

# Welcome and Agenda

**Introduction to FETraining**

**Overview of the NAFEMS e-Learning Course**

**Introductory Optimization FE Analysis**

# Introduction to FE Training



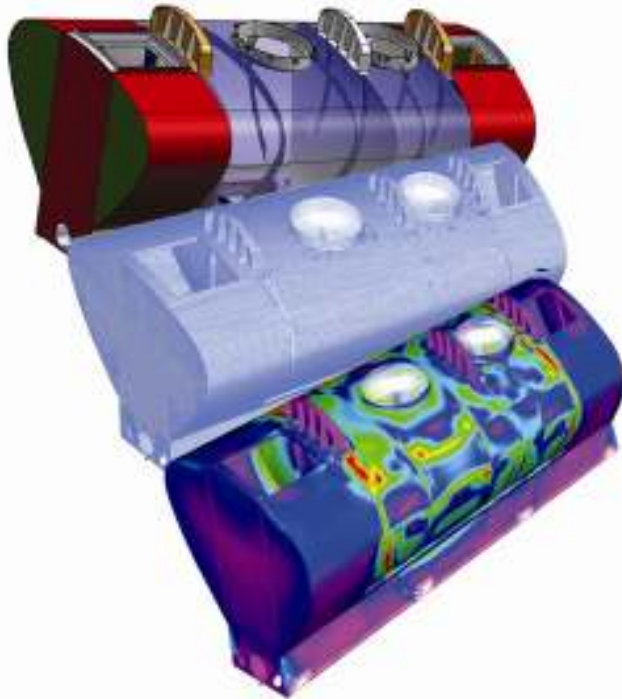
Primary Skill Set:  
NASTRAN  
PATRAN  
FEMAP

## Tony Abbey

BSc Aero. Eng. University of Hertfordshire, UK  
MSc Struct. Eng. Imperial College, London

- Started at BAC Warton, UK in 1976
- Worked in UK Defence Industry for 20 years; Hunting Engineering, BAe Systems, RRA
- Joined MSC.Software as UK support and Training Manager in 1996
- Transferred to MSC.Software US in 2000
- Joined Noran Engineering 2003
- Formed FE Training in 2007

# Intro to FE Training: Consultancy Solutions



**Statics**

**Dynamics**

**Composites**

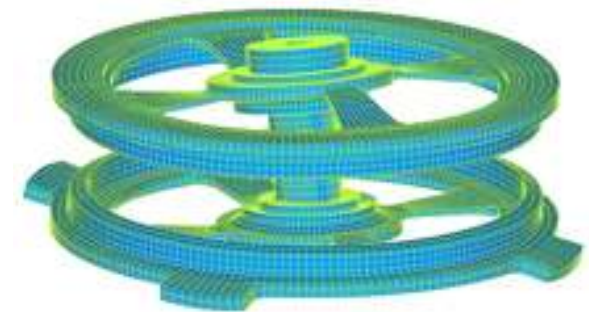
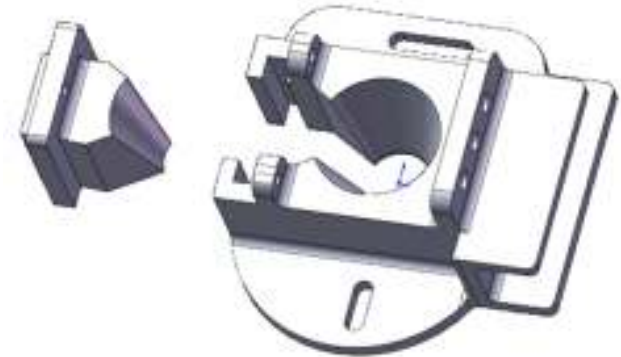
**Non-Linear**

**Fatigue**

**Fracture Mechanics**

**Thermal**

**Aero Elasticity**



**Details**

Email : [tony@fettraining.com](mailto:tony@fettraining.com)

[www.fettraining.com](http://www.fettraining.com)

# Intro to FE Training: Training Solutions



**Live Training On-Site or Public Courses**

**E-Learning – multiple or one-on-one**

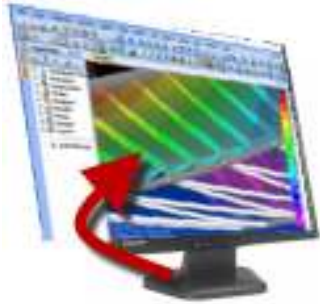
**Details**

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# Intro to FE Training: Training Solutions



E-Learning classes with NAFEMS

- Dynamics in FE Analysis (7 sessions)
- Composites in FE Analysis (4 sessions)
- Nonlinear FE Analysis (4 sessions)
- Optimization in FE Analysis (4 sessions)

## Details

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[www.fettraining.com](http://www.fettraining.com)

[www.nafems.org](http://www.nafems.org)



**Structural Optimization in FE Analysis**

**October 5th - November 2nd, 2010**

**Four-Week Training Course**

**Members Price: £166 | €200 | \$255**

**Non-Members Price: £257 | €309 | \$395**

**Order Ref:el-0013**

**Event Type:Course**

**Location: E-Learning,Online**

**Date: October 5th, 2010 (4 sessions)**

**[www.nafems.org/events/nafems/2010/el013/](http://www.nafems.org/events/nafems/2010/el013/)**



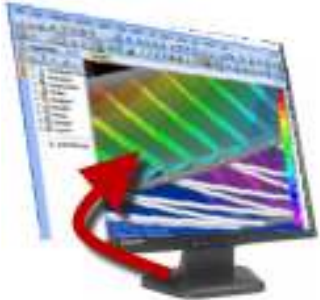
# Overview of Optimization e-Learning Class

## *Optimization Analysis*

Finite Element Analysis has emerged as a tool that can play a vital part in the drive towards the ultimate goal of any manufacturing process; to produce the most effective products in the most efficient manner. This simple statement embraces all of the 'right first time', 'minimum design to test cycles' and other practices that have evolved.

The introduction of a formal structural optimization strategy into this process has met with great success in many industries. It makes the creation of the most effective product that much more attainable.

Traditionally one might think of the Aerospace Industry as the classic example with the goal of keeping weight to a minimum. Indeed the structural efficiencies of modern aircraft owe a lot to optimization methods. However it would be wrong to think of this as always a strength and stiffness against weight minimization task. The interaction of Aerodynamics, Aeroelasticity, Structures, Performance, Operating Cost and many other disciplines all have to play a role in the overall vehicle design.



# Overview of Optimization e-Learning Class

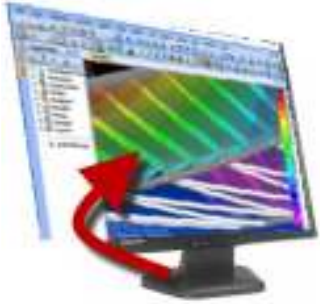
## *Optimization Analysis (continued)*

This gives the clue as to the broader nature of structural optimization across all industries. The objective does not need to be weight minimization. It could be, for example driving down the overall vibration amplitude of a hairdryer, whilst keeping away from unpleasant harmonic frequencies. Weight has still to be monitored, and we can place an upper limit on this – but the other factors are more important and will feature directly in the optimization analysis.

Similarly other disciplines can play a role in structural optimization. In the case of pump housing, we want this to be stiff and strong enough to do the job, with minimum weight. However the cost of manufacture is important so a parametric penalty function can be introduced which ‘steers’ the weight reduction to a compromise solution which is cheaper to machine.

How do we define the penalty function in the above case? Well, that’s where the ingenuity of the analyst comes in! Knowing how to set up the optimization task and how to obtain innovative solutions with the tools provided is a key to success in FEA Structural Optimization.





# Overview of Optimization e-Learning Class

## *Why an e-learning class?*

In the current climate travel and training budgets are tight. To help you still meet your training needs the following e-learning course has been developed to complement the live class.

The e-learning course runs over a four week period with a single two and half hour session per week.

Bulletin Boards and Email are used to keep in contact between sessions, mentoring homework and allowing interchange between students.

E-learning classes are ideal for companies with a group of engineers requiring training. E-learning classes can be provided to suit your needs and timescale. Contact us to discuss your requirements.

We hope that small companies or individuals can now take part in the training experience.

# Introductory Optimization FE Analysis Webinar

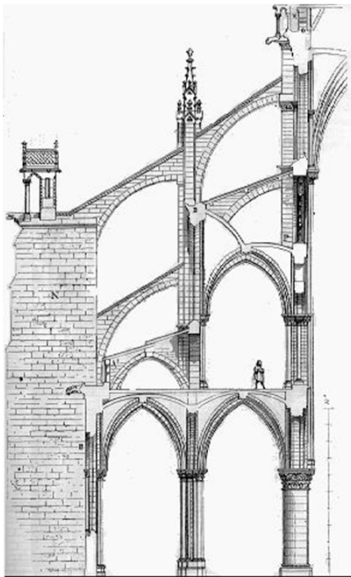
## Agenda

- **Background and History of Structural Optimization**
- Putting Optimization in perspective – what are we trying to achieve?
- Sizing, Shape and Topology – the three fundamentals
- Multi-Disciplinary Optimization (MDO) – trying to put it all together
- Nonlinear Optimization – altered perspectives!

# Introductory Optimization FE Analysis Webinar

The earliest optimization of structures was based on observation and experiment

Maybe the wheel started as square – but evolved into a round shape after corners got knocked off and the performance improvement was noted!



Flying Buttresses evolved from early design failures, and then careful consideration of the load paths required to transfer heavy masonry roof vaulting

12<sup>th</sup> Century optimization – but we don't see the ones that didn't work!

# Introductory Optimization FE Analysis Webinar

Victorian Engineers carried out remarkable achievements in design optimization.

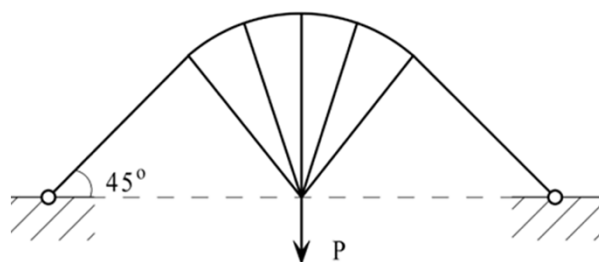
I.K. Brunel was a leader in evaluating many forms of design solution – suspension bridges, tubular span, lattice etc.

Evaluating strengths and weaknesses of candidate designs is key to optimization

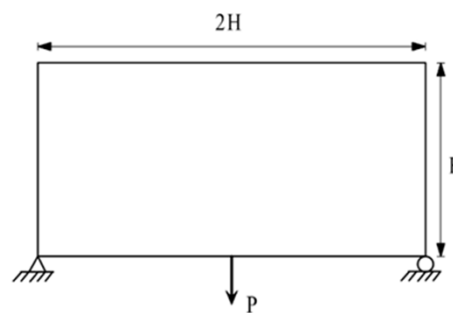


# Introductory Optimization FE Analysis Webinar

Many early workers used analytical methods to investigate optimum structural solutions



(a) A Michell type structure



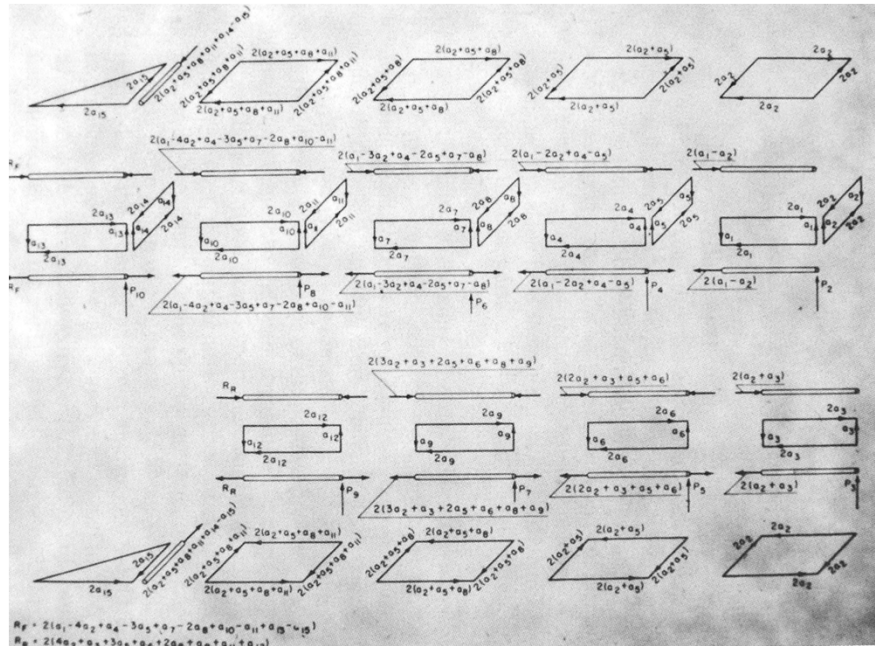
(b) Design domain

Barnes Wallis Geodetic Structures

Airships to Aircraft



# Introductory Optimization FE Analysis Webinar



Evolution of numerical solutions and advent of computing power meant growth in Optimization methods in parallel with FEA

# Introductory Optimization FE Analysis Webinar

An early approach to optimization, and one of the first to be automated was the Fully stressed design (FSD) method.

- This used the observed fact that for a determinant structure, the minimum volume could be achieved by forcing the stresses at all points to be simultaneously at the required upper limit.
- At the same time it was recognized that all loading cases must be catered for simultaneously.
- The method has definite disadvantages when used in non-determinant structures and it can be shown that it is far from optimum in many cases.

This was an early example of the Optimality Criteria method, which was popular for a long time.

- This method uses an a priori criteria to determine the conditions for optimality. Many of the methods are heuristic and based on classical or empirical approaches.
- However when tailored well they give very fast efficient solutions

# Introductory Optimization FE Analysis Webinar

A more rigorous school of techniques evolved, known as Mathematical programming.

- No a priori assumptions are made in this approach
- Gradient based methods dominate ( these are described shortly)
- Methods include:
  - Linear Programming
  - Integer Programming
  - Nonlinear programming
  - Quadratic Programming
  - Dynamic Programming
  - Etc.

Many FE solvers with in-built optimization routines use the gradient based methods with sophisticated sub-searching, filtering and approximation techniques.



# Introductory Optimization FE Analysis Webinar

A different approach to optimization has evolved over the last decade or so

- The increase in computing power and improvement in FEA solution efficiency are one of the primary factors in the growth of this range of methods. Solutions in order of 1000's
- In addition these techniques adapt well to driving an FEA process ( and indeed other co-simulation processes) from an external 'wrapper' program

Methods included in this Category are:

- Genetic Algorithms
- Simulated Annealing
- Neural-Network methods
- Design of Experiments
- Fuzzy Logic
- Etc.

# Background and History of Structural Optimization

## Classical Problem Statement:

- Design variables  $Find \{X\} = (X_1, X_2, \dots, X_n)$
- Objective function: Minimize/M aximize  $F\{X\}$
- Subject to:
  - ◆ Inequality constraints:  $G_j(X) \leq 0 \quad j = 1, 2, \dots, L$
  - ◆ Equality constraints:  $H_k(X) = 0 \quad k = 1, 2, \dots, M$
  - ◆ Gage constraints:  $x_i^L \leq x_i \leq x_i^u \quad i = 1, 2, \dots, N$

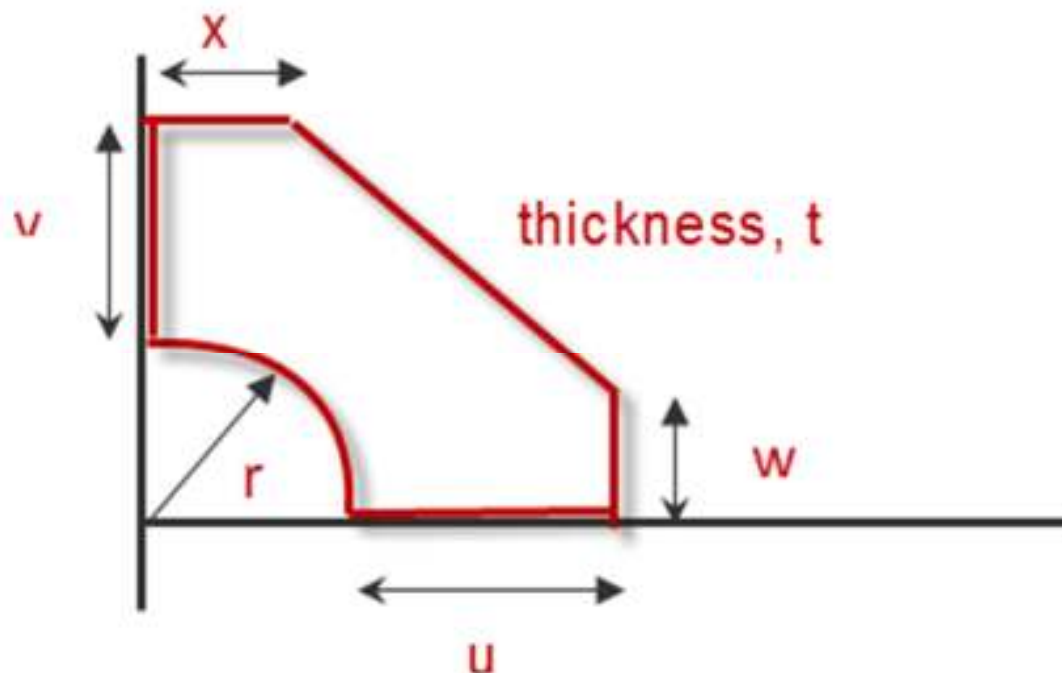
## Background and History of Structural Optimization

Example: A gusset is designed to carry bending loads around the joint between a floor and a wall.

The design aim is to minimize the localized bending stresses at the floor/wall joint

The key dimensions of the gusset are shown

Each one of these can vary



# Background and History of Structural Optimization

- Design variables  $Find \{X\} = (r, u, v, w, x, t)$
- Objective function: Minimize Weight  $\{X\}$
- Subject to:
  - ◆ Inequality constraints: Stresses in the gusset, floor and walls:
 
$$\sigma_{\min steel} \leq \sigma_{gusset, floor, wall} \leq \sigma_{\max steel}$$
  - ◆ :

# Background and History of Structural Optimization

Gage Constraints:

$$u_{\min} \leq u \leq u_{\max}$$

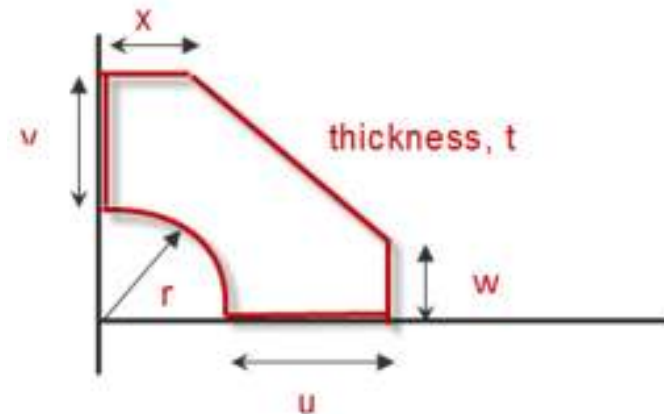
$$v_{\min} \leq v \leq v_{\max}$$

$$w_{\min} \leq w \leq w_{\max}$$

$$x_{\min} \leq x \leq x_{\max}$$

$$r_{\min} \leq r \leq r_{\max}$$

$$t_{\min} \leq t \leq t_{\max}$$



Design Constraints

$$w \leq r + v$$

$$x \leq r + u$$

**These can be problematic in a complex CAD model !**

# Background and History of Structural Optimization

- Design variables  $\{X\} = \{r, u, v, w, x, t\}^T$
- Minimize:  $\rho v = \rho(\text{Area} * t)$

Response constraints:

$$\sigma_{\min steel} - \sigma_{gusset, floor, wall} \leq 0$$

$$\sigma_{gusset, floor, wall} - \sigma_{\max steel} \leq 0$$

:

Design Constraints

$$-r - v + w \leq 0$$

$$-r - u + x \leq 0$$

Gage constraints

$$u_{\min} - u \leq 0 \quad u - u_{\max} \leq 0$$

$$v_{\min} - v \leq 0 \quad v - v_{\max} \leq 0$$

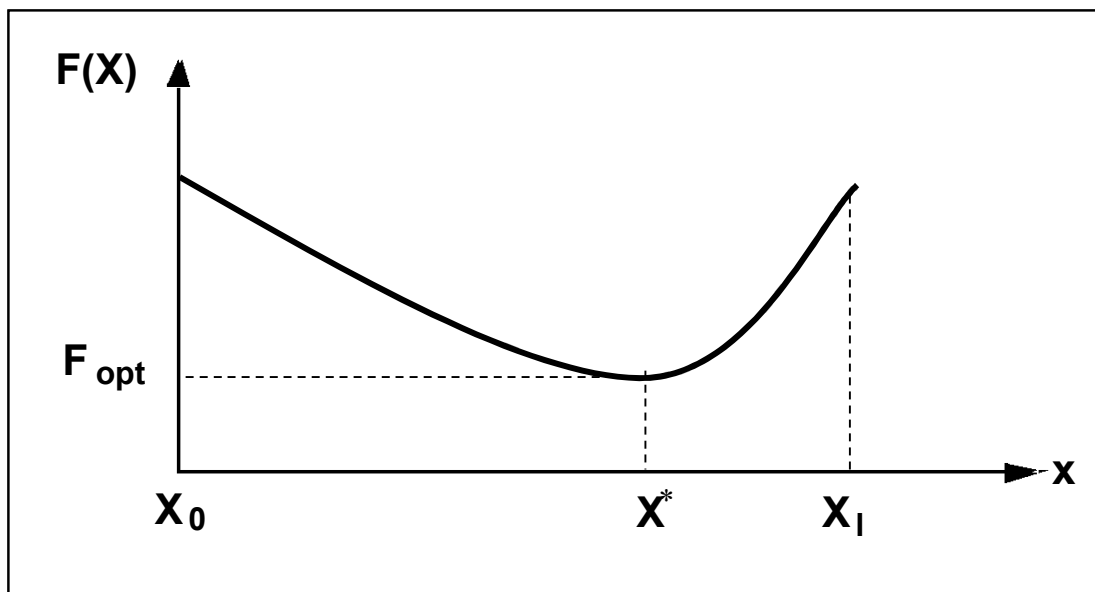
$$w_{\min} - w \leq 0 \quad w - w_{\max} \leq 0$$

$$x_{\min} - x \leq 0 \quad x - x_{\max} \leq 0$$

$$r_{\min} - r \leq 0 \quad r - r_{\max} \leq 0$$

$$t_{\min} - t \leq 0 \quad t - t_{\max} \leq 0$$

# Background and History of Structural Optimization

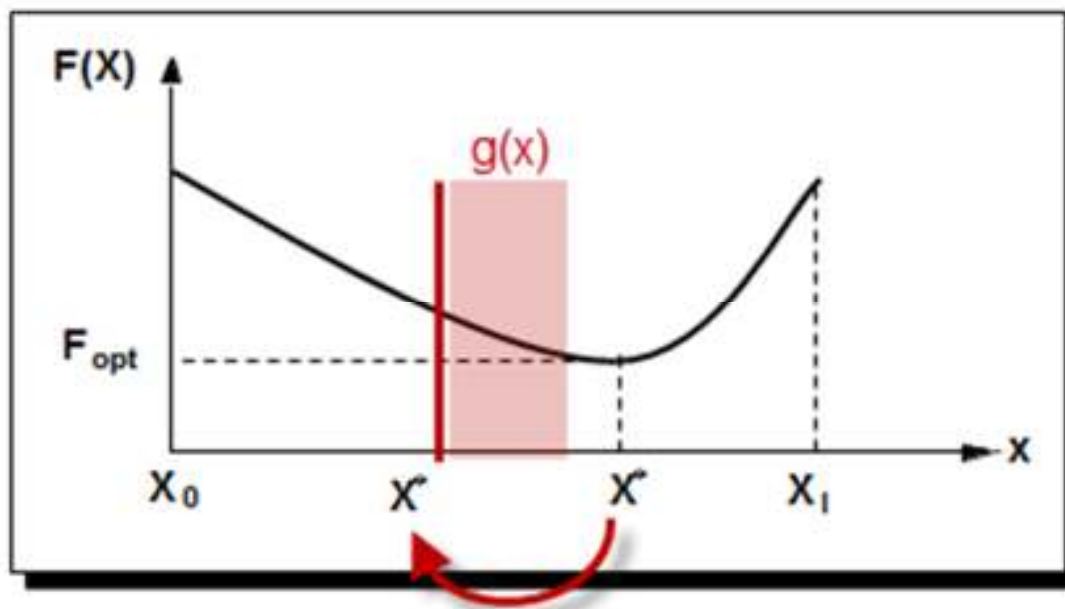


We can imagine the search for the minimum if we consider a 1d search

There is one design variable,  $x$  and an objective function  $f(x)$

We can find the unconstrained optimum  $x^*$  with a numerical search

## Background and History of Structural Optimization



Now we have put in an upper constraint  $g(x)$  and forced the optimum to sit against the constraint 'barrier'

This is now a constrained optimization problem



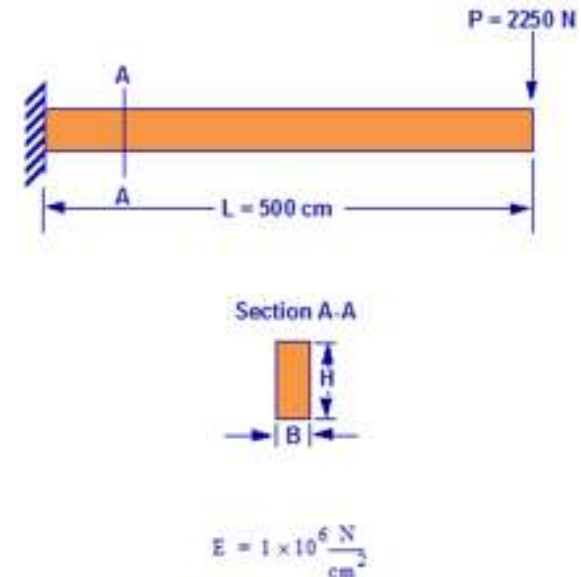
# Background and History of Structural Optimization

In practice we will not have a one dimensional ( one design variable) problem,

There may be hundreds or thousands of design variables in a real problem

For simplicity, imagine a problem with two design variables:

Minimize the weight of a beam subject to a cantilever loading



# Background and History of Structural Optimization

- Minimize  $V = B \cdot H \cdot L$
- Subject to:

$$\delta = \frac{PL^3}{3EI} \leq 2.54 \quad \text{Tip Deflection}$$

$$\sigma = \frac{Mc}{I} \leq 700 \quad \text{Bending Stress}$$

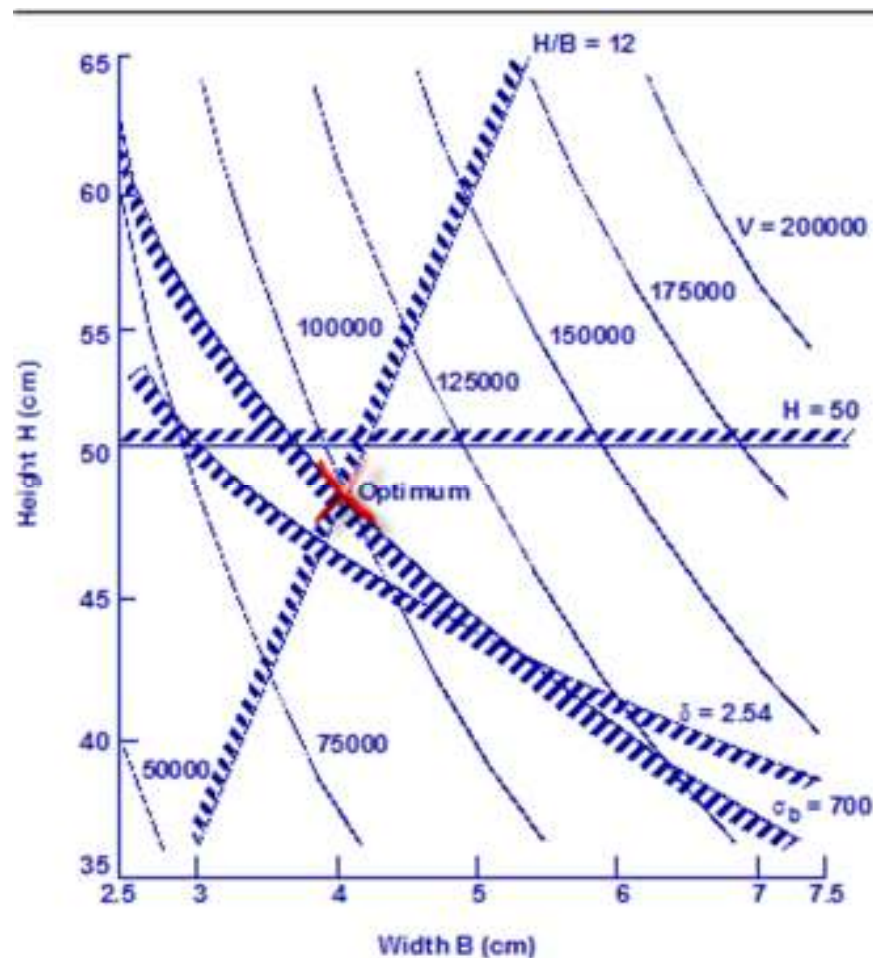
$$\frac{H}{B} \leq 12 \quad \text{Aspect Ratio}$$

$$\left. \begin{array}{l} 1 \leq B \leq 20 \\ 20 \leq H \leq 50 \end{array} \right\} \quad \text{Gauge Requirements}$$

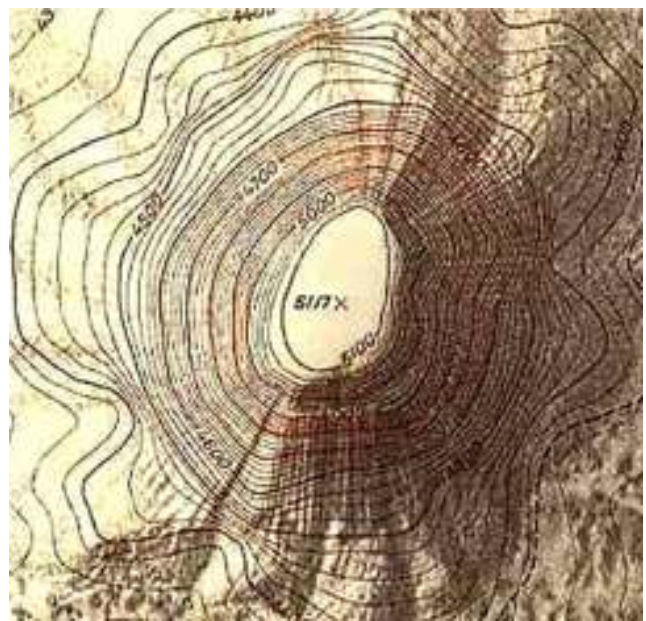
# Background and History of Structural Optimization

The design space in 2d

- Variables H and B
- Objective function contours
- Constraint Boundaries



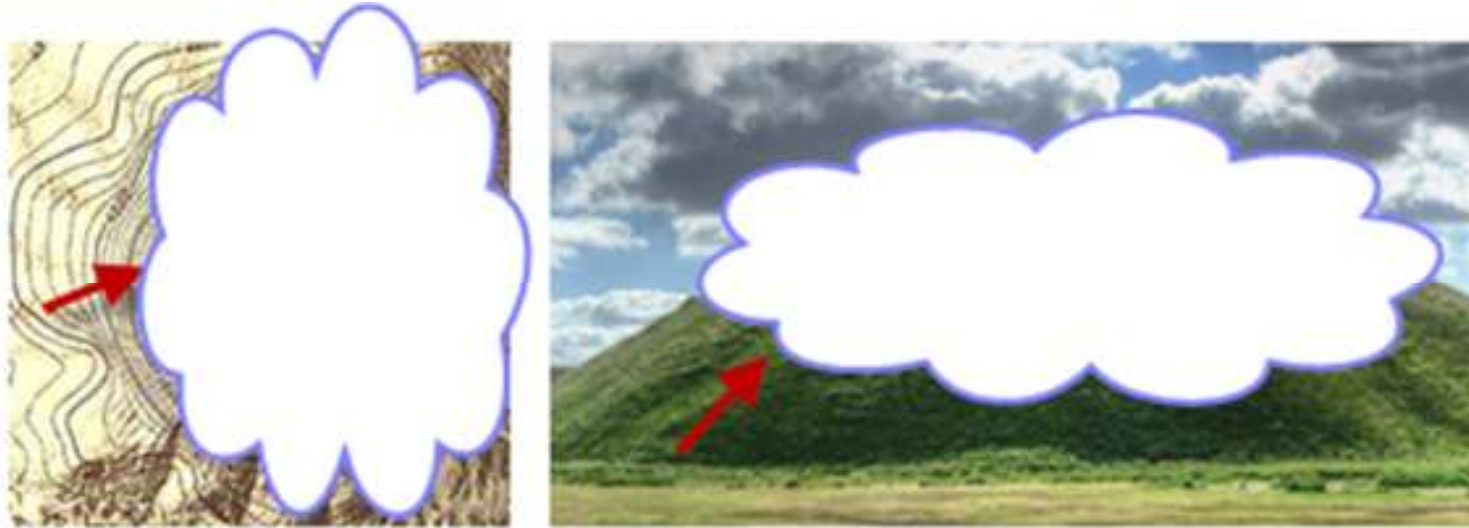
# Background and History of Structural Optimization



We can think of the objective function as a hill, with a corresponding contour map

For the moment assume we want to **maximize – to climb!**

# Background and History of Structural Optimization

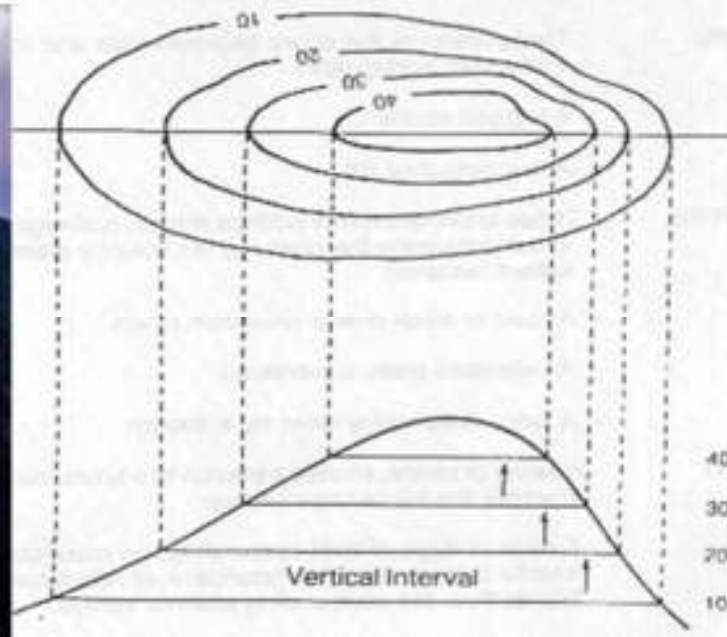


Imagine it is a foggy day and you can't see the top

You will rely on the contours on the map, or the slope of the ground in front of you

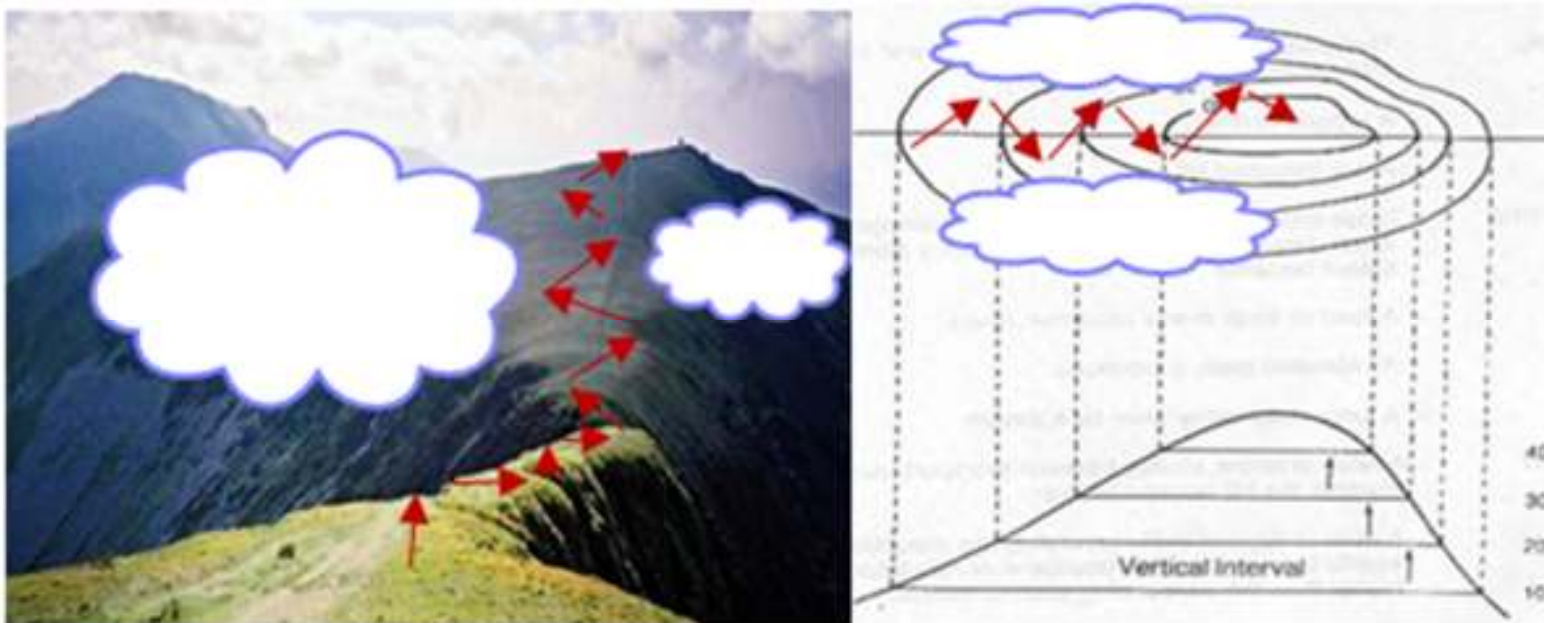
One approach ( very tiring) would be to go for **steepest ascent**

# Background and History of Structural Optimization



That's perfect on a good day when we have lots of information!

# Background and History of Structural Optimization



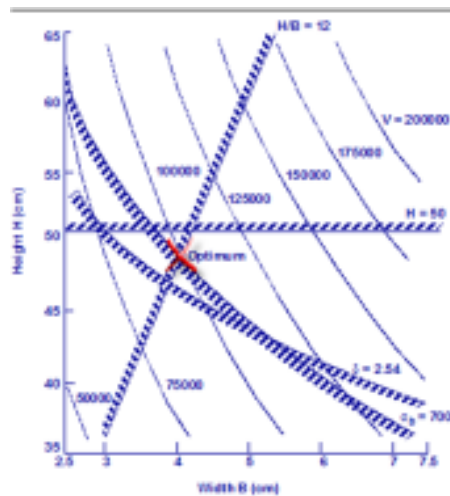
A bad day means little information based on local gradients!

***No GPS allowed here***

# Background and History of Structural Optimization

There are many methods based on this GRADIENT APPROACH

Many years of numerical research have found efficient ways of climbing in the fog



Even with constrained optimization problems!



# Background and History of Structural Optimization

Two huge breakthroughs with Gradient Methods:

Number 1:

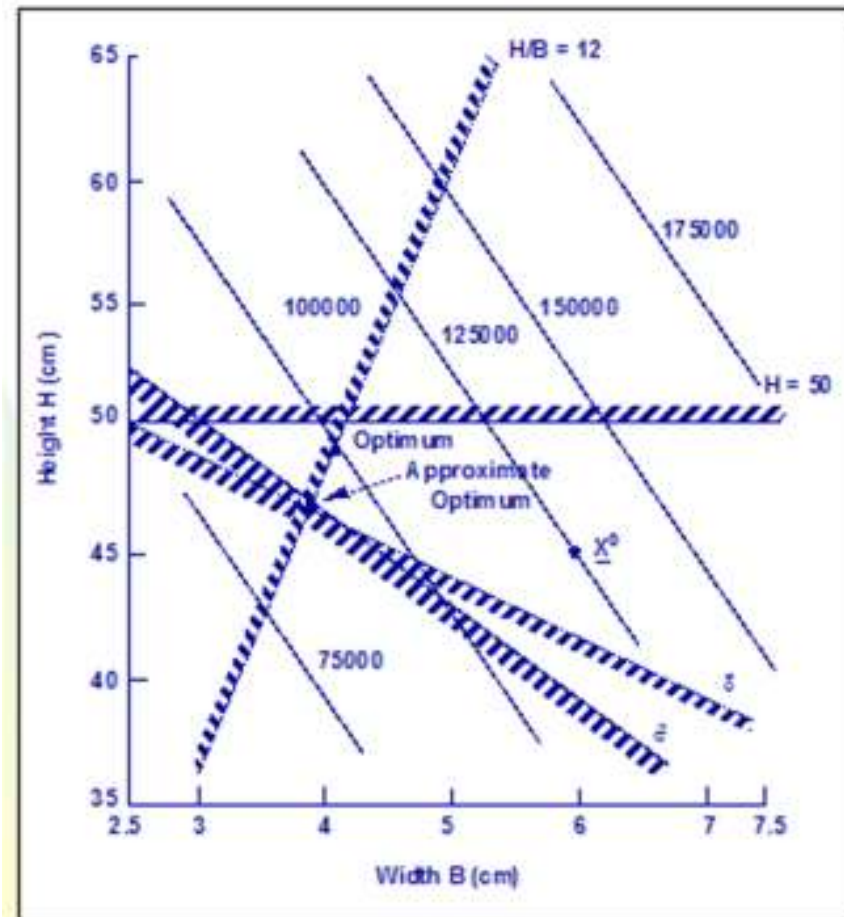
Linearize the design space

You aren't going to walk too far – *approximate it!*

This makes function evaluations easy

Fast and efficient numerically

Pop up from time to time to see how the real world is doing



# Background and History of Structural Optimization

**Two huge breakthroughs with Gradient Methods:**

**Number 2:**

Design Sensitivity Analysis

Evaluate the gradients of the objective function ( the hill) with respect to the design variables

Evaluate the gradients of the constraints (the walls) with respect to the design variables

This is done analytically INSIDE an FE solver

It means we don't have to do a pair of FE analyses around a design point, for every gradient we want to find. That used to be very expensive.

Typically looking for 20 -30 analyses maximum

# Background and History of Structural Optimization

Big drawback of the Gradient Methods:



We don't know for sure if we are at a local or global maxima/minima – without a good search for them all

# Background and History of Structural Optimization

## Alternatives to the Gradient Methods:

The alternatives to the gradient method use some type of response surface fit to a large quantity of data.

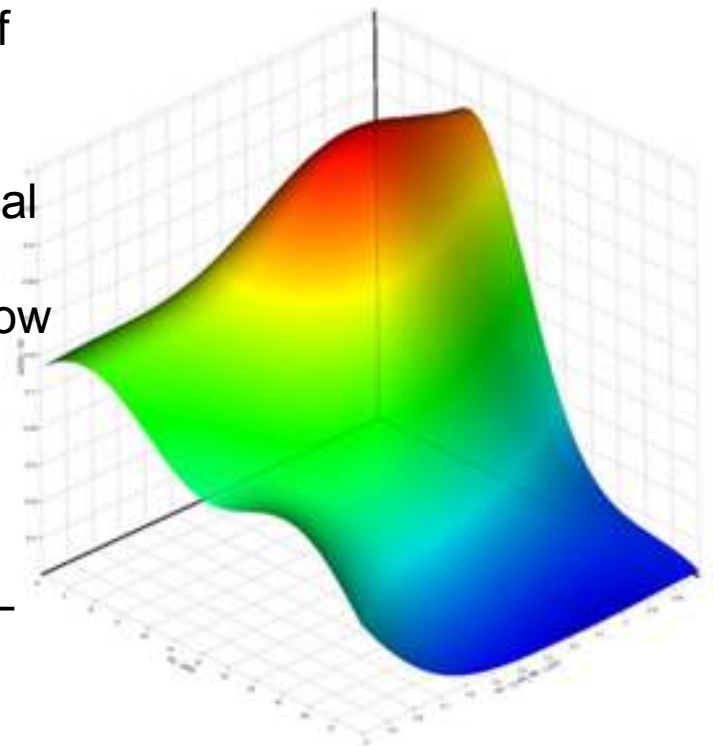
A Design of Experiments method would be typical

There is more global information available to allow searching for the optimum

Finding Global optima is now more likely

However the cost in analyses can be very high – order of 1000

Strategies are used to help predict where to search most effectively



# Background and History of Structural Optimization



## Class Topics taking this further

- Optimality conditions
- Convex functions
- Conditioning methods
- Case Studies and practical examples
- Alternative methods such as GE, DOE, AI

# Introductory Optimization FE Analysis Webinar

## Agenda

- Background and History of Structural Optimization
- **Putting Optimization in perspective – what are we trying to achieve?**
- Sizing, Shape and Topology – the three fundamentals
- Multi-Disciplinary Optimization (MDO) – trying to put it all together
- Nonlinear Optimization – altered perspectives!

## Putting Optimization in perspective

### *what are we trying to achieve?*

This section is used to illustrate some of the points raised throughout the E-Learning course

The Optimization Objective Function has to be defined as part of the problem statement

However the wider question of the ANALYSIS objective should not be ignored

- Why are we doing this analysis
- What do we know about the physics of the situation
- What level of structural sophistication should we be aiming for (nonlinear, multi-physics, high fidelity mesh, full assembly) or (simple assessment to bound the problem)
- What is the timescale, computing resource, tool set, skill level
- Etc.!

## Putting Optimization in perspective

### *what are we trying to achieve?*

The Optimization problem must be well posed to avoid problems

- The loading environment must be fully defined, missing load cases will almost certainly result in structure designed to fail
- The boundary conditions, mesh quality, material characteristics etc. must be the best available – we are honing the design armed only with this information
- The whole approach to optimization should be about a robust practical solution, not eking out the ‘best’ solution to within a fraction of a percent
- Innovation is required in optimization – an objective function may be best considered as a constraint and a constraint can be turned into a goal seeking objective!
- Very often design space is flat, with few stimulations to get the structural response moving – new ideas are then required



# Putting Optimization in perspective



## Class Topics taking this further

- Hints and tips on optimization
- Case studies
- Innovative synthetic responses and objectives

# Introductory Optimization FE Analysis Webinar

## Agenda

- Background and History of Structural Optimization
- Putting Optimization in perspective – what are we trying to achieve?
- **Sizing, Shape and Topology – the three fundamentals**
- Internal solver solutions or external drivers – pros and cons
- Multi-Disciplinary Optimization (MDO) – trying to put it all together
- Nonlinear Optimization – altered perspectives!
- Robust Optimization – broadening the spike of a single point solution!

# Sizing, Shape and Topology

## Sizing Optimization

In this class of problem physical properties of the elements are used as design variables

At its simplest level this means

- Areas of Rods
- Dimensions or Cross sectional properties of Bars and Beams
- Thickness of plates
- Offsets of plates, bars, beams

Design variables can however become more sophisticated by linking individual element properties together

# Sizing, Shape and Topology

## Sizing Optimization

### Simple Example: 3 bar truss

#### Analysis Model Description

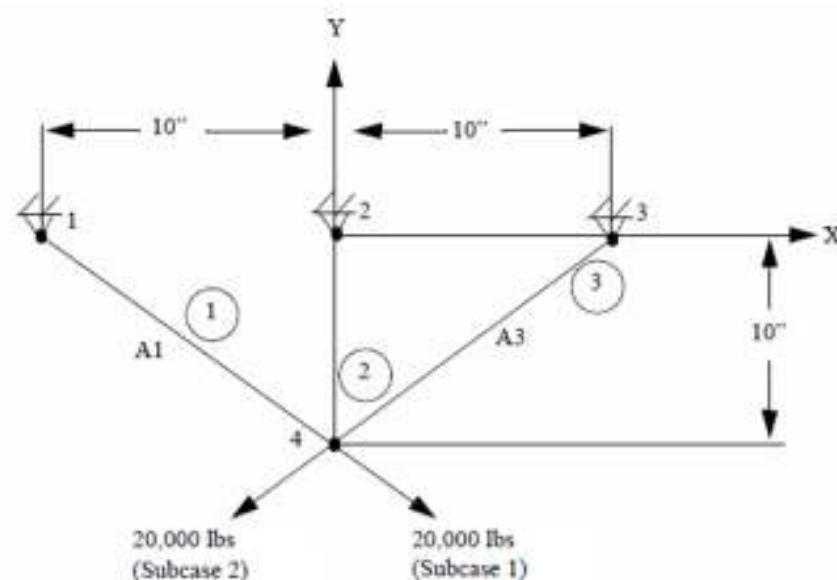
Three rod (pin-jointed) structure in the x-y plane  
 Symmetric structural configuration with respect to the y axis  
 Two distinct 20,000 lb load conditions

Material:  $E = 1.0E7$  psi  
 Weight density =  $0.1 \text{ lbs/in}^3$

#### Design Model Description

Objective: Minimization of structural weight  
 Design variable: Cross-sectional areas  $A_1$ ,  $A_2$ , and  $A_3$   
 Design variable linking such that  $A_3 = A_1$

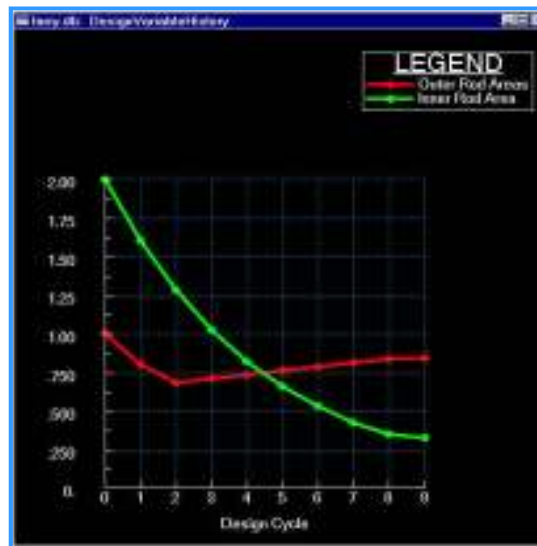
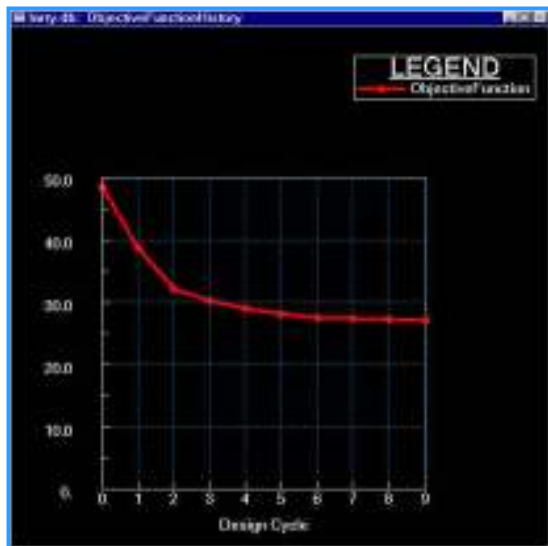
Constraints: Allowable stress: Tensile 20,000 psi  
 Compressive = -15,000 psi  
 Displacement:  $\pm 0.2$  at grid 4 in x end of y directions



# Sizing, Shape and Topology

## Sizing Optimization

Simple Example: 3 bar truss



# Sizing, Shape and Topology

## Sizing Optimization

Linked example with Multi-disciplinary optimization: plate

$$\begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_8 \end{Bmatrix} = \alpha_1 \begin{Bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ \vdots \\ 1.0 \end{Bmatrix} + \alpha_2 \begin{Bmatrix} 1.0 \\ 0.875 \\ 0.75 \\ \vdots \\ 0.125 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} 1.0 \\ 0.7656 \\ 0.5625 \\ \vdots \\ 0.0156 \end{Bmatrix}$$

### Analysis Model Description

2 x 8 array of CQUAD4 elements

Material:

E = 1.0E+6 psi

Poisson ratio = 0.33

Weight density = 0.1/in<sup>3</sup>

Two static load conditions:

Tip loading:

Two 500 lb loads in -z direction

Pressure loading:

Uniform at 7.5 lb/in<sup>2</sup>

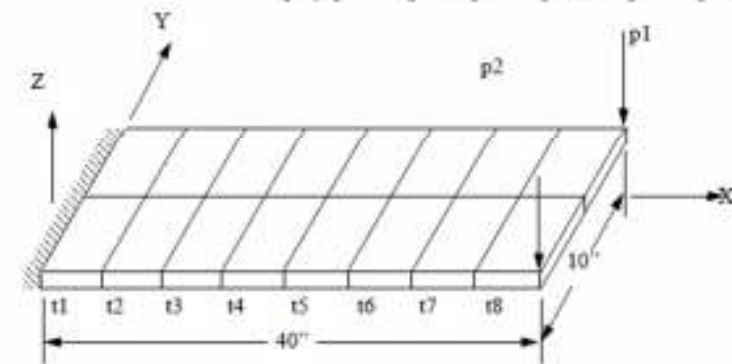
One normal modes analysis

One modal frequency analysis

Two 50 lb loads in z-direction

Tip loading:

Structural damping of 10%



### Design Model Description

Objective:

Structural weight minimization

Design variables:

Basis functions:

Constant, linear and quadratic in x direction to describe plate element thicknesses

Constraints:

ANALYSIS = STATICS

Tip Displacement ≤ 2.0

von Mises Stress ≤ 29ksi

ANALYSIS = MODES

First natural frequency ≥ 1.5Hz

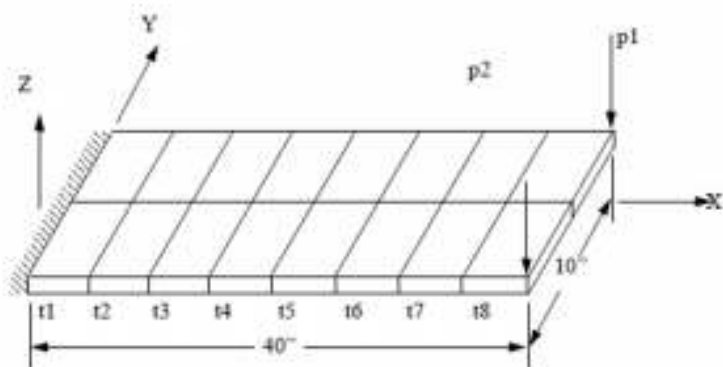
ANALYSIS = MFREQ

Magnitude of tip displacement ≤ 2.0

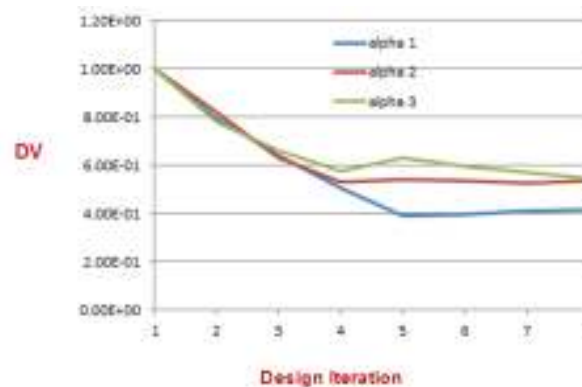
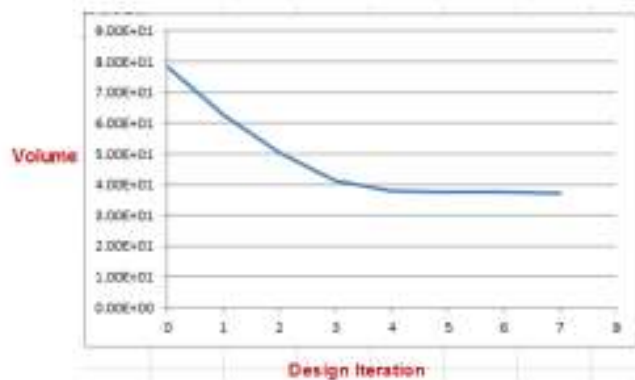
# Sizing, Shape and Topology

## Sizing Optimization

Linked example with Multi-disciplinary optimization: plate



$$\begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_8 \end{Bmatrix} = \alpha_1 \begin{Bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ \vdots \\ 1.0 \end{Bmatrix} + \alpha_2 \begin{Bmatrix} 1.0 \\ 0.875 \\ 0.75 \\ \vdots \\ 0.125 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} 1.0 \\ 0.7656 \\ 0.5625 \\ \vdots \\ 0.0156 \end{Bmatrix}$$



# Sizing, Shape and Topology

## Shape Optimization

In this class of problem the 2d or 3d element mesh is distorted

The mesh keeps the same topology, i.e. element connectivity

Various strategies are available to distort the mesh

- Direct definition of nodal vectors
- Use of basis vectors from deformed shapes (auxiliary models)
- Parametric definition of parent geometry
- Growth functions via homogeneous stress distribution and mass reduction targets

Design variables are based on the above list, or in the last case are spawned automatically

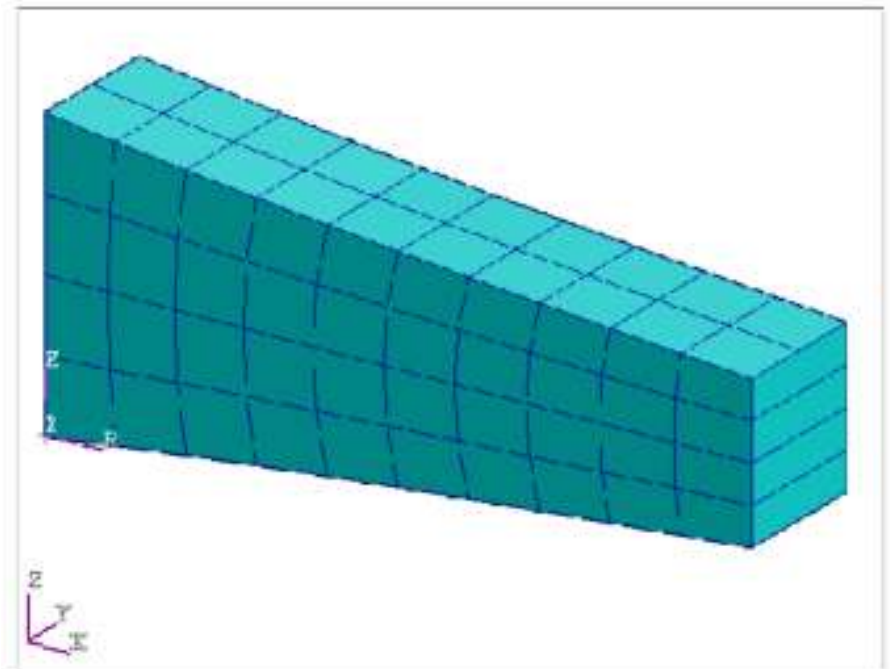
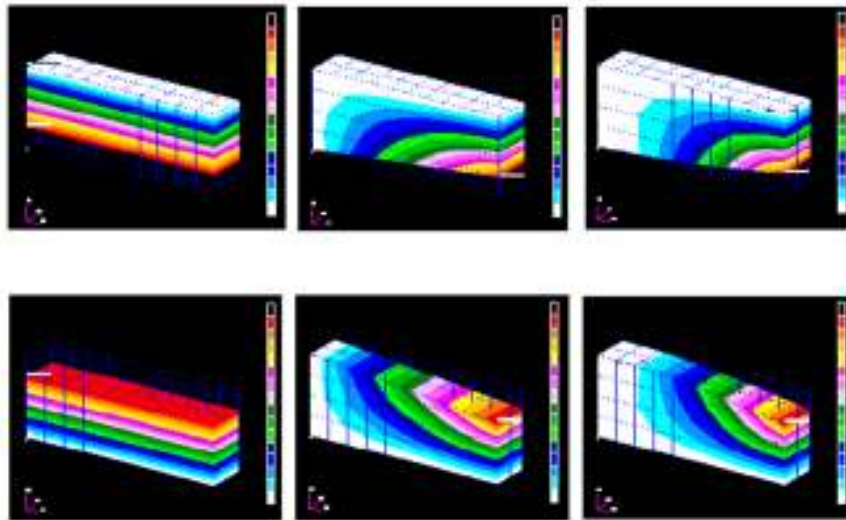


# Sizing, Shape and Topology

## Shape Optimization

Cantilever beam weight minimization  
subject to stress and deflection  
constraints

### Shape basis Vectors

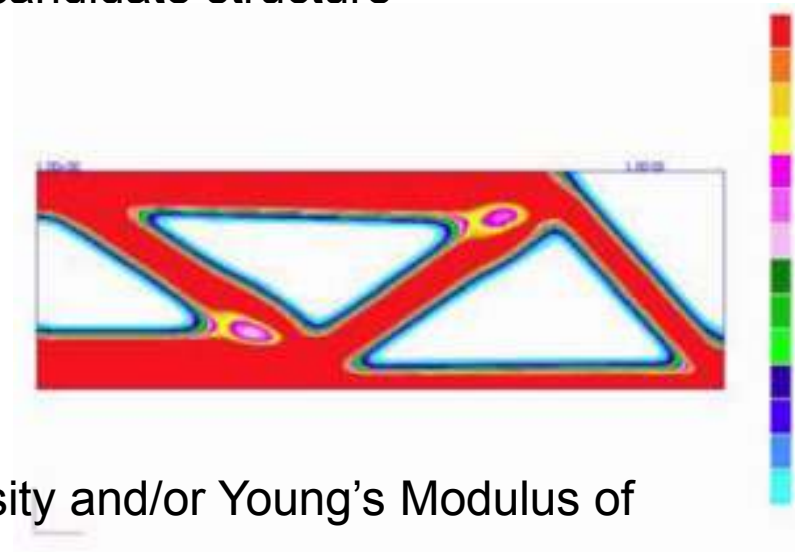
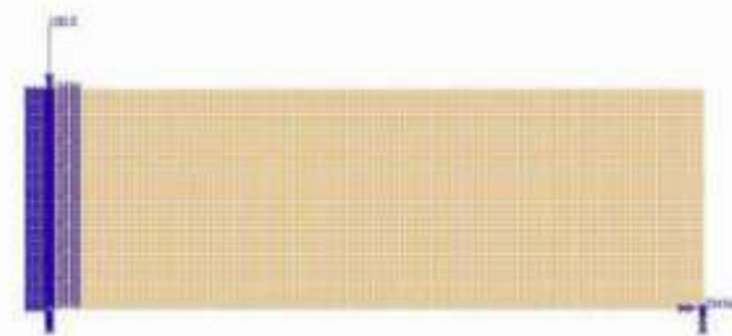


# Sizing, Shape and Topology

## Topology Optimization

In general, keeps the same mesh ( i.e. same Topology)

Whittles away the material to leave a candidate structure



All elements are still present, but density and/or Young's Modulus of 'discarded' elements is driven down

Stress patterns show remaining viable load paths and structure

# Sizing, Shape and Topology

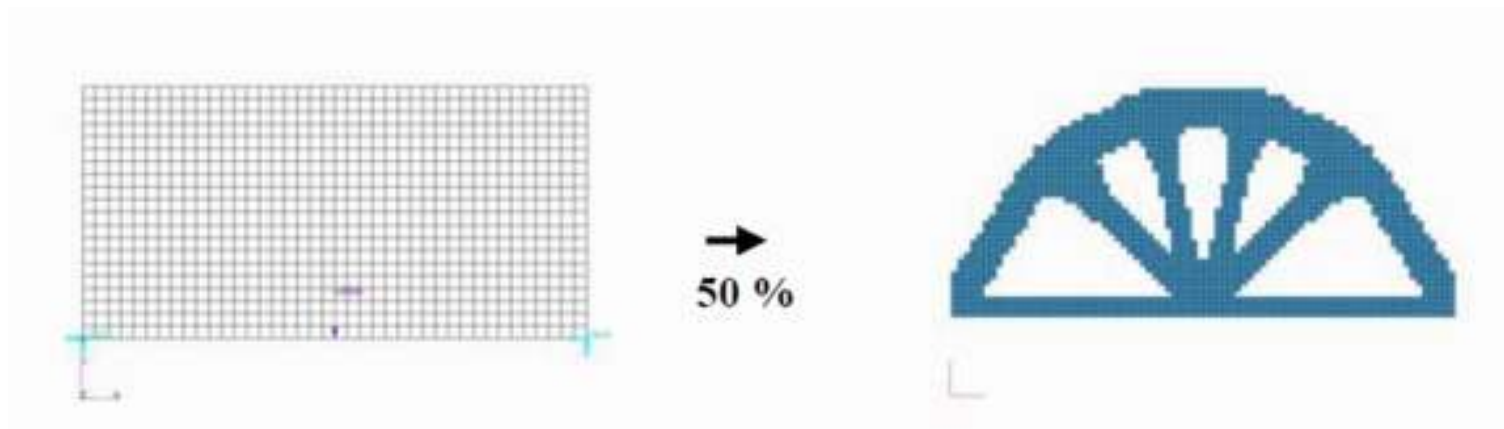
## Topology Optimization

A design space is defined

A target mass or volume reduction is defined as a constraint

Load and structural constraint positions are deemed frozen

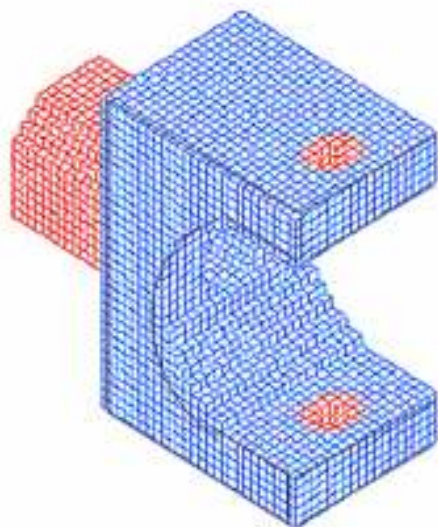
Other zones can be frozen or biased



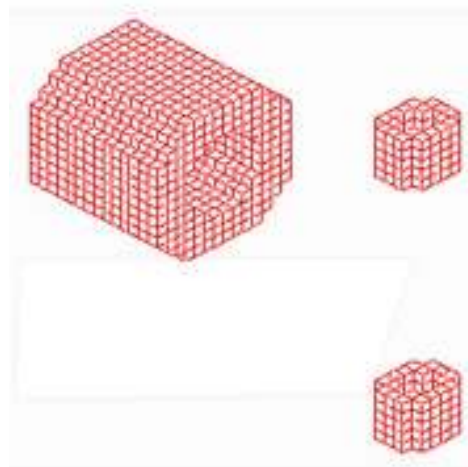
# Sizing, Shape and Topology

## Topology Optimization

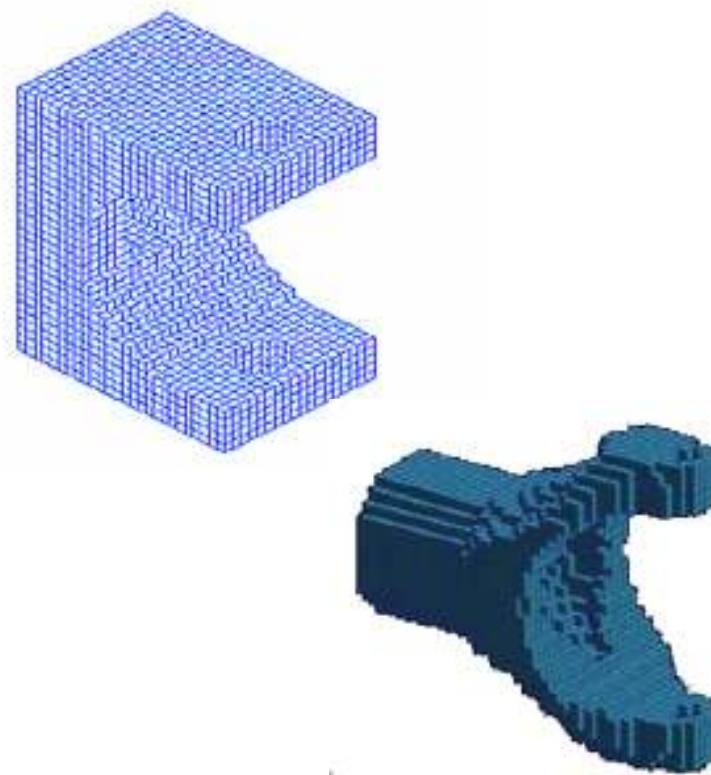
Total design space



Frozen design space



Variable design space



# Sizing, Shape and Topology

## Topology Optimization

The actual objective function depends on the Topology Optimizer formulation

These include

- Maximizing homogeneousness (smoothest possible stress gradients)
- Minimizing Compliance (minimum energy solution)

Each element spawns an internal design variable, this will differ based on the formulation used, large numbers of design variables used

- Homogeneousness - stress functions
- Compliance – element density

# Sizing, Shape and Topology

## Topology Optimization

To assess load paths and innovative design concepts, target mass reduction can be used in isolation

***What would it look like if we reduced 80%, 60%, 40% 20% volume!***

***Results may be very different configurations..***

To design production structures, formal constraints on strength and stiffness can be applied. Target weight reductions are defined, but need to be more tuned; to realistic goals

***I set my mass target to 5%, but it breaks every time!***

Various volume constraints



VOLUME = 0.7



VOLUME = 0.5



VOLUME = 0.3

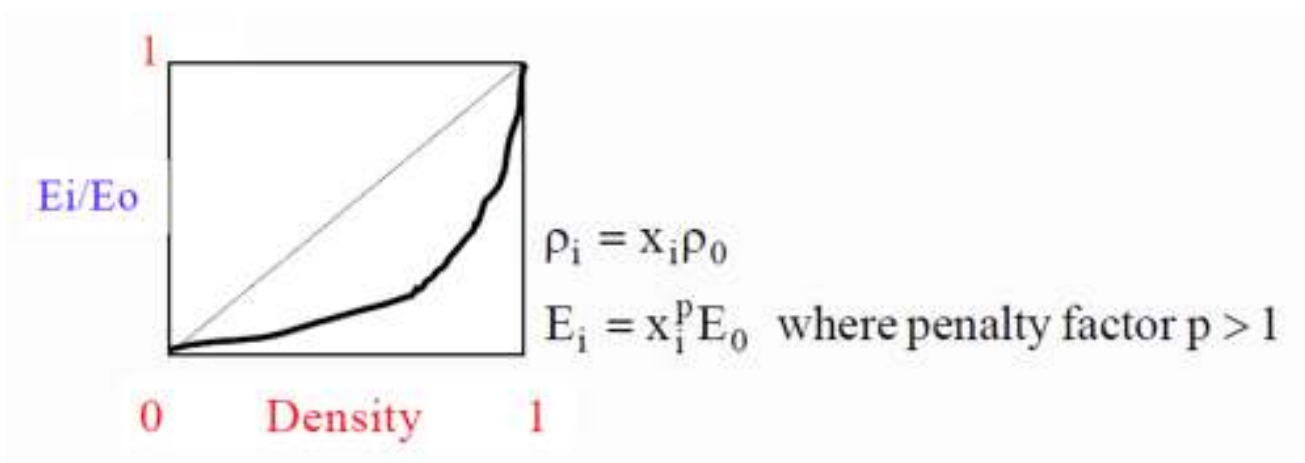


VOLUME = 0.1

# Sizing, Shape and Topology

## Topology Optimization

How do we get designs that show such distinct banding of load paths, rather than patches of fuzzy 'soft' structure?



One approach is to put a penalty bias to favor Design Variables close to 0.0 or 1.0

# Sizing, Shape and Topology

## Topology Optimization

Gotcha's (1)



As the mesh size is refined, then candidate designs can get very high fidelity:



4,800 Elements



120,000 Elements

The structure is trying to achieve a classical 'Michell' or Geodesic type form  
The solution is to define a minimum structural member size

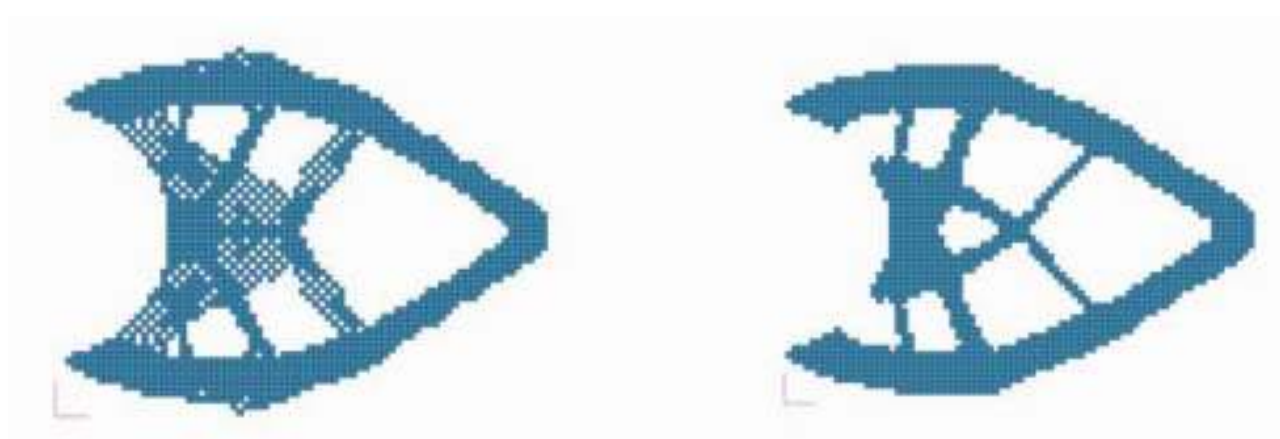


# Sizing, Shape and Topology

## Topology Optimization

Gotcha's (2)

Adjacent Elements may flip/flop between on or off in the design solution:  
Sometimes this is known as checker boarding



The structure on the left is well defined, but full of voids. Using various filtering strategies, the structure on the right can be achieved

# Sizing, Shape and Topology

## Topology Optimization

Gotcha's (3)

The 'lego brick' result of the topology optimizer must be converted to meaningful geometry

Raw result



CAD ready transformation



***This is the real future of topology optimization – allowing easy migration into the parametric CAD world and subsequent sizing and shape optimization***

# Sizing, Shape and Topology



## Class Topics taking this further

- Deeper dive into theories behind Topology methods
- Case Studies
- Statics, Buckling and Dynamics solutions

# Introductory Optimization FE Analysis Webinar

## Agenda

- Background and History of Structural Optimization
- Putting Optimization in perspective – what are we trying to achieve?
- Sizing, Shape and Topology – the three fundamentals
- **Multi-Disciplinary Optimization (MDO) – trying to put it all together**
- Nonlinear Optimization – altered perspectives!

## Multi-Disciplinary Optimization (MDO)

**Multi-disciplinary optimization** (MDO) attempts to solve the optimization problem across a wide range of analysis disciplines.

The key objective in MDO is to include all of the disparate analysis disciplines at the same time, rather than solving sequentially.

Typical disciplines in aerospace include Aerodynamics, Structures, Aeroelasticity, Performance, Cost Analysis etc.

If each discipline is optimized sequentially, and then an optimum derived from these solutions it is well known that a suboptimal solution will be found.

It is very difficult and very expensive to truly combine all analysis disciplines under one optimization solution. Some disciplines have good synergy such as aeroelasticity and structures if the wing aerodynamic shape is fixed.

Extensive research is underway in both analytical methods and process development to achieve MDO

## Multi-Disciplinary Optimization (MDO)

Many MDO problems are truly Multi-objective, meaning that there competing objective functions.

In an aircraft design the conflicts of performance, strength, aerodynamics, aeroelastics, costs etc. all provide a multi-objective optimization problem.

A multi-objective problem will have many possible solutions. The best solution is where the best trade-off between objectives is found.

Identifying the best solution, and the sensitivity of that solution to each of the multi-objectives is the goal of this type of optimization.

One approach to the problem is to apply weighting factors to each of the objectives and to hence form a combined objective. This however means that heuristic decisions are made a priori which can strongly influence the optimum

A more rigorous mathematical approach is to establish a Pareto Frontier

# Multi-Disciplinary Optimization (MDO)

## SSC-454 ULTIMATE STRENGTH AND OPTIMIZATION OF ALUMINUM EXTRUSIONS Report SR-1457 2008

The relationship between weight and strength is complex, and there may be many local minima of structural weight that will be encountered before a truly global minima of weight for a given strength level is found.

This type of problem is typically difficult to address with optimization techniques that use derivatives of the objective function to search for minima.

When investigating the trade-offs between weight and structural strength, the problem is further complicated because the result will no longer be a single minima but, rather, a Pareto set consisting of designs where the strength can no longer be improved without a corresponding increase in weight.

When plotted on an axis of structural strength vs. weight, this Pareto set will form a Pareto frontier, or a curve connecting designs that represent the maximum strength obtainable for a given weight.

# Multi-Disciplinary Optimization (MDO)



(a) Extruded stiffener construction

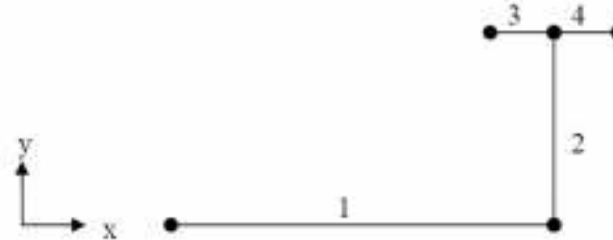


Figure 44: Optimization Variables for Extruded Stiffener Construction

Table 7: Design Variables for the Extruded Stiffener Panel

Design Variable	Lower Bound	Upper Bound
Plate thickness (plate 1)	2mm	14mm
Web thickness (plate 2)	2mm	14mm
Web height (plate 2)	20mm	150mm
Flange thickness (plate 3&4)	2mm	14mm
Flange width (plate 3&4)	10mm	100mm
Number of stiffeners	1	22



# Multi-Disciplinary Optimization (MDO)

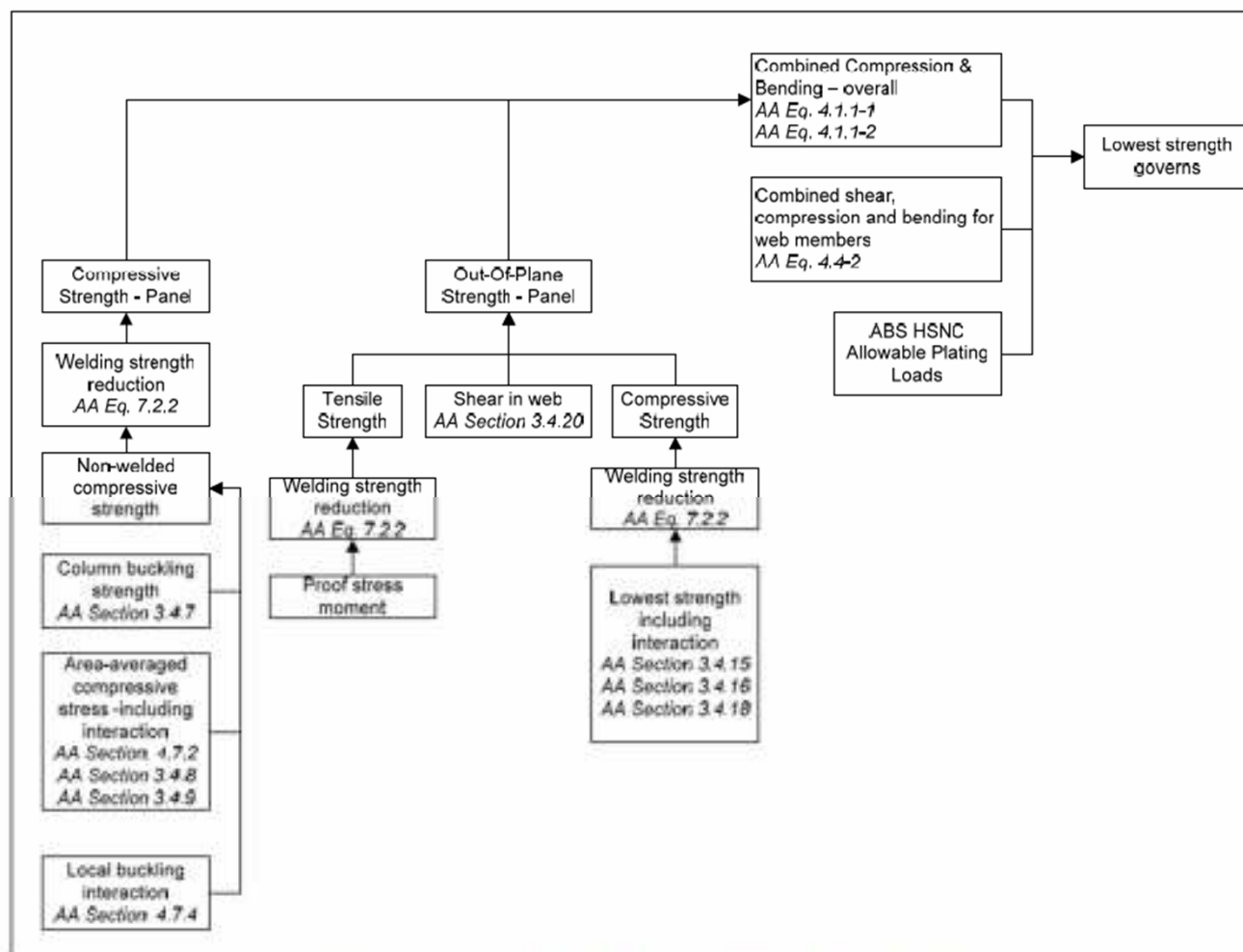


Figure 47: Strength Calculation for Optimizer

# Multi-Disciplinary Optimization (MDO)

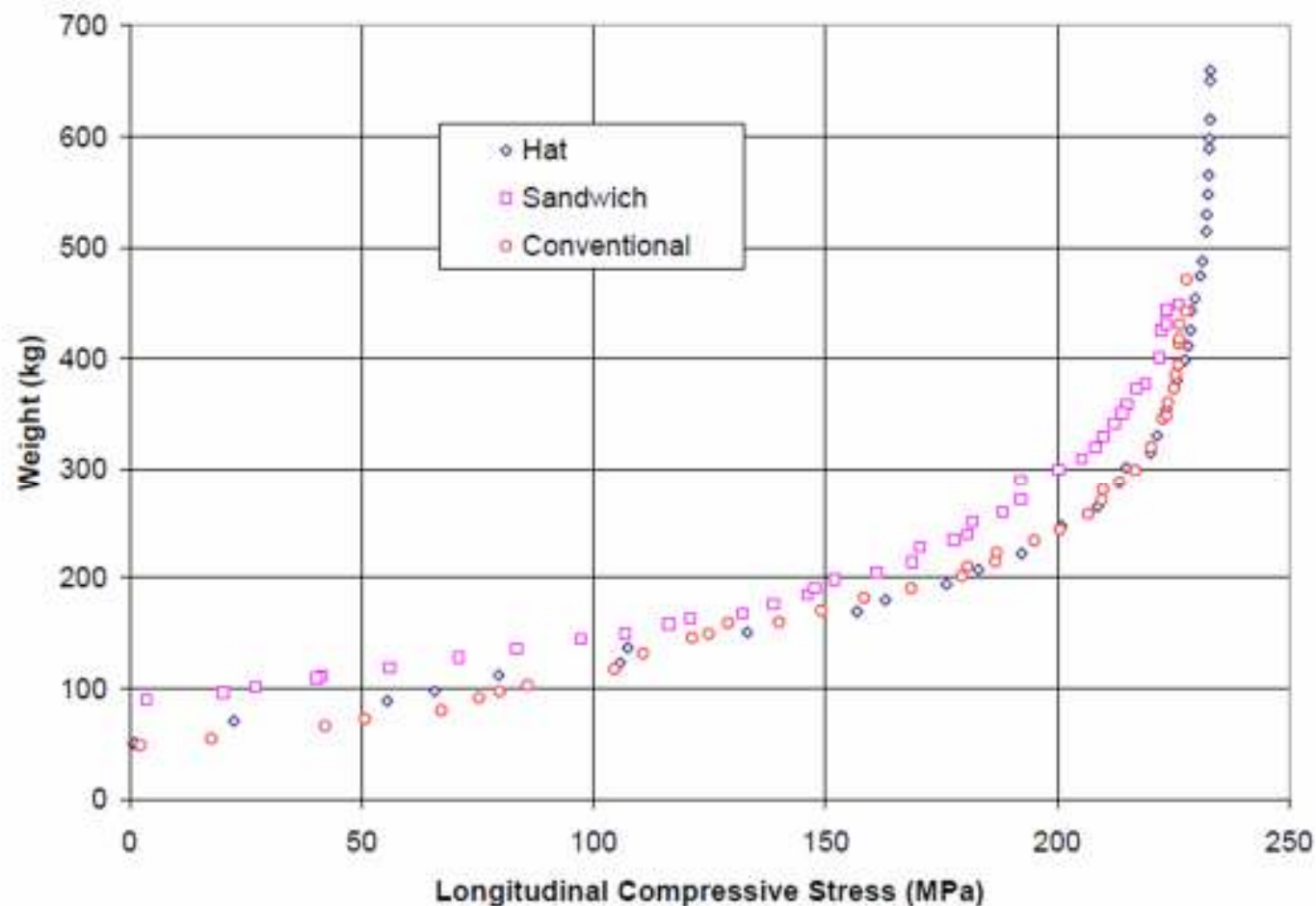


Figure 49: Pareto Fronts – Tier 3 Strength Deck, Panel Length = 2,400mm

# Multi-Disciplinary Optimization (MDO)



## Class Topics taking this further

- Case Studies
- Objective linking and weighting

# Introductory Optimization FE Analysis Webinar

## Agenda

- Background and History of Structural Optimization
- Putting Optimization in perspective – what are we trying to achieve?
- Sizing, Shape and Topology – the three fundamentals
- Multi-Disciplinary Optimization (MDO) – trying to put it all together
- **Nonlinear Optimization – altered perspectives!**

# Nonlinear Optimization

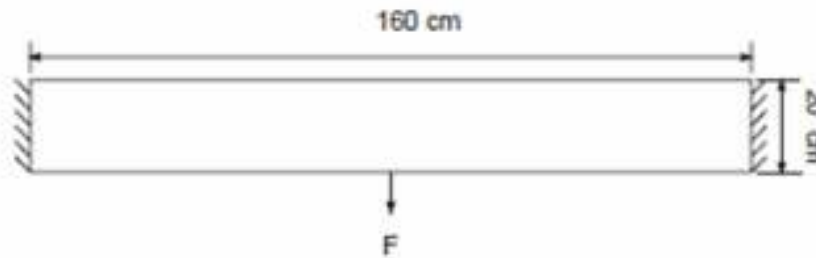
Most Optimization work to date has focused on structurally linear problems

- In the real world many solutions rapidly become nonlinear
- The use of nonlinear optimization is very limited outside of DOE and other 'external' driving methods to an FE solution
- Nonlinear analysis is by definition expensive in CPU time and data output – multiple solution space only adds to this!

A very brief overview of some nonlinear results is given

# Nonlinear Optimization

## Cantilever Beam



Design domain for long slender beam.



Typical 'Michell' type linear solution



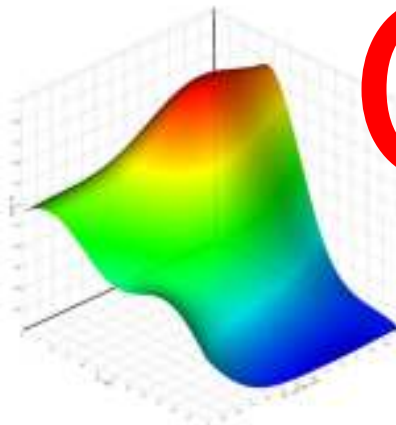
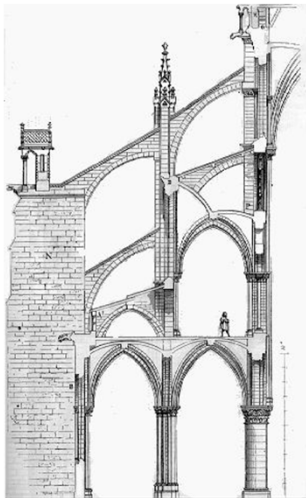
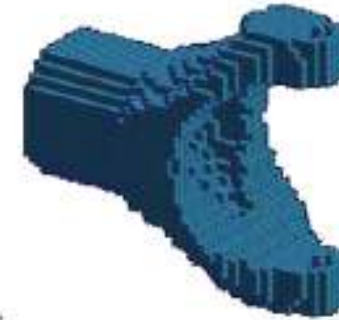
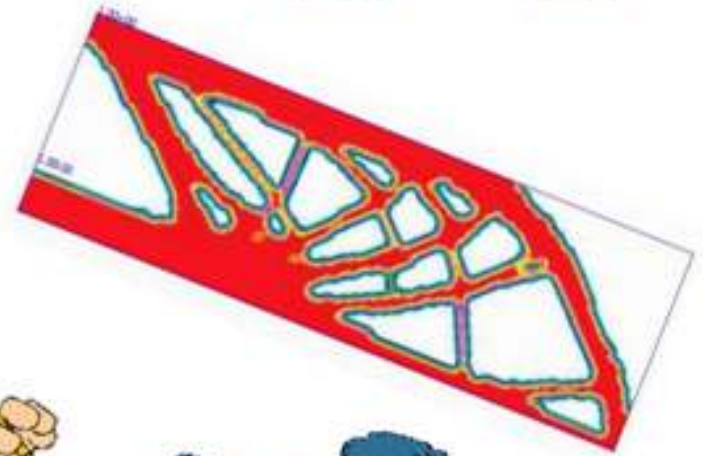
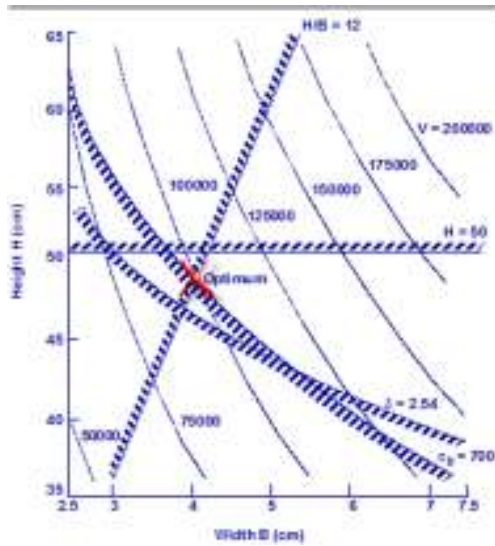
Continuous beam type solution, biased to nonlinear load path

# Nonlinear Optimization



## Class Topics taking this further

- Review of some current methods
- Application to crash type loading



# Q and A

[www.nafems.org](http://www.nafems.org)

[www.fetraining.com](http://www.fetraining.com)