be preferable:

- Modal damping – viscous damping, but linked to each mode shape
- Structural damping – overall or material based single slope frequency dependent damping
- Rayleigh damping – stiffness dominated region and mass dominated region
- Coulomb damping – based on ‘stick – unstick’ friction
- Nonlinear damping – in nonlinear solutions, can be dependent on many types of response including displacement, velocity, acceleration

Etc …

The real world damping is a complex phenomena and is not fully understood. Testing and tuning against test results is the best approach.
Damping

What level of damping should I use?

Now to answer the question, with a very rough rule of thumb and using critical damping ratio:

Vibration absorbing material could be 10% or greater – but*

Composite structure  3% - 10%

Rusty structure, friction clamps throughout  5% - 10%

Clean metallic structure bolted, riveted joints throughout  3% – 6%

Clean integrally machined structure  2% to 4%

Clean room integrally machined specially designed  1% to 2%

* Anything over 10 % is reaching the limits of linear analysis
Damping

Bottom line is that a lower bound estimate is conservative, and upper bound is not.

Always best to present a set of responses, based on two or three damping levels.
Damping

Class Topics taking this further

• Damping theories in response analysis
• Damping case studies
• Checking damping levels
• Compound approach to damping
Introductory Dynamic FE Analysis Webinar

Agenda (continued)

Damping
What level of damping should I use?

Transient Analysis background
– that looks simple, stepping through time?

Frequency Response Analysis
– how is that different from transient analysis?

Quick peek at Shock Spectra and Random analysis
Transient Analysis Background

– that looks simple, stepping through time?

In general a transient analysis is most intuitive and we like to introduce response analysis using this approach.
Transient Analysis Background

The Dynamic equation of motion is $\Delta t$

$$ [M] \{\ddot{u}(t)\} + [B] \{\dot{u}(t)\} + [K] \{u(t)\} = \{P(t)\} $$

The response is solved at discrete times with a fixed time step.

Using central finite difference representation for $\{\ddot{u}(t)\}$ and $\{\dot{u}(t)\}$ at discrete times

$$ \{\ddot{u}_n\} = \frac{1}{2\Delta t} \{u_{n+1} - u_{n-1}\} $$

$$ \{\dot{u}_n\} = \frac{1}{\Delta t^2} \{u_{n+1} - 2u_n + u_{n-1}\} $$
Transient Analysis Background

Some practical points to consider:

It is often easier to think in milliseconds (ms) - .001s = 1ms.

The time step chosen should be sufficiently small to capture the highest frequency of interest in the response. For example, if this value is 100 Hz, each time period is .01s (10ms) so we need at the very least 5 steps to capture the response, i.e.. $\Delta t = .002s$ (2ms). The preferred minimum number is 10 steps per period.

The accuracy of the load input is similarly dependent on the time step chosen, so a loading with a 1000 Hz input will need at least $\Delta t$ of .0002s (.2ms) and preferably .0001s (.1ms)

Shock loading will have very high frequency content, either as an applied load, or coming through a contact in non-linear

The smaller the value of $\Delta t$, the more accurate the integration will be, this may override the previous comments.
Transient Analysis Background

It may be cost effective in a large model to decrease $\Delta t$ at a time of critical interest (say under impulsive loading) and increase it later. However, for linear transient analysis, changing $\Delta t$ can be CPU expensive.

Normally it will take a few runs to tune the model overall, and the effectiveness of this technique can be investigated.

The number of times steps needs to be adequate to ensure all over swings are captured and that the response is decaying at the cut off point, with no surprises.

Reduce output to key nodes or elements to act as ‘test’ points. Only when model is thoroughly debugged then ask for full output.
Transient Analysis Background

**QA – check frequency content matches what you expect** (the building block approach)

Use the graphical method shown, or a Fourier Analysis.

- The time for 3 periods of oscillation is:
  \[ 0.0330 - 0.0105 = 0.0225 \text{s} \]

- Time period:
  \[ \frac{0.0225}{3} = 0.0075 \text{s} \]

- Frequency:
  \[ \frac{1}{T} = 133.3 \text{ Hz} \]

![Graph showing displacement over time with labeled points for loaded and free states.]

Legend:
- Node 56: Displacements, Translational, ZZ
Transient Analysis Background

**QA – check damping values**

Use logarithmic decrement on successive peaks method to confirm total damping at that DOF

\[ G = 2\zeta = \frac{d}{\pi} \]

Where:

\[ d = \ln\left(\frac{d_1}{d_2}\right) \]
Transient Analysis Background

QA – *watch for aliasing*

Time steps are too coarse and misleading frequency content is seen

Peak responses are missed
Transient Analysis Background

When and how to go nonlinear in impact:

Rule of thumb

- Bearing type contacts
- Mother in Law hits parking spot post at 15 mph
- Nigel Mansell hits Indy Car wall at 200 mph
Transient Analysis Background

Class Topics taking this further

• Direct and Modal methods
• Further damping assumptions
• Transient Case Studies
• Cook book approach for time step estimation
• Fourier Analysis of frequency content
• Large model strategies
Introductory Dynamic FE Analysis Webinar

Agenda (continued)

Damping
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→ that looks simple, stepping through time?

Frequency Response Analysis
→ how is that different from transient analysis?

Quick peek at Shock Spectra and Random analysis
Frequency Response Analysis

– how is that different from transient analysis?

- Transient response is intuitive and can be visualized as a time history of an event.

- Frequency response is best visualized as a response to a structure on a shaker table, Adjusting the frequency input to the table gives a range of responses.
Frequency Response Analysis

- We now apply a harmonic forcing function: \( p \sin \omega t \)
  - Note that \( \omega \) is the DRIVING or INPUT frequency

- The equation of motion becomes:
  \[
  m\ddot{u}(t) + b\dot{u}(t) + ku(t) = p \sin \omega t
  \]

- The solution consists of two terms
  - The initial response, due to initial conditions which decays rapidly in the presence of damping
  - The steady-state response as shown:

\[
U(\omega) = \frac{p}{k} \frac{\sin(\omega t + \theta)}{\sqrt{(1 - \omega^2/\omega_n^2)^2 + (2\zeta\omega/\omega_n)^2}}
\]

- This equation is described on the next page
Frequency Response Analysis

- This equation deserves inspection as it shows several important dynamic characteristics:

\[ u(\omega) = \frac{p}{k} \left( 1 - \frac{\omega^2}{\omega_n^2} \right)^2 + \frac{(2\zeta\omega / \omega_n)^2}{1 - \frac{2\zeta}{\sqrt{1 - \zeta^2}} \sin(\omega t + \theta)} \]

- Phase lead of the response relative to the input
- At \( \omega = \omega_n \) this term = 0.0
  - With no damping present this results in an infinite response
- At \( \omega >> \omega_n \) both terms drive the response to 0.0
- At \( \omega = \omega_n \) this term = \((2\zeta)^2\)
  - and controls the scaling of the response
  - From this is derived the Dynamic Magnification Factor \( \frac{1}{2\zeta} \)
Frequency Response Analysis

Summary:

- For $\frac{\omega}{\omega_n} \ll 1$
  - Magnification factor $\rightarrow 1$ (static solution)
  - Phase angle $\rightarrow 0^\circ$ (response is in phase with the force)

- For $\frac{\omega}{\omega_n} \gg 1$
  - Magnification factor $\rightarrow 0$ (no response)
  - Phase angle $\rightarrow 180^\circ$ (response is out of phase with the force)

- For $\frac{\omega}{\omega_n} \approx 1$
  - Magnification factor $\rightarrow \frac{1}{2\zeta}$ (dynamic magnification factor)
  - Phase angle $\rightarrow 90^\circ$ (response is in transition)
Frequency Response Analysis

Magnification Factor = $1/2\xi = 1/G = 50$
Static Response = $p/k = 0.01$
Peak Response = 0.5 at 1.59 Hz

Note:
Use of a Log scale helps identify low order response
Frequency Response Analysis

Some practical points to consider:

Frequency Response calculation points (sometimes called spectral lines):

Must be at each natural frequency to ensure that the peak responses are captured

Must be spread around each natural frequency to capture a good ‘shape’

A general spread of points is required to capture the overall trend of the curve

The number of calculation points will increase CPU cost and output quantities

As with transient – debug the model using key nodes and element responses before running full output requests

Use log scales to help visualization

Avoid responses or definitions at 0.0 hz - $\log_{10} 0.0 = 1.0$ !
Frequency Response Analysis

Class Topics taking this further

- Modal and Direct methods
- Further damping methods
- Large model strategies
- Case studies
Introductory Dynamic FE Analysis Webinar

Agenda (continued)

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Webinar

Agenda (continued)

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*What level of damping should I use?*

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*how is that different from transient analysis?*

Quick peek at Shock Spectra and Random analysis
Quick peek at Shock Spectra and Random analysis

Random vibration is vibration that can be described only in a statistical sense. Its instantaneous magnitude at any time is not known; rather, the probability of its magnitude exceeding a certain value is given.

Examples include earthquake ground motion, ocean wave heights and frequencies, wind pressure fluctuations on aircraft and tall buildings, and acoustic excitation due to rocket and jet engine noise.

Random environment is characterized by a Power Spectral Density (PSD)

FE solvers perform random response analysis as post processing to unit frequency response.
Quick peek at Shock Spectra and Random analysis

- **Response at a DOF due to unit input (TF):**
  - Graph showing a peak in response at a specific frequency.
  - Formula: $g / (in/s^2)$
  - Unit input

- **PSD Input Definition:**
  - Graph showing a flat response across a range of frequencies.
  - Formula: $g^2/Hz$
  - $(in/s^2)^2/Hz$

- **PSD out = PSD in * TF^2:**
  - Graph showing a transformation of response through frequency.
  - Formula: $g^2/Hz$
  - $(in/s^2)^2/Hz$

- **PSD response at DOF:**
  - Graph showing the output power spectral density at a specific DOF.
Quick peek at Shock Spectra and Random analysis

- Area = $g^2$ (Mean Square)
- $\sqrt{\text{Area}} = g$ (RMS)
- One Standard Deviation

First Moment = Apparent Frequency (Hz)
Quick peek at Shock Spectra and Random analysis

Shock Spectra

Transient Input

Tuned Spring-Masses

Accn (g)

Time (s)

1 hz
2 Hz
3 hz
4 Hz
5 hz
Quick peek at Shock Spectra and Random analysis

A substructure or piece of equipment with frequency 3.5 Hz would expect to see a peak base acceleration of 1.4 g

![Shock Response spectra](image)

No coupling is assumed between primary and substructure
Quick peek at Shock Spectra and Random analysis

Class Topics taking this further

• Description of Random background
• PSD theory and practical evaluation
• Case study with full random output
• Introduction to Vibration fatigue
• Shock spectra case studies and further theory
Conclusions

CHECK LIST FOR NORMAL MODES PRIOR TO DOING FURTHER ANALYSIS

RBM’s - are they as expected

Is the frequency range adequate (we will discuss this more in the section on modal effective mass)

Are the modes clearly identified

Is mesh density adequate

Is the element type appropriate

Is the mass distribution correct

Is coupled vs. lumped mass important

Are the internal joints modeled correctly

Are the constraints modeled correctly

Do the results compare with hand calcs, previous experience or test
Conclusions

CHECK LIST FOR RESPONSE ANALYSIS

Make sure the modes are reasonable

Transient - Plan time steps and duration

Frequency response – Plan frequency range and spread

Identify forms of damping and what methods to simulate

Plan a exploratory key point approach to the results, don’t output all data

Transient results – check damping, frequency content, aliasing, duration

Frequency response results - check resonant frequencies captured and good spread of points, check magnification factor against damping and low frequency results approaching static

All forms of response – check driven nodal response against input
Thank you!

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