Uncertainty quantification and value of information challenge problem: Electrical problem

Introduction

It is difficult to execute model validation exercises when uncertainties due to variation in parameters and operational conditions have to be accounted. A challenge problem is devised to develop statements of measures of confidence that the provided model can meet the functional requirements. Information provided from various sources for a specified set of conditions (intended application) will be used to assess the reliability of the device based on the functional requirements. The purpose of the challenge problem is to showcase and share different approaches to UQ methods among SWG members. Methods are not judged and this is not a competition to find the true solution.

A typical electronic component may be represented by an equivalent R-L-C (resistive, inductive and capacitive) series circuit. The electric transients that occur within the system are of interest. This particular challenge problem was chosen because the fundamental equations based on Kirchhoff's current and voltage laws are well known and R-L-C parameters are usually readily available to electrically characterize components or systems. Further, variation and uncertainties from sources is presented in different forms and such responses can be easily generated using readily available tools.

Requirement

The requirement states that at time t=10 ms, defined as the time taken after the input voltage activates (nonzero voltage), the voltage at the capacitor should be greater than 0.9V.

$$V_C(t=10\text{ ms}) > 0.9\text{ V}$$

Also, the voltage rise time t_r, defined as the time from 0%-90% of the input voltage, should be less than 8 ms.

$$V_C(t=t_r\text{ ms}) = 0.9\text{ V, where } t_r < 8\text{ ms}$$

Electric model

The series R-L-C equivalent circuit is shown in Figure 1. Equations representing this model are stated in the equations section below. Also, It is assumed that all the initial conditions are at zero ($i_L(0)=0$, $Vc(0)=0$, $dVc(0)/dt=0$). Some of the expected responses for the voltage at the capacitor are shown in Figure 2.

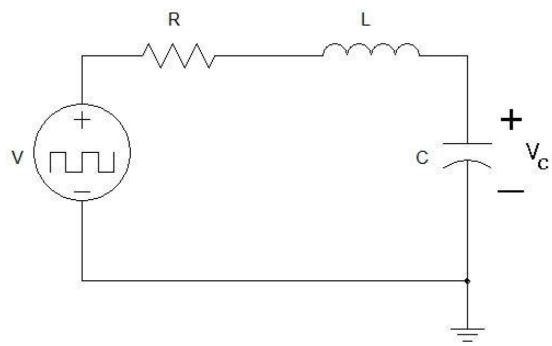

Figure 1: Equivalent R-L-C circuit model

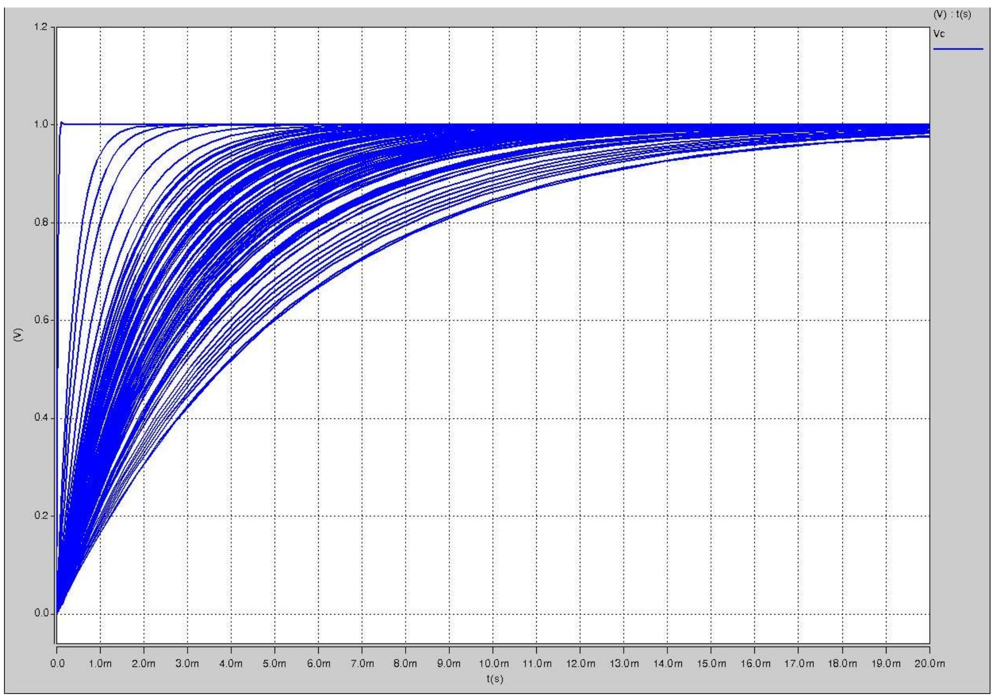

Figure 2: Capacitor voltage, V_C

Parametric characterization

Four cases of uncertainty estimates on the R-L-C parameters are presented.

Case A: Intervals

Known intervals are obtained as follows

	Resistance (Ω)	Inductance (mH)	Capacitance (μF)
Interval	[40, 1000]	[1,10]	[1,10]

Table 1: R-L-C parameters for Case A

Case B: Multiple intervals

Known intervals are obtained from different sources as follows

	Resistance (Ω)	Inductance (mH)	Capacitance (μF)
Source 1, Interval	[40, 1000]	[1, 10]	[1,10]
Source 2, Interval	[600, 1200]	[10, 100]	[1,10]
Source 3, Interval	[10, 1500]	[4, 8]	[0.5, 4]

Table 2: R-L-C parameters for Case B

Case C: Sampled points

Sampled data points are obtained as follows

	Resistance (Ω)	Inductance (mH)	Capacitance (μF)
Sampled Data	{861, 87, 430, 798, 219, 152, 64, 361, 224, 614}	{4.1, 8.8, 4.0, 7.6, 0.7, 3.9, 7.1, 5.9, 8.2, 5.1}	{9.0, 5.2, 3.8, 4.9, 2.9, 8.3, 7.7, 5.8, 10.0, 0.7}

Table 2: R-L-C parameters for Case C

Case D: Incomplete data

Only limited information is available as follows

	Resistance (Ω)	Inductance (mH)	Capacitance (μF)
Interval	$[40, R_U]$	$[1, L_U]$	$[C_L,10]$
Other information	$R_u > 650$	$L_u > 6$	$C_L < 7$
Nominal value	650	6	7

Table 4: R-L-C parameters for Case D

Uncertainty Quantification (UQ) Tasks

Task 1: Assess reliability of the device meeting the functional requirements described in the requirements section above.
Task 2: Quantify the value of information for each case.

Equations

System transfer function

$$\frac{V_c}{V} = \frac{1/(LC)}{s^2 + (\frac{R}{L})s + \frac{1}{LC}}$$

Equation 1

Overdamped

$$V_c(t) = V + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Equation 2

Critically damped

$$V_c(\mathrm{t}) = \mathrm{V} + (A_1 + A_2 t)e^{-\alpha t}$$

Equation 3

Underdamped

$$(V_c(\mathrm{t}) = \mathrm{V} + A_1 \cos \omega t + A_2 \sin \omega t)e^{-\alpha t}$$

Equation 4

Where ω, $s_{1,2}$ and α are defined as shown in equations 5, 6 and 7 respectively. A_1 and A_2 are determined from the initial conditions in equations 8 and 9

$$\omega = 1/\sqrt{LC}$$

Equation 5

$$s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Equation 6

$$\alpha = r/2L$$

Equation 7

$$V_c(0) = 0$$

Equation 8

$$\frac{dV_c(t)}{dt} = 0$$

Equation 9