THE STATE OF THE ART IN PRODUCT & SYSTEM OPTIMISATION EMERGING FROM FENET PROJECT

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SUMMARY

The paper is intended to provide an introductory level overview of the State of the Art of Product & System Optimisation, derived in large measure from the reports delivered during the four years of the FENet Thematic Network.

1: GENERAL INTRODUCTION TO PSO

This paper provides an introductory level overview of 'Product and System Optimisation' as a technology area addressed by FENET and makes extensive reference to material prepared within the project. The obvious starting point [1] is the title of the thematic area and the distinction between product and system. In the FENET and wider engineering analysis community it is understood that what is being considered are things that are technical in nature and require the use of Finite Element Analysis (FEA) at some stage in their design.

An engineering product is seen as an assembly or an individual part whose form is determined through the design process, such as the wing of an aircraft or a single rib of the wing or even a single rivet fastener connecting the rib to the wing. This product has to perform in such a manner as to meet design requirements and to survive within its operating environment for its design life. The emergence of computational methods in the areas of structural mechanics and fluid dynamics over the past half-century has led to the present day situation in which substantial reliance is placed upon such numerical simulations for the purpose of predicting product performance characteristics and for building safety cases.

In parallel with developments in the areas of analytic simulation, corresponding progress has also been made on tools to support the design process but here the take-up has not been as widespread. While in the well established world of "simulation" the principles are clear (build a model able to reproduce numerically the physics of a phenomenon), in the PSO arena the driving force it to first establish a feasible design and subsequently to improve the design. As will be shown, the formulation of a design optimisation process is open to interpretation to a far greater extent.

In conjunction with the above definition of a product, the word 'system' encompasses both the behaviour of the product within an assembly to perform a wider set of functions and the production/manufacture/processing that goes into making the product. This may also require extensive use of Finite Element Analysis as the resulting strength, stiffness and longevity of the product can be as much dependant upon the process(es) by which it is made, as upon the design itself. Most of these complex processes are Multi-Physics and their optimisation goals are multi-criteria.

What the PSO thematic area within FENet sought to achieve was firstly to investigate the methodologies that are currently available for PSO and secondly to review the range of applicability for each method and, if possible, to define which techniques are best suited to any given circumstances.

In section 2 of this paper, PSO is placed in a historical context by reviewing the development of the subject, structural optimisation in particular, up to the point of the widespread availability of digital computer resources. The section also introduces a classification of the forms design optimisation problems might take. Section 3 is more mathematical in nature and provides a basic introduction to the techniques of optimisation traditionally referred to as mathematical programming.

The central element of this State of the Art Review is to be found in section 4 which links the optimisation strategies in widespread use to the forms of design problems introduced in section 2. Finally the emerging role of process integration frameworks is discussed in section 5.

2: OVERVIEW OF DESIGN OPTIMISATION

A HISTORICAL CONTEXT

The concept of structural optimisation, in particular, is not new. Galileo [2] attempted to determine optimal shape of a variable depth beam, despite the fact that it was only during the following century that Parent [3] and Lagrange independently identified the significance of the neutral axis in bending theory of beams and so were able analyse the loaded beam correctly.

A major theoretical advance was made by Maxwell [4] in 1869, establishing a theorem showing that, for fully-stressed layouts of pin-jointed frames under a single applied load case, the following must hold:

$$\sum_{\substack{\textit{tension}\\ \textit{members}}} \ell_i \bar{\sigma}_i^t - \sum_{\substack{\textit{compression}\\ \textit{members}}} \ell_i \bar{\sigma}_i^c = \text{constant},$$

where ℓ_i represents the lengths of the bars and $\bar{\sigma}_i^t, \bar{\sigma}_i^c$ are the values of the maximum allowable stresses in tension and compression respectively. This theorem may be used to show that equivalence, in terms of mass, of alternative layouts such as those shown in figure 1.

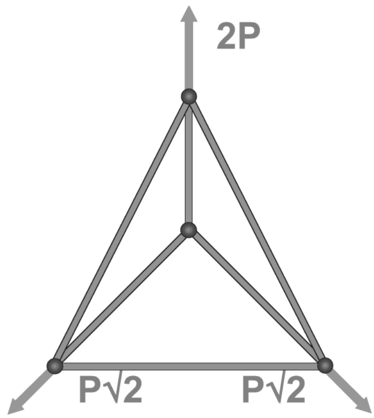

Figure 1: Alternative optimum layout for pin-jointed truss

Even now, this arbitrariness in the structural form of the optimum design has implications for those developing or using both shape and topology optimisers.

In 1904 Michell [5] developed the idea to generate families of solutions featuring orthogonal arrays of pin-joined bars such as the rather elegant arch shown in figure 2 below. This structure still provides a useful benchmark problem for modern-day topology optimisers. It is also of interest to note that the arch forms a mechanism and would not meet any criterion requiring a robust design.

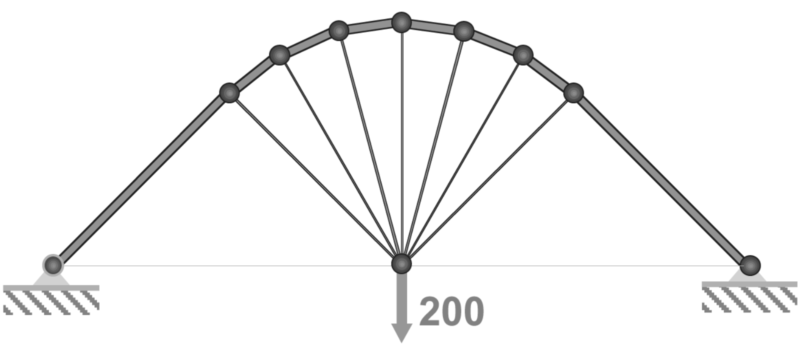

Figure 2 Example of Michelle structure

By the time of the Second World War, the design of light alloy compression structures was of interest and Cox *et al* developed solutions to various aeronautics problems [6], later published within a volume of the ESDU data sheets.

Optimisation studies at that time could be characterised as addressing structures of generic interest which may be described by sets of closed-form equations, so allowing the optimum to be determined by assuming the simultaneous solution of constraint equations as equalities.

For example, in the case of a compression strut, figure 3, buckling is governed by the following equations:

Euler buckling $\sigma_e = \frac{\pi^2 E}{8\ell^2}\left(D^2 + t^2\right) \geq \sigma$

Local buckling $\sigma_c = \frac{0.4Et}{D_0} \geq \sigma$

where $\sigma = \frac{P}{\pi Dt}$

from which the diameter D and wall thickness t may be determined

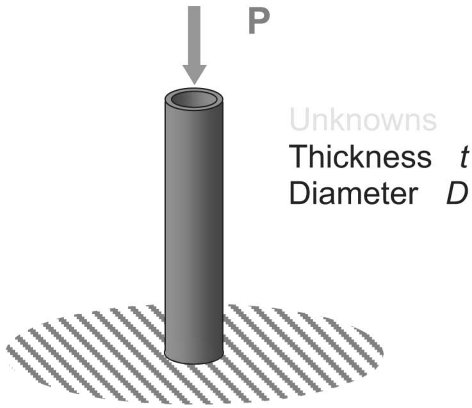

Figure 3: Compression strut

Razani [7], however, showed that the strategy of simultaneously satisfying stress constraints to optimise a structure for strength was flawed when more than one load case is present. The exclusion of an explicit objective function for the optimisation, be it mass or cost, makes sub-optimal designs inevitable and any modern ‘maximum compliance’ approach to topology design is also vulnerable to similar problems.

The state of the art was to change totally during the 1960s with the advent and increasingly widespread availability of the digital computer. Numerical optimisation methods were also developed, under the name of Mathematical Programming [8], to support the Operations Research community. At the same time, the finite element method was being developed, not least by Zienkiewicz and his team at Swansea [9]. By the time of the 1972 conference on the Optimisation of Structural Design [10] held in Swansea, a surprisingly large range of issues of current interest were under consideration including the treatment of discrete variables and design for reliability.

During the 1960s, Schmit had advocated that structural design optimisation problems should be formulated and solved using mathematical programming methods in combination with the finite-element method. Despite this, even by 1974, he reported that “large-scale structural optimisation capabilities developed by combining finite element structural analysis with mathematical programming algorithms have required long running times to optimise problems that are only of modest proportions” [11,12] and observed that “some investigators had abandoned the generality of the mathematical approach to renew effort on fully-stressing concepts and optimality criteria methods”.

By the time the NATO ASI on Structural Optimisation [13] was held in Liège in 1980, however, Petiau was able to report a decade of optimisation applications at Dassault built round the use of Elfini. An increasingly multidisciplinary content was becoming evident with both 2D aerodynamic shape optimisation and aeroelastic considerations appearing.
Sobieski introduced issues of software architecture and user interaction in the chapter

'Black-box to programming system'. The following year saw the launch of the RAE program STARS as a commercially available optimisation system [14,15], targeting the design of aerospace structures. The system was interfaced to external finite-element codes, NASTRAN in particular, with modules scheduled through the use of a special purpose script known as the 'command data file'. In recent years, the issue of Process Integration has become a major area of development with a range of commercial frameworks now available to provide graphical interfaces for the definition and execution of linked simulation tools.

Certainly the methods which found early applications for large-scale structural optimisation tended to address highly-idealised design problems, amenable to solution with very few function evaluations. Only in recent years has it become feasible to contemplate the use of methods such as genetic algorithms or Monte Carlo methods for robust design, which may require 1000s of simulation runs.

B FENET PERSPECTIVE

A survey of the members of the FENET network revealed, as expected, that all commercial companies who used FEA did so for the main purpose of improving their products in some way. This desire led them to use some form of product improvement process. From the responses to the survey it was clear that there are as many processes for product improvement as there are products. Indeed it appears that whilst FEA is a very definite process based on the unchanging laws of mechanics, the user's only choice is which FEA package to use, the range of formulations of design problems for Product and System Optimisation (PSO) is very wide and the solution techniques are even less prescribed.

It is almost certain that the full scope of the problem posed by the development of meaningful products lies well beyond the capacity of any computational process. The problem must therefore be reduced either by partitioning design decisions relating to the definition of the product so that they are determined at a lower level by the need to meet performance goals arising from different disciplines or by applying prior knowledge to reduce the number and complexity of design freedoms.

C FORMS OF DESIGN OPTIMISATION

Design optimisation may be generally seen as the search for a product that is 'better' in some way than those already existing. To achieve this, the designer needs to have in mind a set of potential forms that his product might take, together with any requirements that it must satisfy and the measures of performance that he may use to discriminate between alternatives to determine which he may consider 'best'. To apply an optimisation strategy, these alternatives need to be characterised by a set of parameters.

There were four distinct forms of optimisation identified within an early FENet report [1] as applicable to the structural form of a product. Each one tends to require a different solution strategy. The forms of optimisation are described below in order of increasing generality. In a given design situation any combination of these forms can be present, together with further properties not related to the geometric form of the product.

i) SIZE OPTIMISATION.

The structure is defined by a series of sizes and dimensions. Combinations of these sizes and dimensions are sought that achieve the optimisation criteria. There are two major categories of problems in size optimisation.

Firstly, it is discrete structures, including pin- and rigid-jointed structures, that have received to most attention over the last forty years. A structural layout is defined and its loads and support conditions are prescribed. The sizes of the members are adjusted according to the optimisation goal(s). If member sizes are allowed go to zero then they are unnecessary and a much reduced structure can be produced with changed topology, this situation is sometimes called layout optimisation.

Socondly, continuum structures including aircraft style structures such as wing layouts comprising spars and ribs, stiffened panels and carbon fibre laminates have olso received considerable study. The structure can be describes as series of sizes or parameters, such at stiffener pitch, skin thickness and ply-angle. Optimisation techniques are then used to find the combination of design variables that give a minimum weight design subject to a variety of constraints.

ii) SHAPE OPTIMISATION

Structural shape optimisation comes in two distinct forms, one where there is some small region of detail than needs to be sculpted such that the maximum local stress is minimised; this is referred to as local shape optimisation. Secondly the whole profile of a structure can be investigated to determine what is best; this is referred to as global shape optimisation.

With local shape optimisation the topology of the structure is known and there is some aspect of detail that is giving rise to a high stress, such as a fillet or a notch. The object of shape optimisation is to find the best shape that will have the best stress outcome.

Shape is not only of interest to the structural designer. It is also the principal driver for aerodynamic or hydrodynamic design optimisation. Here the objective is likely to be the minimisation of drag, combined with requirements for the generation of lift.

iii) TOPOGRAPHY OPTIMISATION

This is the least studied form of structural optimisation. In its simplest form it can be the drape of a shell surface in space that best meets the design criteria. This is an interesting area of development that can be applied to the design of stadium roofing and other tent-like structures.

iv) TOPOLOGY OPTIMISATION.

Topology optimisation exists where the actual form of the structure is unknown in advance. Only the spatial extent in which the structure is to exist may be known, together with the optimality criteria and design constraints that are to be applied.

v) NON-GEOMETRIC OPTIMISATION

Whilst the above categories of design focus upon the geometric form of the product other characteristics may also be of importance. Examples could include damping characteristics, material selection and gains in actively controlled systems.

3: OPTIMISATION THEORY

Before covering the use of optimisation algorithms in PSO in section 4, it is useful to describe and classify the main optimisation methods. Further detail is provided in a recent FENet report [19].

A BASIC CONCEPTS

I) DESIGN VARIABLES

As discussed above, most engineering systems are defined by a set of quantities that can be modified during the project design. Some of these quantities are kept fixed and consequently are called assigned parameters; conversely the remaining quantities that are variable during the project design, are called design variables, and can be set, from a mathematical point of view, in the vector of design variables

$$\mathbf{X} = \begin{pmatrix} x_1 & x_2 & \dots & x_n \end{pmatrix}^T$$

This may be viewed as a vector space in which each point corresponds to a particular design. If there are very few design variables the design may be viewed graphically, as shown in figure 4. The variables may be represented by real numbers where continuous variation is possible or by integers if only discrete options are allowed.

ii) CONSTRAINTS

In most common problems, it is necessary to define other requirements which must be satisfied for the design to be considered valid. For instance, in structural optimisation, it will be necessary to check if the structure is strong enough to resist the loads imposed during service, that is $\sigma \leq \overline{\sigma}$ where $\overline{\sigma}$ is the maximum admissible stress. In optimisation terms, such conditions are known as constraints.

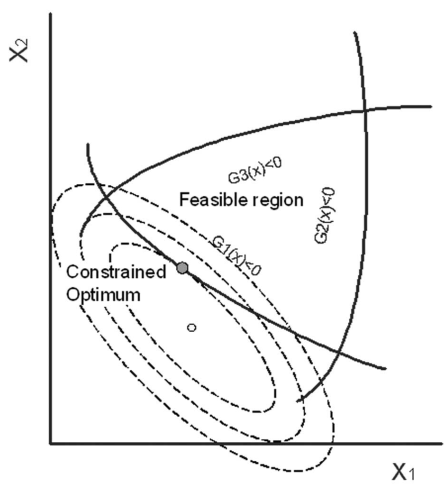

Figure 4: Design space showing constraints and contours of the objective function

Typically, any constraint may be expressed as a function $g_j(\mathbf{X}) \leq 0$, whose surface $g_j(\mathbf{X}) = 0$ divides the design space in two regions: one for which $g_j(\mathbf{X}) \leq 0$ is satisfied, the space of admissible solutions also known as the 'feasible region', and the space of inadmissible solutions for which $g_j(\mathbf{X}) > 0$.

iii) OBJECTIVE FUNCTION

An optimisation methodology aims to find the variant of the designed product that is, in some respects, the best. Generally, the design process will give more than one configuration that satisfies the design requirements. Consequently, it is necessary to have in mind a further criterion by which different configurations may be ranked: this criterion is called the objective function , sometimes known as the fitness function. The choice of the objective function depends on the nature of the optimisation problem; in the turbo-machinery, it may be the thermo-fluid-dynamic efficiency, in structural mechanics the weight of the system, in finance the gain, etc.

iv) OPTIMALITY CONDITIONS

Combining these elements we obtain the generic mathematical formulation of the constrained optimisation problem, namely:

Find X that maximises/minimises	$f(\mathbf{X})$
subject to the constraints	$g_j(\mathbf{X}) \leq 0$, j=1,2…m
and	$h_j(\mathbf{X}) = 0$, j=1,2…p.

Returning to figure 4, above, we show the objective function as a set of contour lines within a hypothetical design space. In the absence of constraints and assuming the variables are continuous, the optimum is given by the condition

$$\frac{ff}{fx_i} = 0, \quad i = 1,2\ldots n.$$

In the presence of the constraints, the maximum of the function found by the optimisation process reduces and the new optimum satisfies the Kuhn-Tucker necessary conditions:

$$\frac{\partial f}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_j}{\partial x_i} = 0, \quad i = 1,2\ldots n,$$
$$\lambda_j g_j = 0, \quad j = 1,2\ldots m.$$

The optimisation problem as illustrated in figure 4 is a non-linear constrained optimisation problem. In the absence of constraints, the problem reverts to an unconstrained problem. A further special case arises if the objective function and constraints are linear in which case the optimisation formulation is described as a linear programming problem. In each case, the simplification results in an optimisation problem which is far easier to solve.

v) MULTI-OBJECTIVE OPTIMISATION

The definition of a single objective function is however not always straight-forward. If, for example, an aerodynamic profile is optimised to minimise drag resistance, the profile found will be a characterised by a near zero lift force, a situation that cannot be accepted by the designer. In such a case, one option is to define an additional objective function, namely lift, to be optimised simultaneously. This form of optimisation, common in the project design, is known as multi-objective optimisation. The classic way to solve this class of optimistion given the two functions $f_1(\mathbf{X})$ and $f_2(\mathbf{X})$, is to combine the objectives by defining the function:

$$f(\mathbf{X}) = \alpha_1 f_1(\mathbf{X}) = \alpha_2 f_1(\mathbf{X})$$

where α_1 and EMBED α_2 are constants that define the relative importance of the objectives with respect to one another. This approach does not offer an ideal solution technique for such problems and; for this reason, new algorithms have been developed whose basic feature is the simultaneous optimisation of the objective functions. This avoids the use of a functional to reduce the problem from multi-objective to single-objective (at least in the initial phases of the optimisation, since in the final phases it is possible to select a single fitness function).

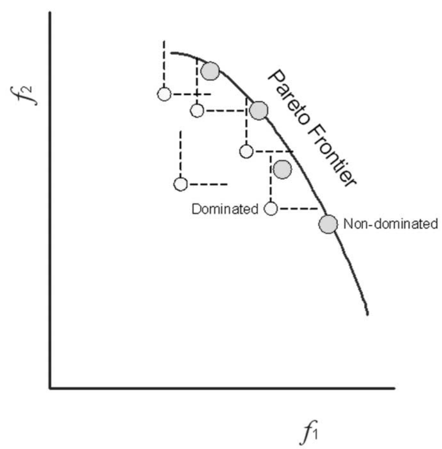

Figure 5: Objective space showing a Pareto-frontier

In such a case, the objective space provides a useful graphical representation of the problem. Here, each point represents one or more designs plotted against their key performance metrics, rather than the values of design variable that define the product as before. The various designs may be categorised as dominated or non-dominated according to the existence, or otherwise, of a design that is better in all respects to the current design. Ultimately, any performance gain can only be achieved by sacrificing some other objective, this being a defining characteristic of the Pareto frontier (figure 5) that provides a bounding envelope passing through the set of non-dominated solutions.

vi) ABSOLUTE AND RELATIVE MAXIMUM
A further consideration of relevance when using optimisation is the possible existence of local optima, such as that shown in figure 6. Many search algorithms will tend to locate the nearest maximum point rather than the absolute maximum. The robustness of an optimisation algorithm relates to the likelihood of it locating the absolute maximum.

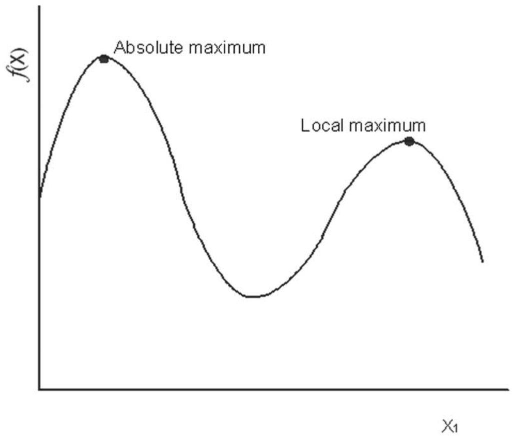

Figure 6: Local versus absolute maxima

B CLASSES OF SOLUTION ALGORITHM

i) Gradient based methods (hill climbing)
Such methods are based on the iterative search of an improved point and use the gradient of the function, as calculated in the previous point, to guide the path of the optimisation. The need to know or to be able to approximate the partial derivatives of the objective function and any constraints is a significant requirement of such methods, both in terms of the calculation cost and the implied limitation to continuously variable problems. The simplest method of this class is the method of steepest decent/accent, attributed to Cauchy, which steps in the direction of the gradient.

The best convergence to the optimum is achieved by the Newton method. This uses first and second derivatives of the objective function to form a local quadratic approximation to the objective function. The maximum of the quadratic function is obtained at a single step by setting the gradient vector to zero, resulting in a set of linear equations. The limitation of Newton's method as a practical algorithm is its requirement for second derivatives.

There are several algorithms, like quasi-Newton, BFGS, Powell, SQP, Conjugate Gradient, etc, which accumulate information from gradients, calculated at past steps, to provide approximations to second derivative terms.

Where constraints are present, a sequence of linearly constrained quadratic approximations (SQP) is commonly used although other methods are available. In the extreme case of an optimum defined largely by constraints, a linear approximation to the objective function and constraint set is sufficient to solve the problem. The simplex Linear Programming method moves from vertex to vertex of the constraint set, progressively replacing one constraint at a time to increase the value of the objective function. Where the problem exhibits some non-linearity the method can be used recursively on a sequence of linear approximations in the SLP method.

A key characteristic of all the above methods is that they act on the basis of local information, making a sequence of improvements until a local optimum is reached. This means that they are not robust in the sense that they are not assured of reaching the point associated with the absolute maximum.

ii) Heuristics
In general heuristic methods combine a strategy for generating new points to search the design space with a selection process to identify the best trials. Among the heuristics algorithms, the most commonly used ones are the Evolutionary Algorithms, and in particular the Genetic Algorithms (GA). They are robust, since they can be used for real and discrete variables, in highly or weakly non-linear problem types, for global search or refinement and they can also address multi-objective problems.

Genetic Algorithms are based on an analogy with the biological evolution, since the configurations defined by the different combination of variables, or individuals, improve their fitness generation after generation. Other implementations of evolutionary algorithms simply take random steps from a single parent to generate a further generation of candidate designs. Strategies to control the step-length are important: the technique of simulated annealing, for example, uses an analogy with the solidification of crystals to regulate convergence.

Other heuristic techniques are simply sets of rules that are intended to lead to an improved point. If they are based on a strategy to satisfy the Kuhn-Tucker necessary conditions above then they should at least lead to a local solution of the optimisation problem, otherwise they are likely to converge to an arbitrary point with no real expectation of optimality, despite exhibiting some desirable characteristic.

4: THE APPLICATION OF OPTIMISATION TO PRODUCT DESIGN

A consensus was reached during project that it is not possible to define a-priori what is the best optimisation methodology but, by classifying the problems being addressed it is possible to derive "rules-of-thumb" that can suggest to the engineer which strategies are likely to be efficient in providing performance improvements for a given optimisation problem. This is summarised in the 'applicability table' to be found in the report [19].

In this section, we pick up on the content of the section 2A, covering historical context, and develop it further. The organisation of this section mirrors that of section 2C in which the forms of optimisation relevant to PSO are outlined. Broadly this takes us from the highly-idealised design problems, amenable to solution with very few function evaluations that characterised early work in the field to far more generic approaches introduced in recent years.

A SIZE OPTIMISATION.

Size optimisation has traditionally been used within the aircraft industry by structural engineers. The starting point is a defined topology and structural shape and the goal is to reduce mass, even at the expense of increased manufacturing cost. Only the thickness or cross-sectional areas of structural members are used as design variables.

i) The stress-ratio method

The simplest technique which may be used for design for strength is the stress-ratio method. This predicts the stress in a structural member by assuming the stress resultant to remain constant, as it would for a statically-determinate structure, thus:

$$\sigma(t) = \frac{N}{t}.$$

The value for thickness taken to correspond to a fully-stressed structural member is therefore given by

$$t^{+} = \frac{N}{\overline{\sigma}} = \frac{\sigma(t)}{\overline{\sigma}}t.$$

This formula is applied recursively in order to satisfy the constraint equations within the optimality conditions. This is an example of a heuristic approach that is known to give rise to reasonably light and strong structures despite the fact that optimality is not assured other than for a statically determinate structure under a single loadcase. The attraction of the approach is that it is not necessary to calculate design sensitivities (gradients) and so it can be used effectively with 100,000s of design variables and many times that number of potential constraints.

ii) Gradient-based methods including SQP

Many size optimisation codes were developed within the Aerospace companies to address a wide range of structural performance constraints including strength, stiffness, local and overall buckling, natural frequency, vibration reduction, static divergence and aeroelastic flutter. Broadly, this is possible because the required design sensitivities are available at little cost. Given the FE equations for statics:

$$\mathbf{Ku} = \mathbf{p}$$

taking partial derivative gives

$$\frac{\partial \mathbf{u}}{\partial x} = -\mathbf{K}^{-1}\left\{\frac{\partial \mathbf{K}}{\partial x}\mathbf{u}\right\}$$

where the load **p** is assumed to remain unchanged. Similar algebraic expressions can be calculated for each of the other constraint types addressed and the gradient of mass, as the objective function, is known explicitly. These programs can be used effectively with 1000s of design variables and, typically, only require 10-20 function evaluations (by FE analysis).

B SHAPE OPTIMISATION

i) Structures – gradient-based methods

Some of the programs developed by aerospace companies are capable of including shape optimisation, although here both the task of choosing a good parameterisation and calculating design sensitivities are not nearly as simple. Problems of this class are typically run with 100s of design variables although 1000s are possible. If analytic sensitivities are not available then a finite difference approach is needed in which one additional analysis is required for each design variable to give:

$$\frac{\partial g_i}{\partial x_j} \cong \frac{g_i\left(x_j + \Delta x_j\right) - g_i\left(x_j\right)}{\Delta x_j}.$$

The need for repeated analysis significantly reduces the number of design variables it is practical to use in the design process.

ii) CFD – gradient-based methods

Shape optimisation is also relevant to aerodynamicists. A typical problem might be to reduce drag at cruise whilst maintaining adequate lift during the low sub-sonic take-off state. Again, the selection of suitable parameterisation for a wing profile is not straightforward and design sensitivities are normally calculated using finite difference approximations as above, although analytic derivatives can be calculated using adjoint methods. An iterative CFD solver will normally re-converge given a small perturbation in far less time than that taken for the reference solution so gradient-based methods are practical even for the computationally expensive Navier-Stokes codes. Typically the number of design parameters would be in the 10s.

Although this section describes the use of CFD in the context of aerodynamics design, the same approach is used in the context of hydrodynamics for ship design.

iii) CFD – heuristic methods

For this last problem many practitioners would choose a heuristic method such as GA. Although the cost of taking a population of designs through several generations may be high, design sensitivities are not required and the robustness of the method is attractive. Moreover the exploration of Pareto frontiers through the use of multi-objective GA (MOGA) follows naturally. Elapsed times for optimisation runs can also be considerably shortened if one is fortunate enough to have access to multiple processor machines since GA is a natural candidate for coarse-grained parallel processing.

C TOPOLOGY OPTIMISATION.

Some of the most innovative applications of PSO are in the area of topology optimisation. These approaches tend to be highly heuristic. Typically the approach adopted is to define a fixed ground mesh of elements which are progressively selected or removed according to some optimality criterion. It is like starting a sculpture with a big block of material and chipping away till a topology emerges that best meets the criteria.

Topology optimisation is appropriate where the actual form of the structure is not prescribed in advance. What is specified is the spatial extent in which the structure may exist, together with the optimality criteria and design constraints that need to be applied.

In Aerospace the use of topology optimisation for the design of ribs in the leading edge of the A380 wing has been well-publicised. In mechanical engineering, topology optimisation has been used to determine the topology of cast components such as automotive suspension arms. An initial design concept can be developed by designing for minimum compliance but, ultimately, there are static and dynamics constraints and manufacturing constraints to do with the costs of competing construction techniques, forging, stamping, welding, investment casting need to be addressed.

The traditional single criterion which has been used to guide to selection of material in topology optimisation is the fully stressed design (FSD), in which each part of the final structure is at the same stress. As discussed above in the context of size optimisation, the absence of any reference to an objective function in the formulation means that there is no reason to expect an optimum design will result. The maximum compliance approaches more closely follow the distribution consistent with the Kuhn-Tucker conditions.

Over the last few years, methods have emerged that can detect an optimal topology under multiple criteria and multi constraints. The criteria consist of stress, stiffness, buckling load, frequency and moment of inertia. Indeed any physical quantity that can be measured together with any physical process that can be analysed can now be included in topology optimisation

There are currently several methods that are effective in solving commercial topology optimisation problems, including: Optimality Criteria [16] based a dual formulation of the original design problem; Homogenization [17] which assumes micro-structural form of the material selected by mathematical programming techniques and Evolutionary Structural Optimisation (ESO) [18] in which elements of a fine FE mesh are slowly removed driven by optimality conditions drawn from the KT conditions with multiple constraints.

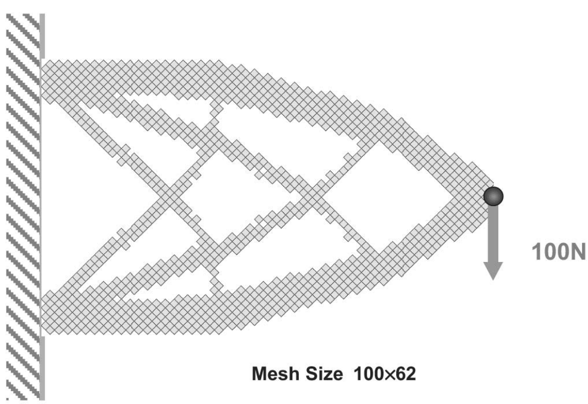

Figure 7: ESO applied to of Michell cantilever problem

D NON-GEOMETRIC OPTIMISATION

As the scope of the PSO process broadens there is an increasing tendency for non-geometric parameters to enter the optimisation. Cost is one example with NASA conducting studies with "cost as an independent variable" (CAIV). Broadly, the output from such a study is cost/performance trade off data in the form of a Pareto frontier which provides critical information to aid decision making.

5: PROCESS INTEGRATION

A PROCESS DEFINITION AND CONTROL

So far in this paper, attention has focussed upon the tools and techniques of PSO and the degree of usage that have been found practical in support of the industrial product design process. A further area of increasing importance, that has not been addressed this far, is the context in which these tools are used. In recent years, a number of process integration frameworks have become available with the potential to make the tools required for PSO far more accessible

From a survey conducted within FENet, it is clear that companies operate a wide variety of internal optimisation processes that suit their product and that these internal processes contribute to the success of their business. It follows that it is more important to give the company the tools to integrate algorithms and processes that best suit them than to try to develop a very general purpose programme that suits nobody.

The most basic integration approach simply requires the ability to run a number of executable programs from the operating system. Greater flexibility can be achieved by using one of a number of scripting languages which have become available including Python, Visual Basic and Java. The Process Integration frameworks take this a step further by offering a graphical environment in which the user can define and run chains of tools, with the exchange of data, to suit his company's objectives.

Some process integration frameworks, such as Optistruct (Altair), BossQuattro (Samcef) and modeFrontier (Enginsoft) placed the major focus on optimisation tools whilst others, including ModelCenter (Pheonix Integration), had their principle focus on Enterprise integration. The market leader in the area, iSight (Engenious), and Optimus (LMS/Noesis) have traditionally occupied the middle ground but an overall convergence of functionality is evident. Process integration frameworks allow simulation tools to be brought together using resources from across the distributed computer networks within the company, or even going outside to access resources across the internet (security permitting), and at the same time offer a wide range of embedded tools and utilities such as Response Surface Methods, statistical tools to support Robust Design and support for graphical presentation.

Legacy codes are 'wrapped' so that data fields from both their input and output files exposed to the system allowing data to be exchanged throughout the process. For the more modern codes, run through the use of a graphical user interface (GUI), it may be necessary to open GUIs for the individual tools incorporated into a process chain. More often, however, the application will be run without user-intervention with control coming through the use of any API offered by the application. Such issues are discussed in a FENet report [20].

A typical process chain may include steps such as accessing key defining parameters for a potential product from a product data model along with a specification of the design requirements and the intended operating environment. These parameters may then be passed into a CAD system to allow the creation of a geometric representation of the design concept. In turn this is likely to be used to support the construction of a number of numerical models used to capture aspects of the product performance.

Once an automated process for running product assessment tools is established, a large number of repeat runs can be carried out to explore the potential design space. Finally, output from the system needs to be brought together to support a decision making within the product improvement process. Provided the number of factors controlling the design is reasonably small the response can also be captured using response surface analysis, below.

B <u>SUPPORTING TECHNOLOGIES</u>

i) Response Surface Methodology

RSM attempts to approximate the output of expensive computational or experimental processes by a sequence of much simpler interpolations in which functions are approximated by response surfaces. Typically it combines Design of Experiments (DOE) with Response Surface Analysis (RSA) [21].

The DOE part of this is a traditional method first developed in Japan for manufacturing quality control. In essence it is a method that, given a set of input parameters with upper and lower limits on the values they may take, determines the minimum number of function evaluation required to provide information relating to the significance of individual variables, in isolation and in combination. Several DOE methodologies exist including: random and quasi-random sampling, factorial DOE (systematic sampling on pre-defined variables intervals) and Orthogonal Arrays (sampling is done according to orthogonal arrays by Taguchi or Fisher).

The RSA methodologies serve to interpolate available data in order to predict, locally or globally, the correlation between variables and objectives. Methods include: RSA with linear coefficients using Polynomials, Taylor Series or Fourier Series and Linear Kriging. Methods with non-linear coefficients include Neural Network and Gaussian Processes.
Response Surfaces are particularly valuable when a large number of repeated studies are required to support multi-objective trade-off studies in which the problem may be reformulated many times or for robust design studies in which Monte-Carlo simulation can be used on the constructed response surface.

The major limitation in the use of RSM leads is the excessive number of response function evaluations required for anything but a very small number of parameters. Also it should be bourn in mind that a response surface model is not a substitute for a proper physics-based model, it can interpolate reasonably well but the quality of extrapolation beyond the available data could be poor.

ii)Robust Design Optimisation.
It is one thing to determine some optimal set of design variables, but another to be able to say that the design is robust. By robust is meant that a small variation in one design variable, or group of variables, will not cause a major shift in the response. In other words the minimum that represents the optimum is as flat as possible rather that a steep sided valley.

The need for Robust Design method appears in many contexts [22]. During the preliminary design process, the exact value of some input parameters is not known. For example, in turbo-machinery this could be the case of the mass flow rate or the inlet pressure, or in aeronautics, the flight speed, the angle of attack, the air temperature, etc. Consequently, the aim is try to look for a solution that exhibits as weak dependence as possible on the unknown input parameters. Another important concern in the design optimisation is to find a solution that is insensitive to small manufacturing process errors. In some cases traditional single-point optimisation tends to 'over-optimise' the solution, producing a final design that offers good performances at the design point but has poor off-design characteristics. Many numerical methods have been developed for optimal design under uncertainty in the input parameters: [23-26].

iii) Reliability Based Design Optimisation.
These methods can examine ranges of design data with probabilistic effects on occurrence and report on the most sensitive issues. As discussed above, there can often be a difference between the design for an object and how it is eventually made, parts can be undersize or over, holes can be in different locations; material values, strength allowables in particular, can vary widely. The term Stochastic FEA is being used for this type of situation, in recent years the word fuzzy data has been used. Other techniques used here include Monte-Carlo Methods whereby data sets that span the range of potential variation are shot through the solver(s) and the resulting cloud of results again scanned for sensitivity or insensitivity. In addition the range of physical processed incorporated into the design can include, fluid, thermal, electro-magnetic as well as the traditional structural.

6: CONCLUSIONS

It is evident that Product and System Optimisation does not represent a single technology, so there are no simple measures of technological maturity and industrial uptake available for this State of the Art review. Rather the subject may be characterised by a range of techniques, beginning with highly restrictive but computationally efficient techniques for size optimisation, through somewhat heuristic approaches to topological design and, finally, reaching the new generation of process integration tools capable of supporting design at a systems engineering level.

For each aspect of the technology, guidance may be given as to the likelihood of gaining meaningful results in the light of past application. The lack of shared functionality has also made the development of benchmarks difficult for the PSO task within FENet; advice tends to be qualitative rather than quantitive. For example, it is difficult to compare the effectiveness of size optimisation for strength, operating with 100,000 design variables, with a multidisciplinary study using response surfaces that may struggle with 10 variables. To a large extent, the problem formulations, as described, are mapped to the needs of the available solution techniques, rather than the other way round. The extent to which this produces a match with industrial needs varies with industry sector.

Overall, Product Optimisation is reasonably well established in the sense of technology maturity level, although the limited uptake in industrial design means it has yet to make as full a contribution to product improvement as one may wish to see. The position as far as System Optimisation is concerned is less clear.

The first references to the subject in FENet used the term to refer to the manufacture and processing that goes into making the product. Few examples have been found to demonstrate maturity in this area although techniques such as the Design of Experiments would appear to be well-suited to the problem. Most of these complex processes are multi-physics in character it is likely that further development in use of physics-based modelling must precede the wider use of optimisation.

A second interpretation of the term System Optimisation uses it to refer to matching the behaviour of the product within an assembly to meet wider performance goals, in response to customer requirements. Here, the relatively new breed of Process Integration tools contributes to the task, although stronger links between system engineering principles and the product design at a detailed level of are probably required at corporate level as well as that of tool integration. The total current market for Process Integration tools is probably less than a tenth of size it should be for the health and quality of product design.

As a final thought: designing without an understanding of the mathematics of optimisation is like backing horses without understanding probability theory – both are possible …

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