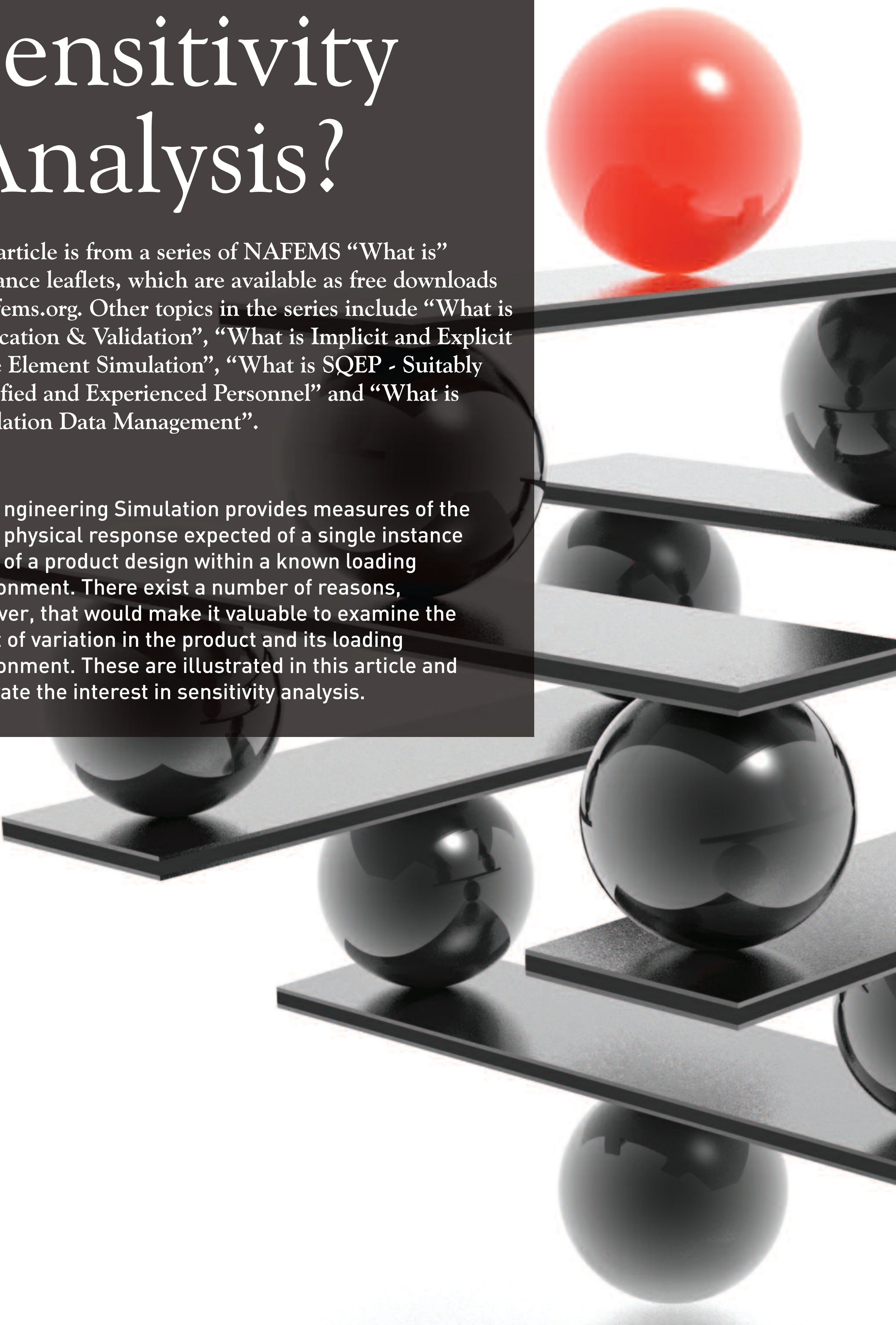
What is Sensitivity Analysis?

This article is from a series of NAFEMS "What is" Guidance leaflets, which are available as free downloads at nafems.org. Other topics in the series include "What is Verification & Validation", "What is Implicit and Explicit Finite Element Simulation", "What is SQEP - Suitably Qualified and Experienced Personnel" and "What is Simulation Data Management".

Engineering Simulation provides measures of the physical response expected of a single instance of a product design within a known loading environment. There exist a number of reasons, however, that would make it valuable to examine the effect of variation in the product and its loading environment. These are illustrated in this article and motivate the interest in sensitivity analysis.



Perspectives of Risk and Decision Errors

Definitions

Mathematically, a sensitivity is a partial derivative of some measure of the performance of a product, such as the aerodynamic drag or the stress at a point, with respect to a parameter representing a physical property of the product or its environment. In this context sensitivity analysis provides a measure of the effect of small perturbations, local to the initial design.

The term may also be used to describe the average of such changes caused by large perturbations so providing a more global measure of change.

The sensitivity analysis can be performed in the deterministic or probabilistic context. A type of sensitivity analysis that is performed in the context of stochastic analysis provides information regarding the relative contribution of noise or random variables to the scatter of design performance metrics; these are frequently referred to as probabilistic sensitivity factors.

Motivation

Design Change

Design processes create a product that performs to specification and minimises a cost function. A common example is minimising product weight or cost without failure occurring under normal operating conditions. This constrained minimisation is a design optimisation problem.

The use of such deterministic sensitivities forms the basis of a number of mathematical methods used for design optimisation. These are typically known by the alternative term of 'gradient-based' methods.

Variability

The product as manufactured will exhibit variation from one instance to the next and will contain departures from the ideal design (typically categorised by the term 'noise'). The impact of such factors upon key per- formance metrics may be estimated through the use of sensitivities and is of critical importance in establishing safety cases for industrial products.

Robustness

Some designs are inherently less sensitive to intrinsic variability (noise). This is often a design goal and needs to be demonstrated.

Design for Six Sigma, for example, prioritises the robustness of the product, measured in terms of its failure rate, over achieving maximum performance.

Poorly known parameters

Some input parameters will simply not be known. For example, in geotechnics the ground conditions vary from point to point and there is a degree to which the properties of the soil are unknown.

The preferred design must either be insensitive to the assumptions made for the unknown parameters or a programme of testing will be needed.

Sensitivity Calculation Differencing

This is the crudest approach to estimating the sensitivity of FE output with respect to design change. Each design parameter is varied in turn and a complete reanalysis is performed. Dividing the change in any FE output by the change in the design parameter provides an estimate of its sensitivity.

A disadvantage of the method is that a large number of analyses may be called for if many sources of variation exist; also if the design change is too big the result will be inaccurate; conversely, if it is too small, the result can be swamped by rounding error.

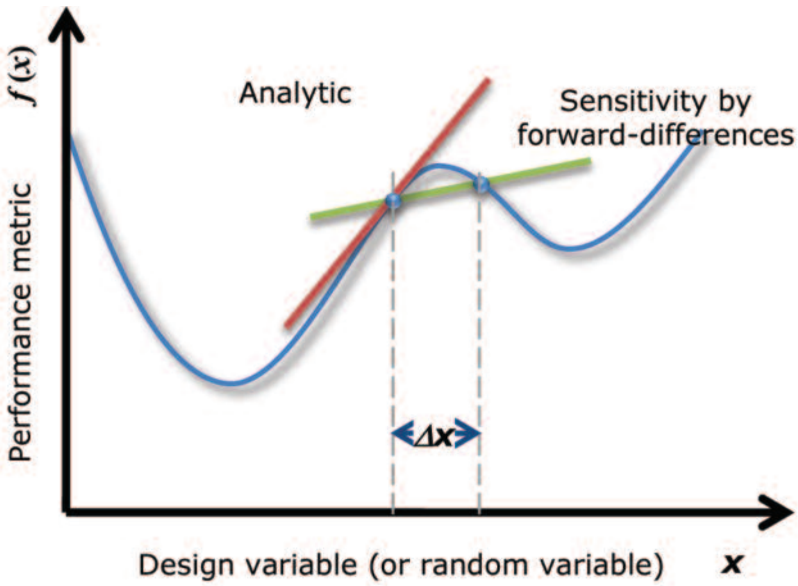


Figure 1

Design of experiments

Design of Experiment (DoE) studies are defined as a test or series of tests in which input parameters (factors) of a process or a system are systematically changed so that the causes of changes in the output response can be identified and understood. The DoE used in sensitivity analysis aims to quantify the sensitivity of the output response to variation of the factors and to identify the factors that cause the largest variation in output response. DoE studies can also provide information about interactions between the input factors and their effects on the output. DoE plays a key role in the assessment of robustness.

The systematic changes to the factors are defined as a set of sample point (design points), chosen to span the entire design space and to enable the effects of each factor on the output response to be separated out. A basic approach to sampling is provided by the Latin Hypercube Design. An illustration of a design space with 2 design variables and 11 sample points is shown below. Larger perturbations spanning a region of the design space can be used to determine the average sensitivity within that region.

After a DoE study is complete, a polynomial expression of the value of an output in terms of all the input parameters, called a regression equation, is obtained. The regression equation is an approximate mathematical representation of the true input-output relationship. The sensitivity analysis is then performed on the regression equation.

Analytic sensitivity – direct

The main difficulty in the analytic calculation of sensitivities arises because the response, as calculated by analysis, is only known as an implicit function of the input variables.

In essence, this approach uses the initial solution as an approximation to the solution of the perturbed equations. Clearly the governing (equilibrium) equations will no longer be satisfied and the errors give rise to small out of balance loads. Solving the original equations with these loads gives the first approximation to the solution of the perturbed system.

The approach is more accurate than differencing and only requires an additional load case to be processed for each design parameter.

An analytic sensitivity and its finite difference approximation are illustrated in figure 1.

Analytic sensitivity – adjoint

This approach requires far greater level of mathematical manipulation and involves solving a linear set of equations, closely related to the original equations but with boundary conditions determined by the performance metrics one wishes to study.

The number of additional load cases to be calculated is related to the number of constraints/performance metrics of interest and is independent of the number of design parameters.

Caution

Any prediction of performance or safety based upon the use of partial derivatives for sensitivity analysis represents an extrapolation. As such, its validity will be limited to the neighbourhood of the initial design. Beyond that, errors may well be large and potentially non-conservative. ■

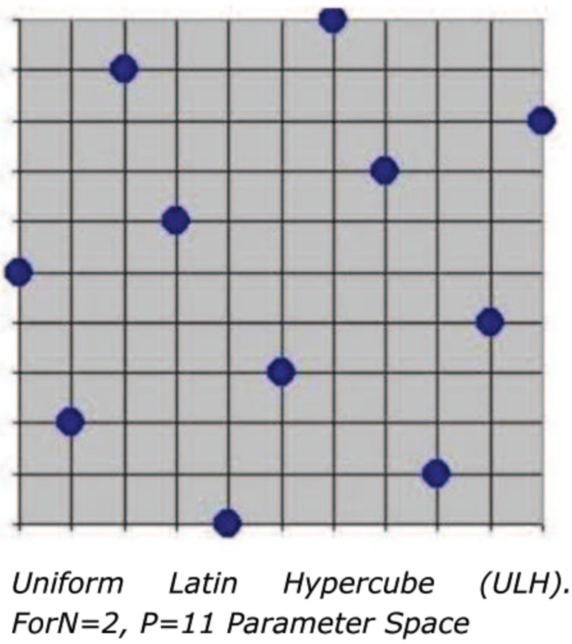


Figure 2