Plotting load paths from vectors of finite element stress results

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THEME

Post-processing of stress results.

KEYWORDS

Load paths, finite elements, stress analysis.

SUMMARY

This paper summarizes a theory that defines pointing vectors and load paths to map load transfer in structures. Pointing vectors are defined from the stress results from a finite element analysis. Contours are plotted parallel to these vectors to define the paths. The paths are applied to the results of a transient dynamic analysis for the first time.

1 Introduction

Modern tools for visualization of simulation results include images of deformation and contour and fringe plots of stresses as 3D static images and animations. The primary goal of a structure is however to support loads. A procedure for displaying how this key function is being resolved has not been a standard post-processing operation of finite elements. The primary reason for this is that applied mechanics and the classical theory of elasticity have not provided a clear definition of load paths. Users have resorted to finite element models based on beam elements so that the force and moment resultants could be tracked through the structure to identify the main load bearing components. Indeed books on stress analysis use the term "load path" without definition [1] and in one case load paths are declared to be a "useful abstraction" [2].

In the paper we repeat a definition of the paths that has been published recently in [3-6]. It is derived directly from the Cauchy stress tensor. "Pointing vectors" are defined from the stress components. Contours through the pointing vector field identify regions that carry a constant load in static analysis, or are equilibrated by inertia forces in transient dynamic solutions. The contours initiated on a loaded surface bound regions carrying a constant load and narrowing of the path identifies stress concentrations. Strings can be initiated from arbitrary locations (mimicking ribbons in a fluid flow) to identify the structure of the load flow without the accuracy required to follow the geometric path carrying constant load. In recent work properties of the paths have been identified and procedures have been defined to create a statically determinate topology to carry the loads in conceptual design [5,6].

The pointing vectors are defined from the stress tensor. Eigenvalues and eigenvectors of the 3x3 matrix of the stress components define the three principal stresses. Similarly there are three pointing vectors that can be assembled from the stress field at a point in a three-dimensional domain. The three pointing vectors correspond to load transfer in the set of orthogonal axes in which the stresses are defined. The path for the transfer of shear along a beam is, for example, accompanied by the path in an orthogonal direction comprising the direct stresses corresponding to the bending moment. A common error in design is to fail to recognise the generation of moments when a load is displaced from its axis, so the appearance of these secondary paths in a load transfer analysis is a welcomed feature.

The axis system for the paths is arbitrary and the stress field can be transformed into any orthogonal system. In addition the principle of superposition that applies to loads in a linear analysis can be applied to define combinations of load paths.

The theory for defining the pointing vectors and plotting the load paths is defined in Section 2. A simple but highly informative example of load paths near a loaded bolt hole in a flat plate is described in Section 3. Previous work has focussed on stresses for a static analysis. In this paper the theory is extended to transient dynamic solutions. Some preliminary plots for a university based racing car project including load transfer from front-end impact are given in Section 4. Section 5 applies the load path theory to define a load bearing topology in conceptual design. Section 6 then gives some conclusions and summarises current developments.

2 Theory for plotting load paths

Paths can be plotted by tracking force by integration on sections across the domain. For example, commencing at the edge of the domain in Figure 1(a) and integrating along the dashed line till a set value of the edge force is obtained, defines one point on a contour that can be created by finding an equivalent point on neighbouring sections. Alternatively, for a structure constructed from beam or truss elements the path can be created by following force resultants from member to member across the domain as indicated in Figure 1(b).

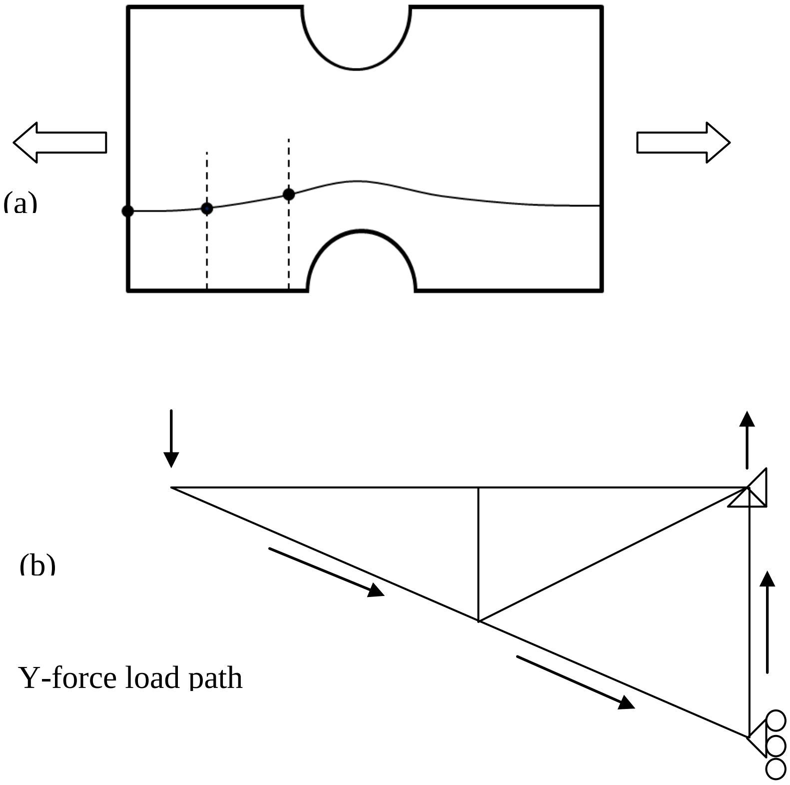


Figure 1: Load paths by following load.

A second approach for plotting load paths across continua is most recently described in [5,6]. The components of stress at a point in a structure form a second order tensor and can be represented in a 3x3 matrix. If each row gives the three stresses acting on a plane whose normal is aligned with one of the coordinate axes, then

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Here σ_{ij} is the shear acting on the plane whose normal is in the i-direction, directed positive in the positive j direction. Eigenvalues and eigenvectors of this matrix provide the principal stresses and principal stress vectors.

Load paths can be defined by plotting contours aligned with total stress “pointing” vectors given by the columns of the stress matrix. Each column of the matrix gives the stress component in the corresponding coordinate direction on the three planes that form the sides of the corner element depicted in Figure .

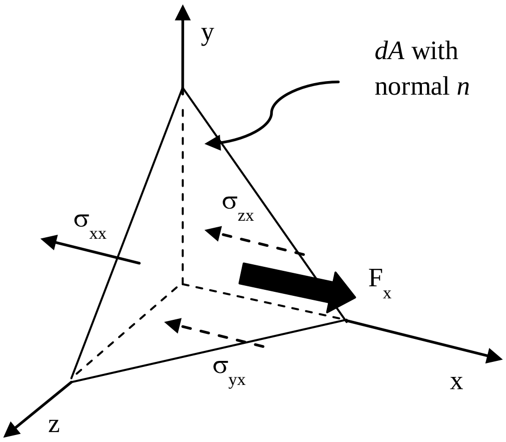


Figure 2: Construction of force components.

The pointing vectors are thus defined at every point in the domain by

$$\boldsymbol{V}_x = \sigma_{xx}\boldsymbol{i} + \sigma_{yx}\boldsymbol{j} + \sigma_{zx}\boldsymbol{k}$$
$$\boldsymbol{V}_y = \sigma_{xy}\boldsymbol{i} + \sigma_{yy}\boldsymbol{j} + \sigma_{zy}\boldsymbol{k}$$
$$\boldsymbol{V}_z = \sigma_{xz}\boldsymbol{i} + \sigma_{yz}\boldsymbol{j} + \sigma_{zz}\boldsymbol{k}$$

The element in Figure gives the components of the pointing vector $\boldsymbol{V}_x$. For example, for a load P_x transferred in the x-direction $V_x = \sigma_{xx} i$. For and load P_x transferred as a shear force in the beam in the y-direction, $V_x = \sigma_{yx} j$ etc.

The forces acting on the arbitrary plane in Figure with normal given by

$$\boldsymbol{n} = n_x\boldsymbol{i} + n_y\boldsymbol{j} + n_z\boldsymbol{k}$$

are obtained by integrating the pointing vectors

$$F_x = \int \boldsymbol{V}_x \bullet \boldsymbol{n}\, dA$$
$$F_y = \int \boldsymbol{V}_y \bullet \boldsymbol{n}\, dA$$
$$F_z = \int \boldsymbol{V}_z \bullet \boldsymbol{n}\, dA$$

where the dot indicates the vector dot product.

Let us define the load path for a force in a given direction for a static problem as a region in which the force in that direction remains constant. For example, if the path in Figure is to define a region in which the force P_x remains constant, the requirement is to determine the curved contour forming an edge along which the normal and tangential edge loads make no contribution to force in the x-direction. This requires that there is no contribution to the x-force on sides AB and CD. On AB this requires

$$F_x\big|_{AB} = 0$$
$$\therefore \int_A^B \boldsymbol{V}_x \bullet \boldsymbol{n}\, dA = 0$$

This is achieved if the normal to the surface is perpendicular to V_x, as the dot product is then zero. Alternatively, this is achieved if the surface tangents are parallel to the vector V_x as indicated in Figure .

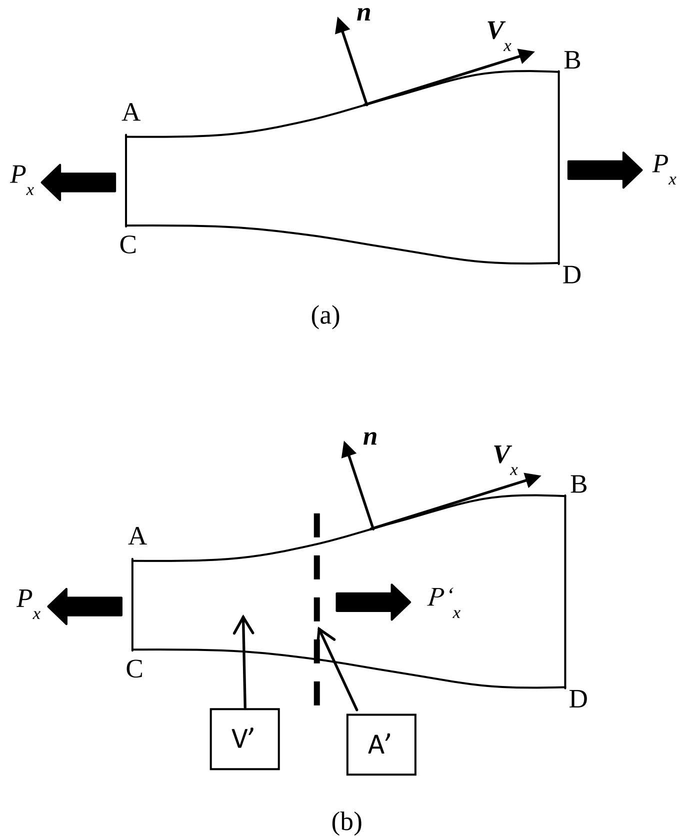


Figure 3: Contours for a path along which Px is constant.

For a dynamic problem inertia forces are introduced and the equilibrium relation

$$\sum F_x = 0 \quad \textit{which implies} \quad \int_{A'} \sigma_{xx}\, dA = P_x \quad \textit{on any vertical section in Figure 3,}$$

becomes

$$\sum F_x = m\ddot{x} \quad \textit{which implies} \quad P_x' = \int_{A'} \sigma_{xx}\, dA = P_x + \int_{V'} \rho\ddot{x}\, dV$$

where V′ is the region to the left of the dashed line.

The vector field of "pointing vectors" for the required load path is first defined by averaging stresses to the nodes in the finite element mesh. The appropriate vector V_x, V_y or V_z is then formed from the stress components at the node. To plot the contours through the vector field a fourth-order Runge-Kutta scheme can be used. The vector can be defined at any arbitrary point by first associating the point with an element and then interpolating from the nodal values.

A scalar spatial discretisation, Δs, is used which represents a small increment along the load path. To ensure that the spatial increment is fixed, the stress pointing vectors are first normalised such that $|\boldsymbol{V_i}| = 1$.

For a normalised vector field, $\boldsymbol{V}$, defined over the mesh domain and an initial point $\boldsymbol{p}_i$,

$$\boldsymbol{dp}_1 = \boldsymbol{V}\Big|_{\boldsymbol{p}_i} \Delta s$$

$$\boldsymbol{dp}_2 = \boldsymbol{V}\Big|_{\boldsymbol{p}_i + \frac{1}{2}\boldsymbol{dp}_1} \Delta s$$

$$\boldsymbol{dp}_3 = \boldsymbol{V}\Big|_{\boldsymbol{p}_i + \frac{1}{2}\boldsymbol{dp}_2} \Delta s$$

$$\boldsymbol{dp}_4 = \boldsymbol{V}\Big|_{\boldsymbol{p}_i + \boldsymbol{dp}_3} \Delta s$$

$$\boldsymbol{p}_{i+1} = \boldsymbol{p}_i + \frac{1}{6}\left(\boldsymbol{dp}_1 + 2\boldsymbol{dp}_2 + 2\boldsymbol{dp}_3 + \boldsymbol{dp}_4\right)$$

where

$\boldsymbol{p}_{i+1}$ is the next point.

$\boldsymbol{V}\Big|_p$ is the value of the vector field, $\boldsymbol{V}$, evaluated at point $\boldsymbol{p}$.

Finally the contour can be colour coded to reflect the magnitude of the vector being plotted.

In three-dimensional applications the paths are further modified if the element has a free face. To prevent the path from exiting the solution domain due to numerical error, the path is projected parallel to the free surface when the angle of vector to the surface is small (say, within 30 degrees of tangency).

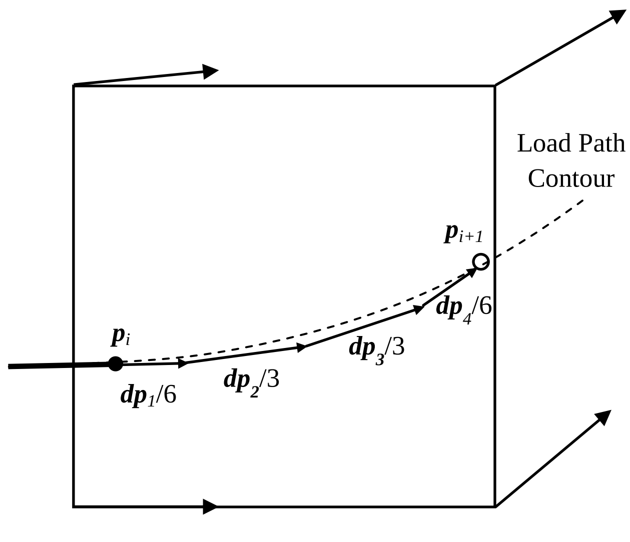


Figure 4: Nodal pointing vectors and Runge-Kutta sampling vectors for path creation.

An additional complication occurs when the path encounters an intersection between surfaces as in a thin-walled assembly. The algorithm selects the surface in which the magnitude of the load path vector is greatest and then proceeds to create a path in that surface. It is clear, however, that the paths can branch. In one version of the algorithm, elements after the branch point on which the load path vector has a magnitude greater than 20% of the input vector, are recorded and saved. After completing a dominant path the algorithm returns to the branch point and advances along the secondary paths.

The paths are usually plotted starting from a loaded surface. In two-dimensions the paths can be spaced so that they define regions carrying the same load. The contours bounding the regions will then converge in regions of higher stress since the load being carried is constant and higher stress is balanced by a lower load carrying area.

3 Simple examples from classical analysis

3.1 Load flow on a pin loaded hole.

An example that has been published to demonstrate the relationship between the load paths and the principal stress trajectories has been a pin loaded hole [3]. The pin-loaded hole is shown in Figure 5. Contours aligned with the tensile and compressive principal stress vectors are shown in Figure 6.

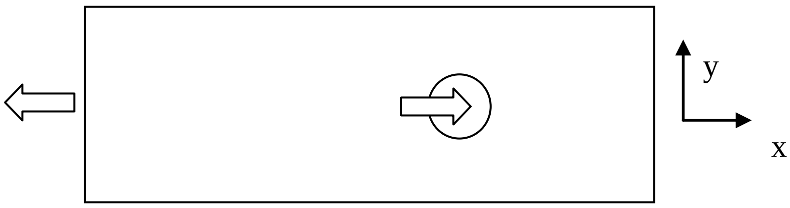
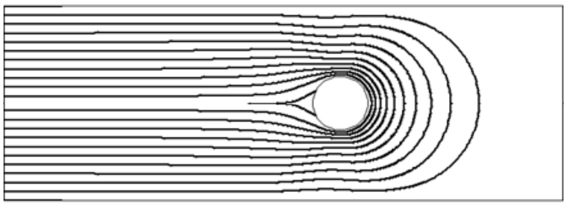
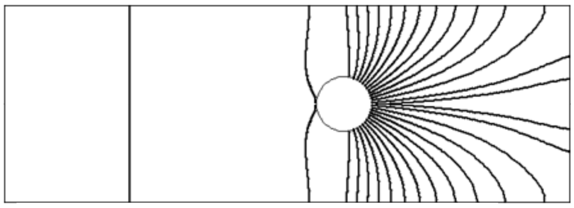


Figure 5: Region with a pin loaded hole.



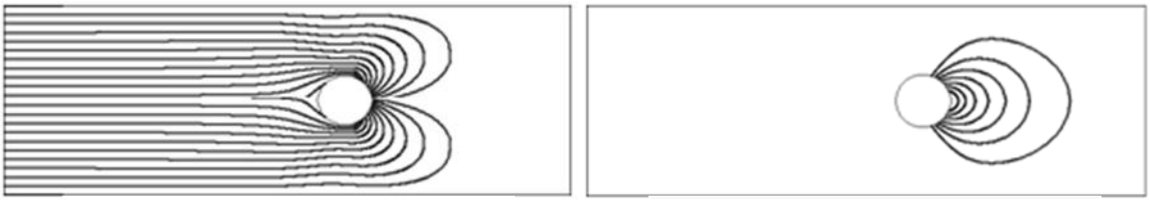
(a) Tensile or major principal stress (σ_{11}) trajectories.



(b) Compressive or minor principal stress (σ_{22}) trajectories.

Figure 6: Principal stress trajectories for a pin-loaded hole in an isotropic material.

To indicate the different interpretation provided by contours aligned with the load path vector, the path plots where applied to the same stress field, as shown in Figure 7. In this example the paths are spaced to carry the same load and the stress concentration on the hole is indicated by the convergence of the paths. The image for the $\boldsymbol{V}_x$ paths, Figure 7(a), shows the primary load bypassing the hole to be reacted on the surface bearing on the pin. The load has been applied normal to the hole surface behind the hole. The $\boldsymbol{V}_y$ paths, Figure 7(b) indicate the secondary load transfer due to the vertical component.



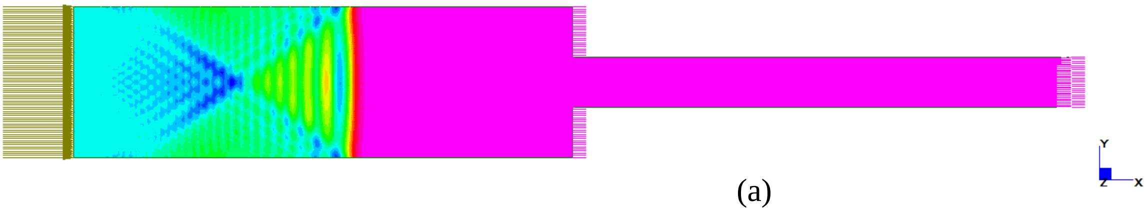
(a) $\boldsymbol{V}_x$ load path trajectories (b) $\boldsymbol{V}_y$ load path trajectories

Figure 7: Load path trajectories for a pin-loaded hole.

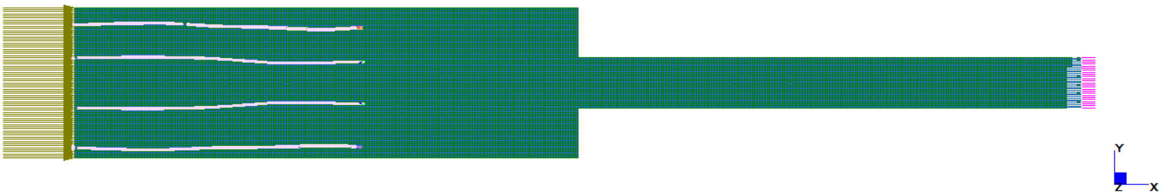
3.2 Load paths on a cantilever with tip axial impact.

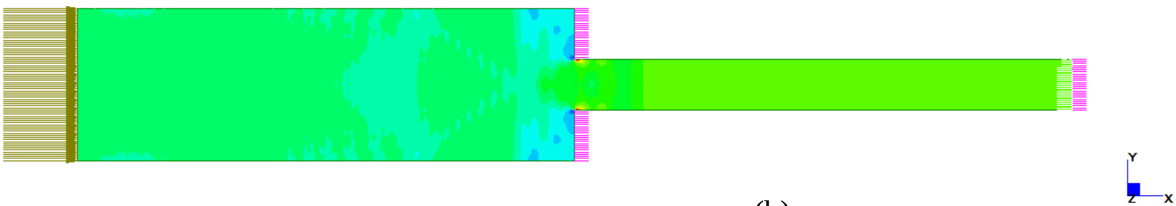
Load paths on a variable-section beam with end impact axial load (P=0 t<0, P=Const t>0)are shown in Figure 8. The shoulders are supported where the cross-section changes.

Figure 8(a) shows the stress wave propagating in the wider section of the bar. The paths are terminated when the magnitude of the pointing vector V_x falls below 0.5 of the value equivalent to the applied load. This occurs at the front of the stress wave where the material in the bar is accelerating. Figure 8(b) repeats the plots shortly after the stress wave has passed the section change in the bar. Reflection of the stress wave by the supported wall introduces new force reactions and so load paths now initiate from the supported wall.



(a)





(b)

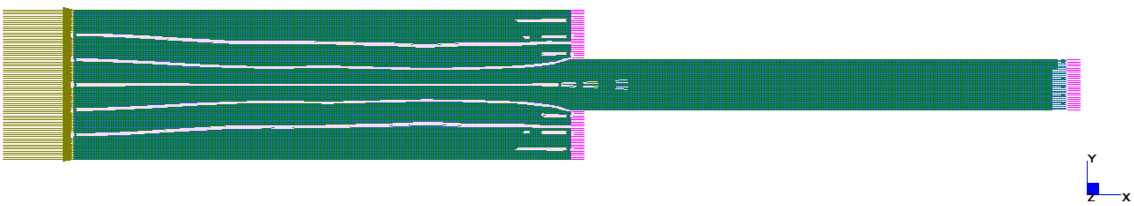
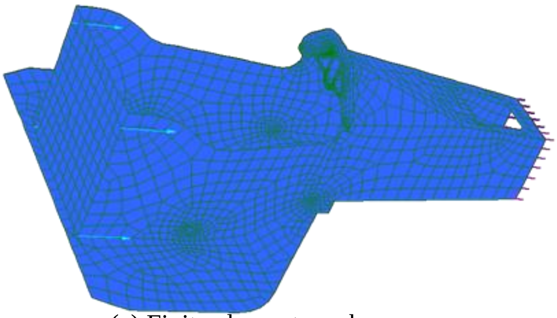


Figure 8: Constant section beam x-path for a static load.

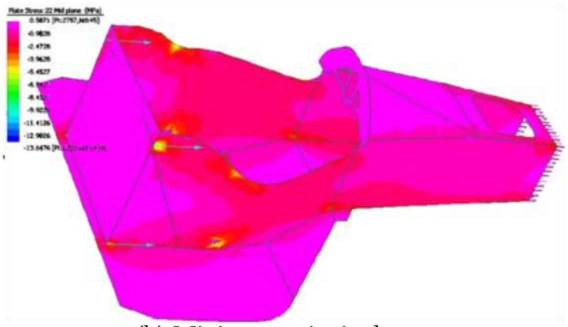
4 Load paths for a vehicle frame

4.1 Static paths in the FSAE car

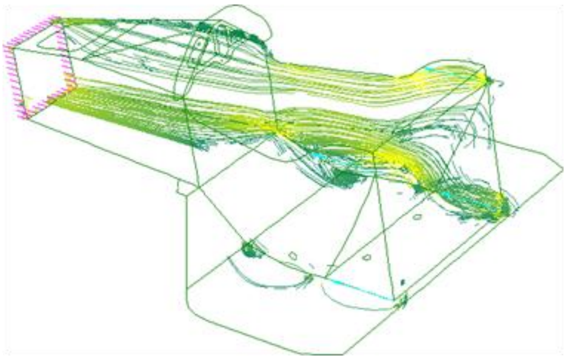
Load paths are plotted in Figure 9 for the shell if a racing car. The front of the vehicle is fixed to simulate front end impact. Loads are applied at the engine mounts at the rear of the vehicle to simulate inertia loads from the engine and rear axle assembly. The finite element mesh is plotted in Figure 9(a). A plot of the minimum principal stress indicates the path of the loads in Figure 9(b). Plots of the load path for the component parallel to the applied loads are given in Figure 9(c). The static analysis was first reported in [6].



(a) Finite element mesh



(b) Minimum principal stress



(c) Fore-aft load paths

Figure 9: Static load paths for the formula SAE car.

4.2 Dynamic paths in the FSAE car

Mass was added to the points of attachment of the engine to the back of the vehicle and all nodes on the vehicle were given an initial velocity of 10m sec^{-1}. A nonlinear transient analysis was conducted to model contact with a rigid wall at the front of the vehicle. Paths from the stress field at a defined instant in time are plotted. Figure 10 (a) and (b) show the paths developing from the front of the vehicle. In these plots the trace is terminated when the magnitude of the pointing vector falls to half of the peak value. The time for a stress wave to travel the length of the vehicle is 30.0E-05 sec. The complete paths (without cut-off) are shown for this time step. In the early part of the analysis the stress wave is advancing too quickly (due to the coarse mesh and implicit algorithm used).

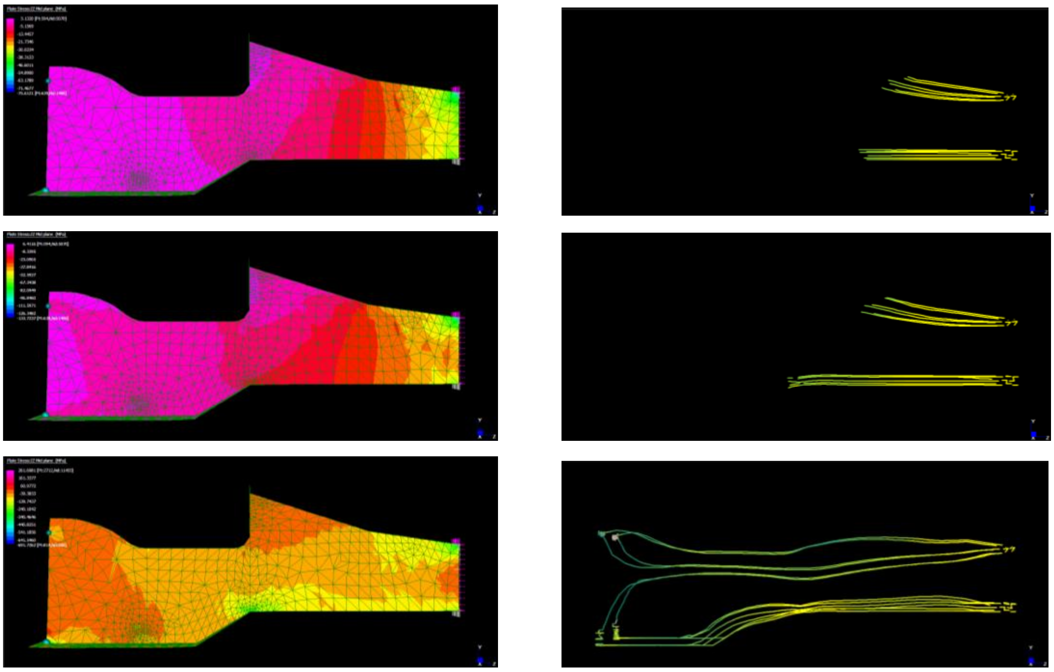


Figure 10: Transient load paths. Top 2.5E-05s after impact, centre 5.0e-05s after impact and bottom 30.0E-05 after impact.

5 Design optimization

Algorithms to define a load bearing topology for a structure have also been presented elsewhere [5]. An evolutionary procedure has been applied. The algorithm reduces the modulus of elasticity in members that are not contributing to the paths while retaining a set fraction of the volume of material. Figure 11 defines the design domain for a cantilever to carry a tip load. Figure 12 shows the designs proposed for retaining a fraction of the volume from 0.1 to 0.9. The last image in Figure 12 gives a measure of efficiency given by the inverse of the product of the volume retained and the strain energy in the structure. The structure with retained volume 0.4 is proposed as the most efficient topology.

6 Conclusions

A theory for plotting pointing vectors and load paths from the stress field following a finite element analysis has been presented. Contours parallel to the pointing vectors identify the load transfer across the domain.

In previous work the paths have been applied to the results of a static analysis. In this paper the theory has been extended to include inertia forces to enable plotting the load transfer in a structure subject to transient dynamic loads such as a crash analysis.

Stress concentrations are identified by convergence of the paths and the paths terminate in a stress wave analysis where the loads are causing acceleration of the material.

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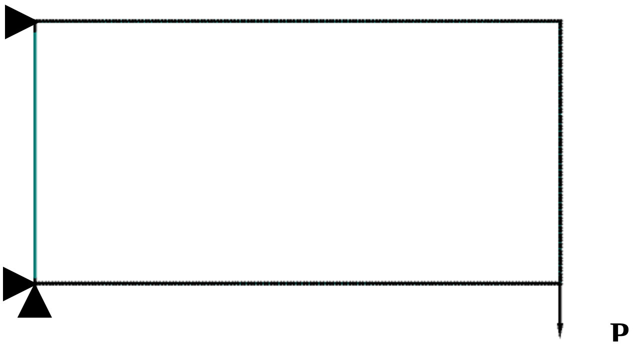

Figure 11: Cantilever beam with tip load.

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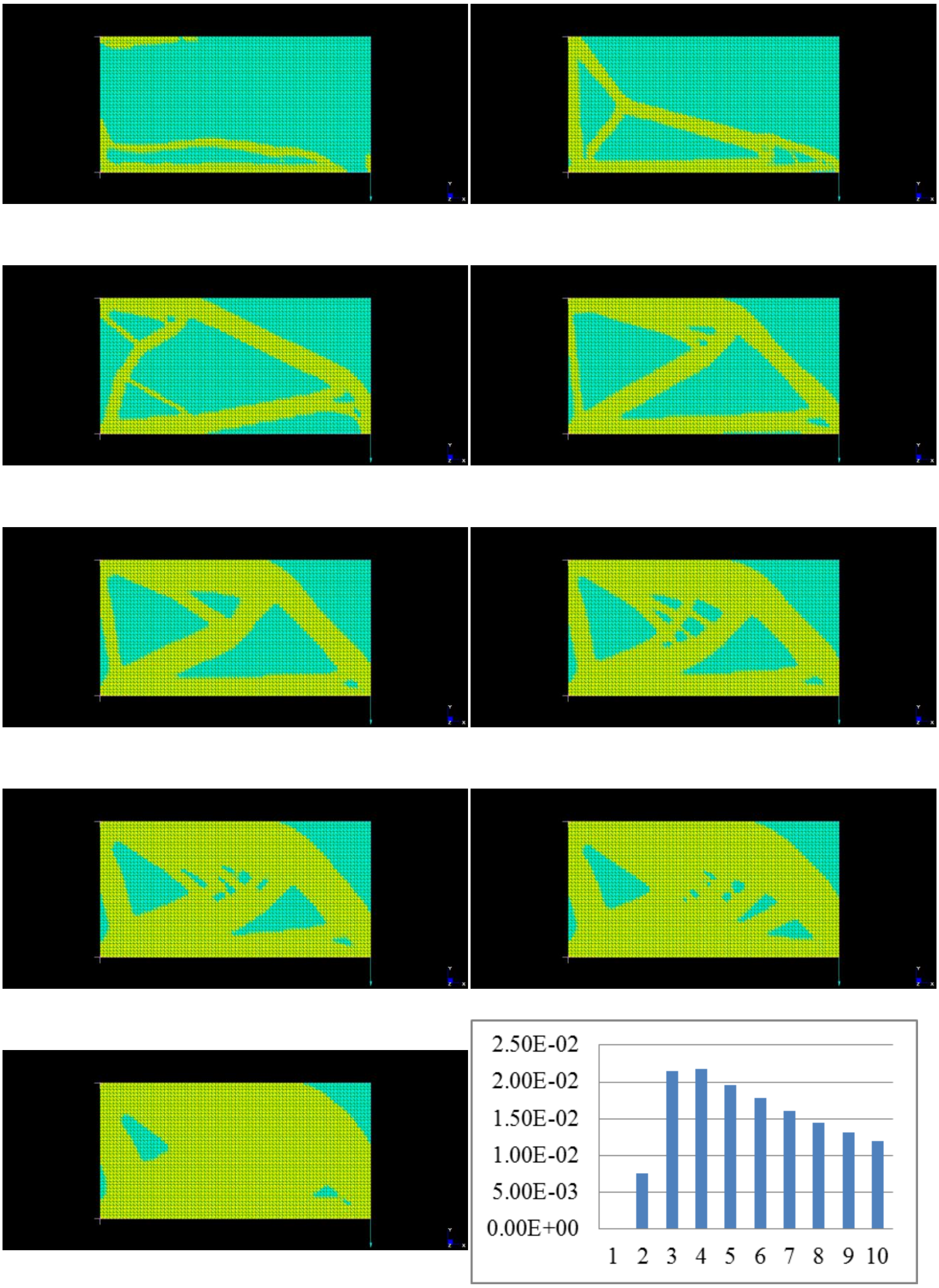

Figure 12: Topology design using load paths. Volume fraction to be retained Top row 0.1 & 0.2. Second row 0.3 & 0.4 etc. Bottom right efficiency measure.