Strength analytical methods revisited, Focus on plastic bending, Correlation between tests, FEM and legacy methods

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Abstract

Historically the plastic bending was analyzed using the integration of the non linear stresses on the critical cross section. The stress-strain curve is usually simplified using the Ramberg-Osgood method (Reference [1]) or in this paper ESDU 73016 (Reference [2])

Today, due to modeling democratization, a lot of structures are analyzed using detailed finite element models. This method can have good results in accurately predicting stresses in the linear domain but it can lead to inaccuracies if the wrong material representation or element type is used.
Furthermore a special attention should also be given to manufacturing tolerances that also impact the strength of the parts.

This paper attempts to address these issues by comparing two cases where DFEM results, the tests, and the analytical methods were compared.

The first case represents a double shear lug joint. A detailed 3D finite element model was created in order to analyze a force required to close a gap between female and male lugs due to nut clamping. These results were compared with a test and an analytical method, reference [1]. The detailed finite element model and the analytical method show a good correlation, but the test results vary, due to manufacturing tolerances and the inability to accurately measure preload at the lugs. This case shows a correct modeling technique in respect to element type and material, but it also shows that an analytical method is accurate as well. A further improvement could be made by taking into account the manufacturing tolerances and having a better instrumentation for the preloads.

The second case represents a beam with a sideways loading. The beam was initially modeled with 2D elements, but this modeling was too conservative as the capacity predicted by the test was higher. Possible reasons for this discrepancy include inability of the 2D FEM to predict the ultimate capacity of the material especially in the web to flange radius area. 3D modeling showed good test correlation. The results from the test and from 3D modeling were used to develop an analytical method for this analysis that can be applied to similar cases.

1. Lug clamping using an analytical method, correlated with non linear DFEM and test

Lug clamping due to nut torquing in a single pin joint is normally avoided by having a design solution with straight bush or shoulder bolt. In this way the nut is clamped against the bolt or bush and the lugs are not clamped. The benefit of this design is that it prevents residual stresses on the lug that can cause stress corrosion. But in some cases this design can't be avoided (Figure 1) and the calculation of the clamped stresses is required. Two methods will be presented here. An analytical, based on the standard formulas, and using 3D non-linear DFEM.

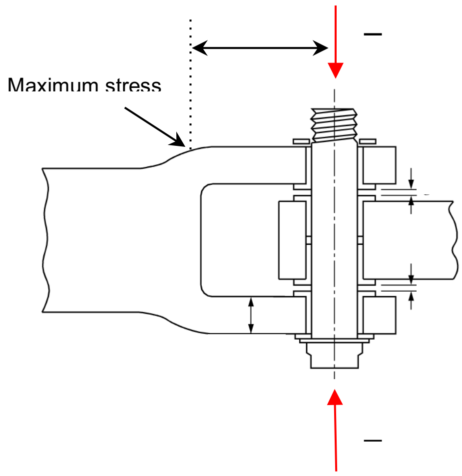

Figure 1: Example of lug clamping

Analytical Method

If the stresses are in the elastic range, the calculation could be done using the standard beam formulas as follows:

$g = \frac{F \cdot L^3}{12 \star EI}$ or $F = \frac{12EI \cdot g}{L^3}$

Assuming the clamped condition of both sides;

$M = \frac{FL}{2}$ or $M = \frac{6EI \cdot g}{L^2}$; $\sigma = \frac{M}{Ix} \cdot \frac{t_f}{2}$ and $I = \frac{b{t_f}^3}{12}$

$$\sigma = \frac{3 \cdot g \cdot E \cdot t_f}{L^2}$$

Where;
g → Gap between female and male lugs
E→Youngs Modulus
I→Moment of Inertia
b →Lug width
L→Lug moment arm
tf→Flange thickness

If the stress level exceeds the yield point the same analysis could be done using the secant modulus, instead of the elastic modulus. In this way the softening of the lug could be assessed due to plastic deformation. Secant modulus is calculated by using the following formula:

$$E_{sec} = \frac{f}{\varepsilon}$$

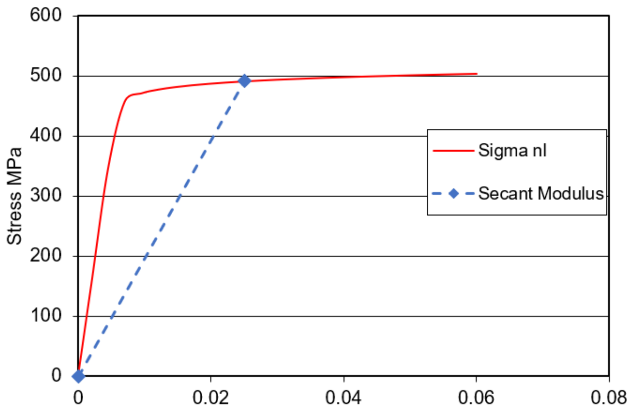

Figure 2: Secant Modulus

As the lug is clamped at both ends then using this boundary condition and the elastic line theory (Figure 3)

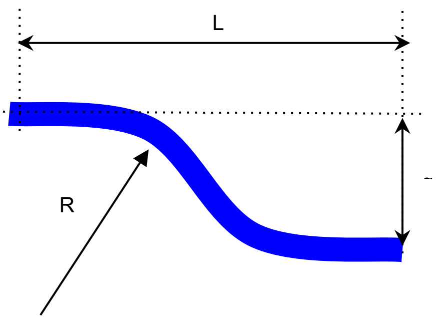

Figure 3: Lug Elastic line

$M = \frac{6EI \cdot g}{L^2}$ and $\frac{1}{R} = \frac{M}{EI} = \frac{6 \cdot g}{L^2}$

The maximum strain at the clamped ends could be calculated by:
$\varepsilon = \frac{t_f}{R}$ the non linear stress is $\sigma = f(\varepsilon)$ while the force required to close the gap is:
$F = \frac{12 \cdot E_{sec} \cdot I \cdot g}{L^3}$

Please note that the strain slope varies along the lug and the maximum value calculated here corresponds to the clamped lug ends.

Analytical method shown here uses the approximation of stress strain curve from ESDU 76016 [2]

$\frac{\varepsilon \cdot E}{fn} = \frac{f}{fn} + \frac{1}{m} \cdot \left(\frac{f}{fn}\right)^m$ where;

fn and m are material constants
f is nonlinear stress
ε is non linear strain

Material constants fn and m can be calculated from σ_y, σ_{ult} , E, and plastic deformation εp

Example:

Material Al 2050 Reference [3]
E=76500MPa

$$\sigma ult = 515MPa$$
$$\sigma y = 457MPa$$
$$\varepsilon p = 0.06$$

Lug Geometry:

tf=5mm → Flange thickness
g=1mm→ Gap
L=24.5mm → Lug length
1/R=0.01/mm → strain slope
$\varepsilon = 0.025$ → maximum strain
$\sigma = 478MPa$ → non linear stress for clamped lug
F=5009N → Force required to close the gap

DFEM Analysis

The same analysis is done using the non linear DFEM.
Both male and female parts were fine meshed as well as the bolt, the bushes and the nut.

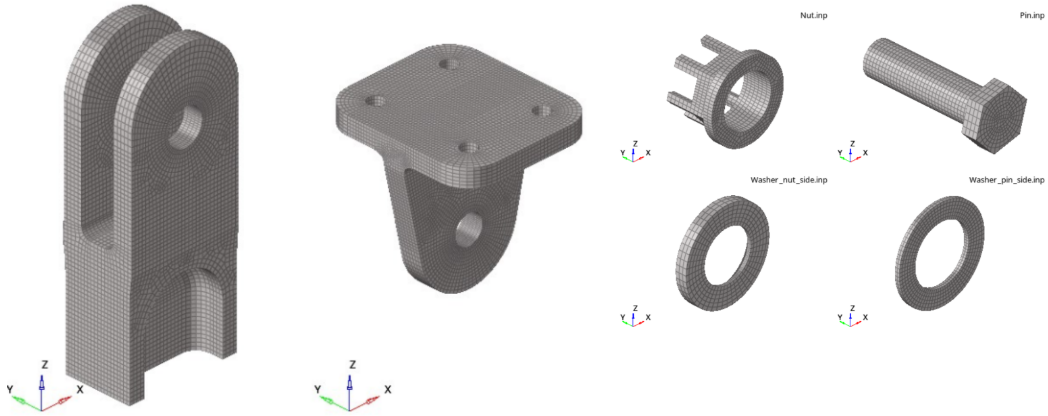

Figure 4: DFEM of the lug assembly

All parts were meshed with hexagonal elements with at least 4 elements through thickness. The lug material was modeled as elastic-plastic and the rest is linear elastic. Contacts were modeled between the bolt and lug shank and between lug faces and the bushes. Maximum pretention load of 15000N was applied.

The results show very good correlation between DFEM and the analytical method. The force required to close the gap was determined to be 5800N compared to 5009N using the analytical method representing about 15% of error. Also, the max stresses show a good correlation with the maximum DFEM stress of 527MPa compared to the 491MPa for the analytical method representing about 7% error. (Figure 5)

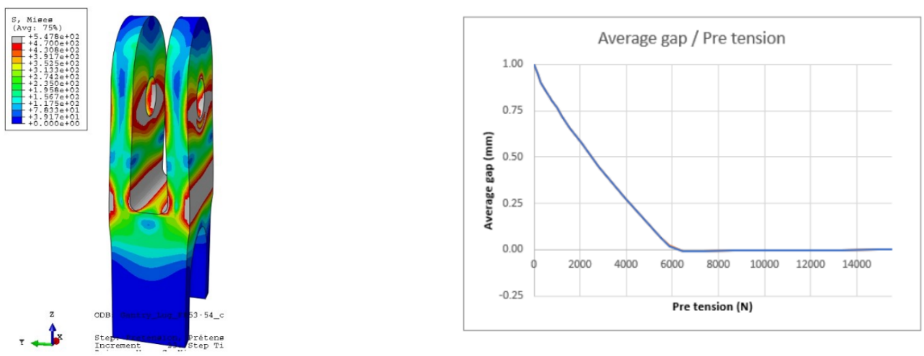

Figure 5: Maximum Stresses and lug deflections

This comparison shows that classical analytical methods can provide reasonable accuracy and quicker answers with much lower costs, compared to DFEM models. However if a more precise and detailed answer is required, the DFEM will provide a better solution.

The test on the actual lug configuration was performed, but it was difficult to establish a correlation between the applied clamping force and the gap between the lugs. This was due to the difficulty to measure the applied force on the bolt. The best solution would be to use the specialized strain gauged bolt but this was not available. Instead it was attempted to measure the preload by reading the applied torque values and then converting it to applied load. This method is known to be inaccurate as it is highly dependent on the friction between the nut and the bolt.

2. Seat rail side loads capacity

Seat rails are used to attach seats and other cabin equipment and transfer the loads from them to the aircraft primary floor structure. The capacity of seat rails to transfer side loads (Fy) is analysed in this paragraph.

This analysis is done by using analytical methods, DFEM and test results. The DFEM is correlated with the test and the analytical solution is then derived from the correlated model.

Analysis

Initial analysis was done using the classical beam stress equations. As the ultimate (failure) capacity is required, plastic bending analysis with the idealization of stress strain curve ([Reference 1+2]) is used.

The load on the seat rail is applied in the middle of the section. This represents the most critical location. To simulate the uniform Fy loading a half of the load is applied on each side of the cross beam.(Figure 6)

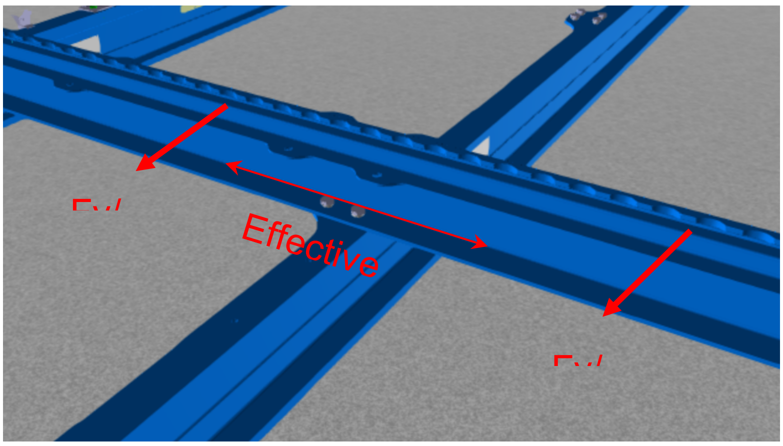

Figure 6: Typical seat rail to cross beam connection with the applied side load Fy

The side load causes rail torsion which is then transferred to the cross beam.
The critical parameter for this analysis is the effective length of the rail that resists this torsion. The usual assumption is that the effective length is equal to the width of the cross beam plus length that corresponds to a spreading angle of 45° (Figure 6).
The previous tests have shown that the effective width is larger than this value. A DFEM is built correlated to the test, and it is used to determine the effective width.
Non linear analysis had to be performed to simulate material yielding and the large displacement that occurred during the test.
Because of the filet radius between the flange of the rail and the web, 3D elements were used to accurately model the section and predict the ultimate rail capacity.
Using these assumptions DFEM matched the test values for ultimate rail capacity.
Finally the effective width is then measured from the model using the length of the rail in radius area with the strains exceeded 0.2%.

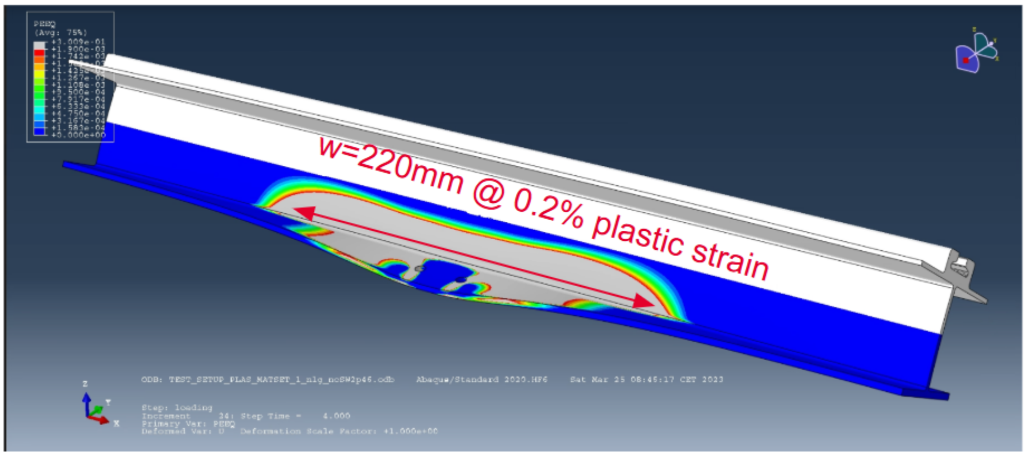

Figure 7: NL DFEM seat rail with Fy loading showing the effective width length

After the correlation phase was over, an analytical method was derived using the effective width and the plastic bending equations. The correct boundary condition for the upper flange was applied, as in reality this flange is supported by the floor panels, while in the test it was free to rotate. (Figure 8)

This case shows how a DFEM together with test results can be used to better understand the stress distribution in the analysed part and refine the analytical method. It combines the precision of the DFEM with the simplicity of the analytical solution to create the best solution.

Example:

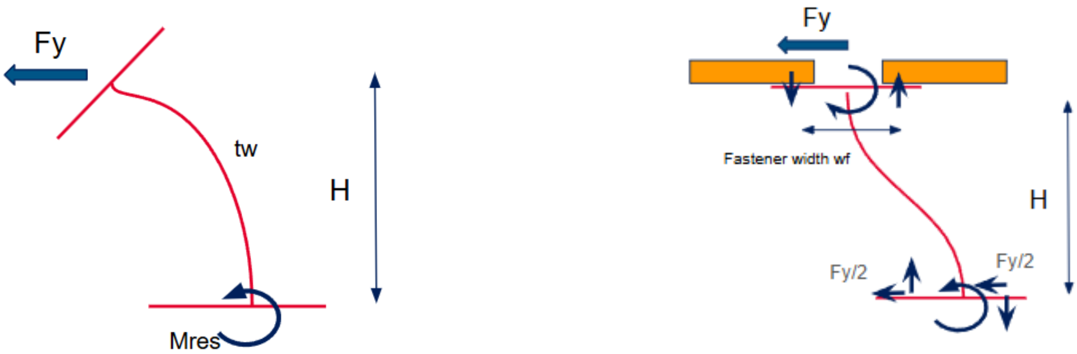

Figure 8: Seat rail test and actual boundary condition

Material 7175-T79511 Extrusion
Ftu=540MPa → Ultimate tension allowable [3]
k=1.4 → Plastic Bending factor Reference [1]
tw=2.4mm→web thickness
H=50mm→Rail Height
$I = \frac{w * t_w^3}{12}$ → Moment of inertia

$$M = \frac{Fy \cdot H}{2}$$

$$\sigma = \frac{M}{I} \cdot \frac{t_w}{2}$$

From $RF = \frac{Fty \cdot k}{\sigma}$=1

$Fy = \frac{4 \cdot Ftu \quad \cdot k \cdot I}{H \cdot t_w}$=5367N

3. Conclusion

This paper shows the advantages and drawbacks of classical analytical methods and DFEMs. In case 1 a classical analytical method is used that gives a conservative answer and a big cost advantage compared to DFEM. In case 2 a more accurate method is needed, so the analytical method is calibrated with a DFEM and test.

4. References

[1]E. F. Bruhn, *Analysis and Design of Flight Vehicle Structures*. 1973.
[2]ESDU 76016, *Generalization of smooth continuous stress strain curv*e 1976
[3] MMPDS-10 Metallic Materials Properties Development and Standardization April 2015