Isogeometric Analysis
Toward Unification of Computer Aided Design and Finite Element Analysis

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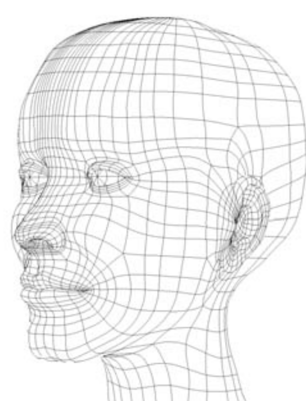
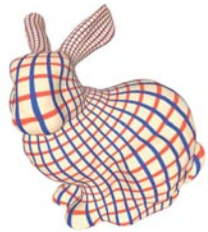
Outline

- Isogeometric analysis
- NURBS
- Structures
- Vibrations
- Wave propagation
- Nonlinear solids
- Fluids and fluid-structure interaction
- Phase-field modeling
- Cardiovascular simulation
- Design-to-analysis
- T-splines
- Conclusions

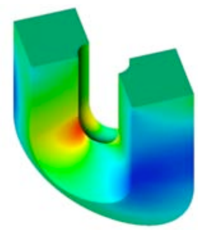


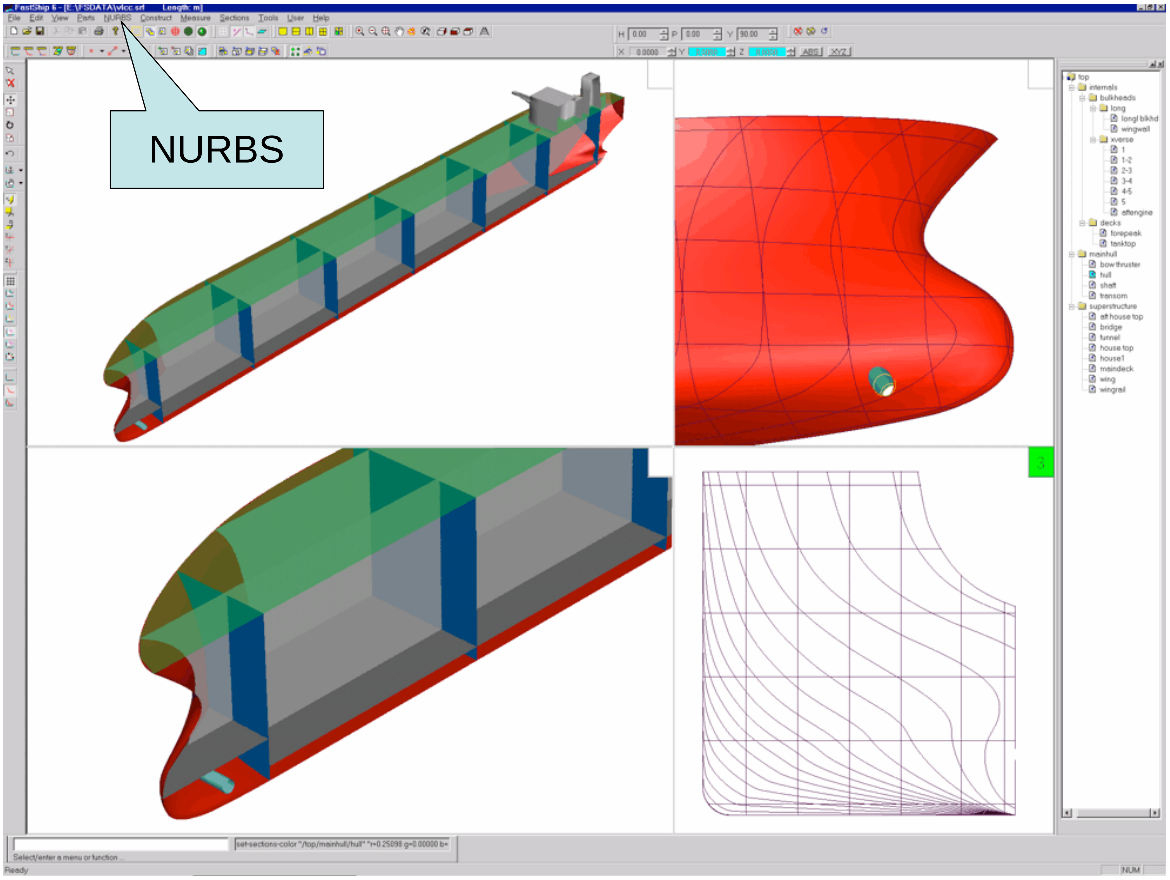
Isogeometric Analysis

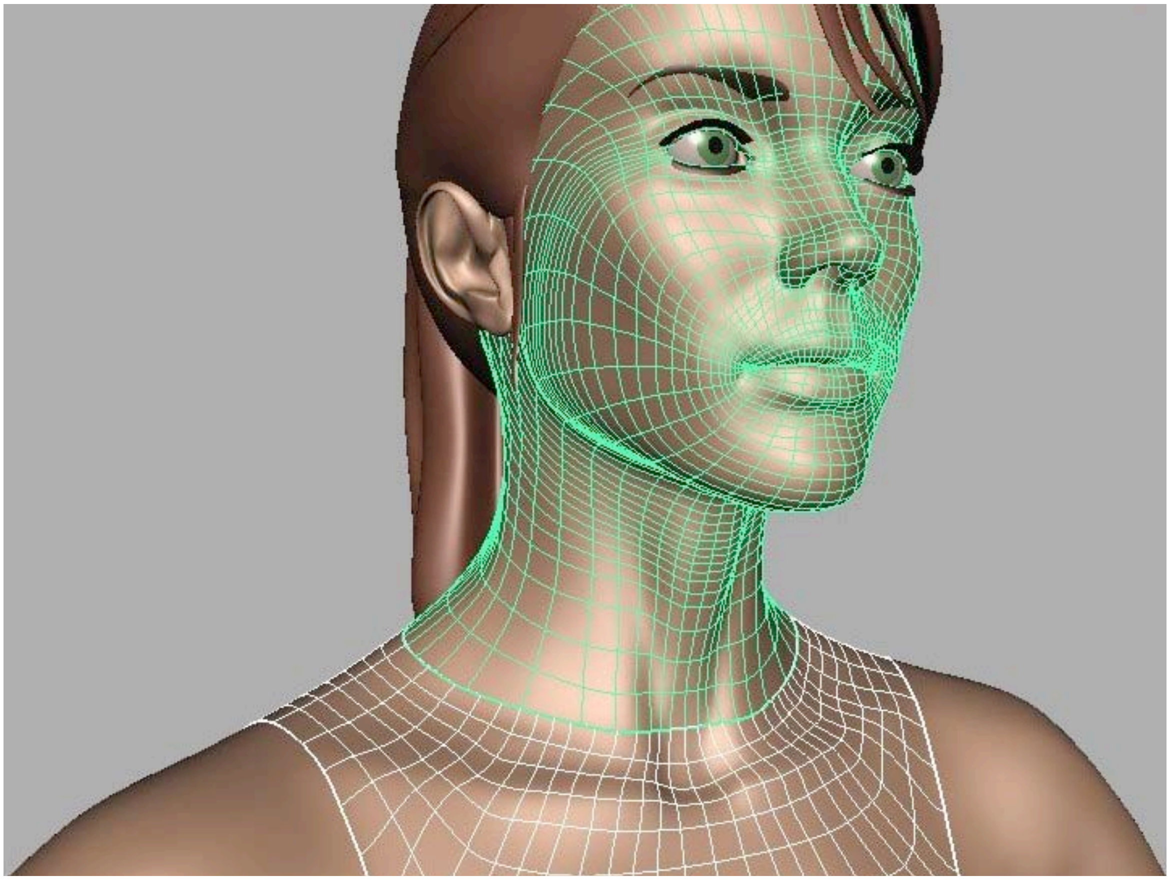
- Based on technologies (e.g., NURBS) from *computational geometry* used in:
  - Design
  - Animation
  - Graphic art
  - Visualization

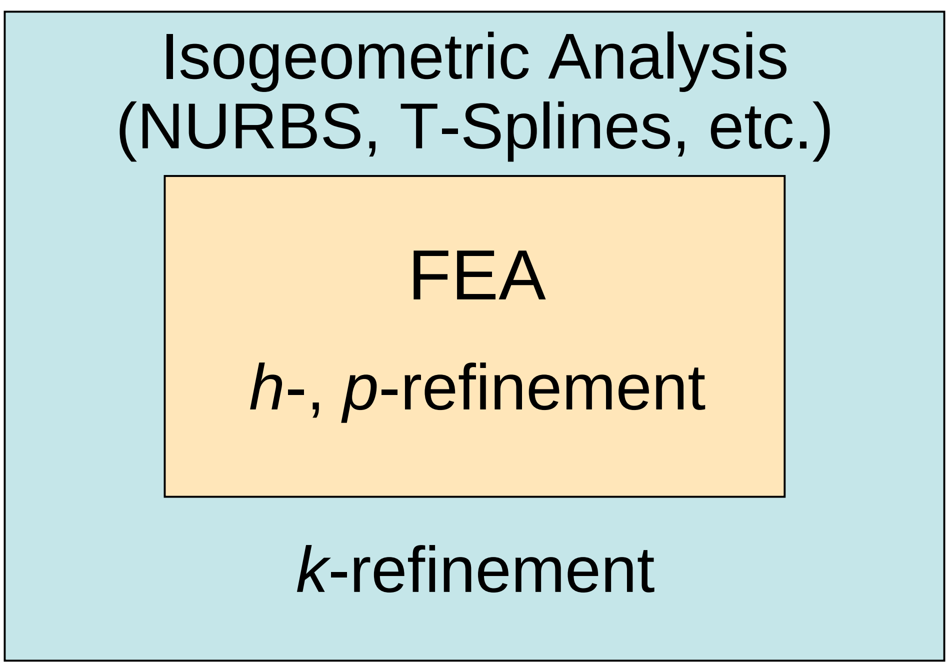


- Includes standard FEA as a special case, but offers other possibilities:
  - Precise and efficient geometric modeling
  - Simplified mesh refinement
  - Smooth basis functions with compact support
  - Superior approximation properties
  - *Integration* of design and analysis




NURBS

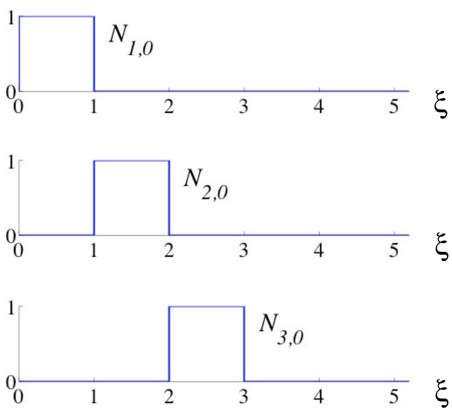


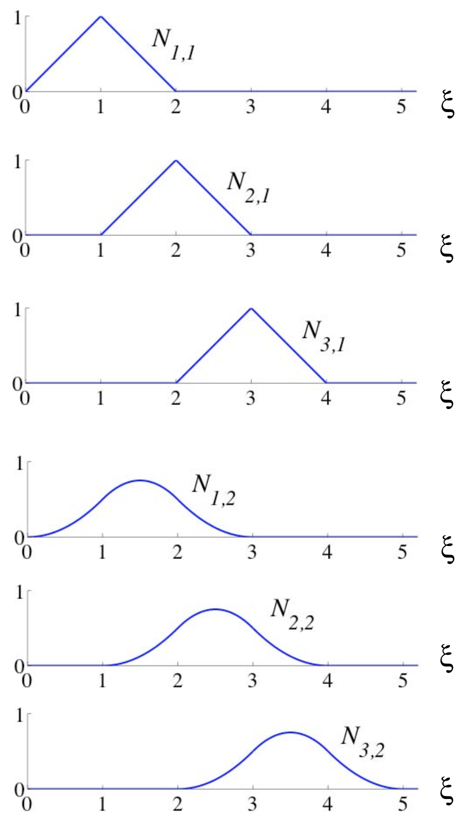

Isogeometric Analysis
(NURBS, T-Splines, etc.)
FEA
h-, *p*-refinement
k-refinement

B-Splines

B-spline Basis Functions

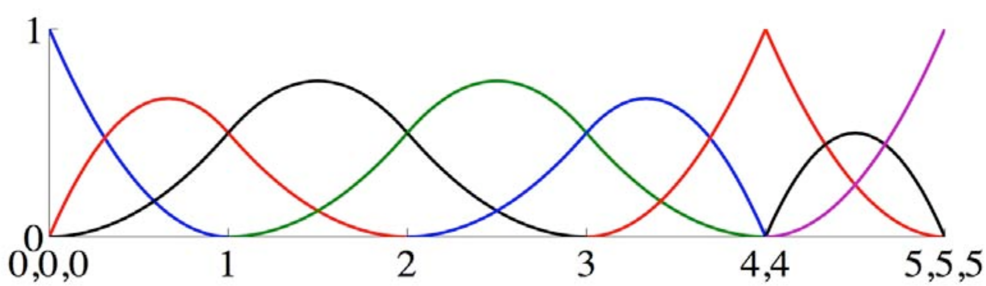
- $$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases}$$
- $$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) \;+$$
$$\frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$





B-spline basis functions of order 0, 1, 2 for a *uniform knot vector:*

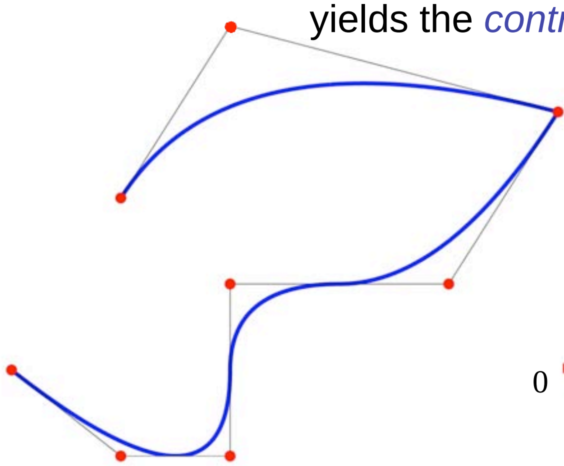
$$\Xi = \{0,1,2,3,4,\ldots\}$$

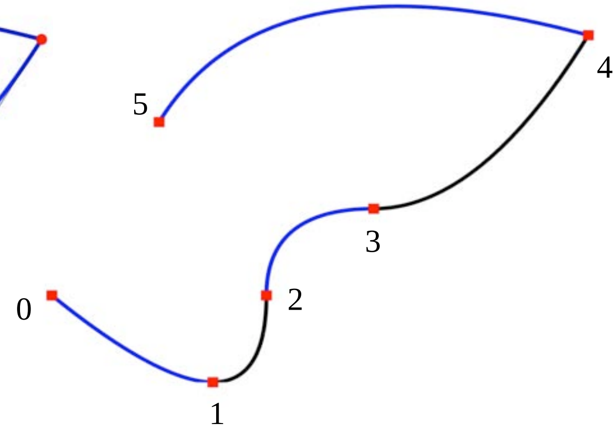


Quadratic (p=2) basis functions for an *open, non-uniform knot vector:*

$$\Xi = \{0,0,0,1,2,3,4,4,5,5,5\}$$

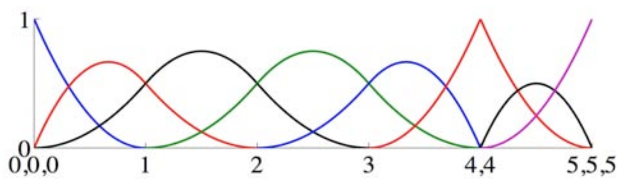
Linear interpolation of control points
yields the *control polygon*



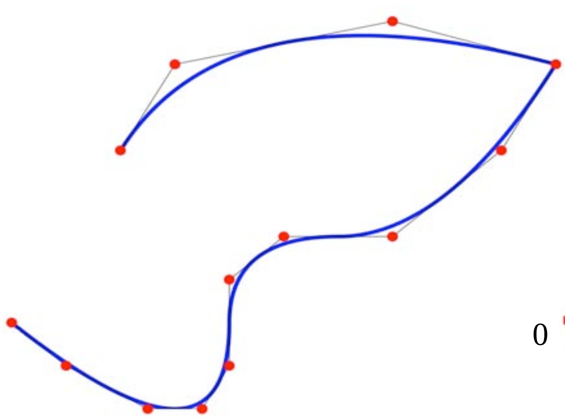


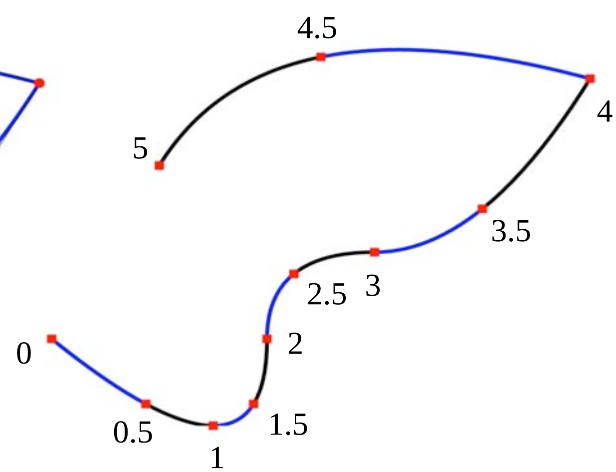
• - control points ■ - knots

Quadratic basis



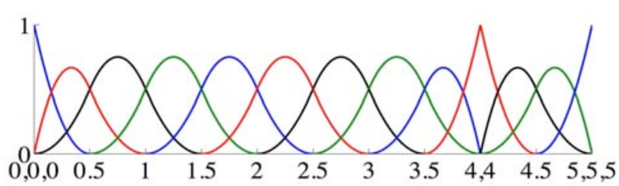
h-refined Curve



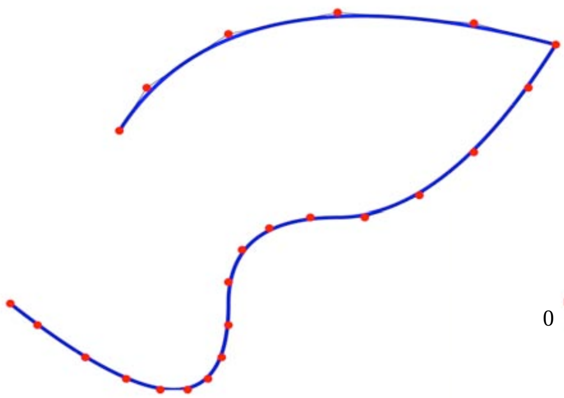


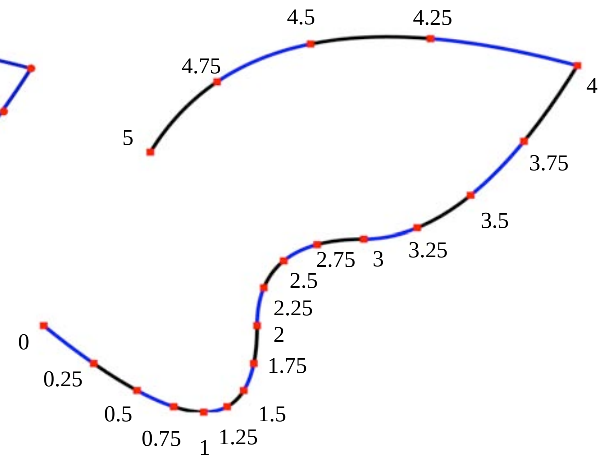
• - control points ■ - knots

Quadratic basis



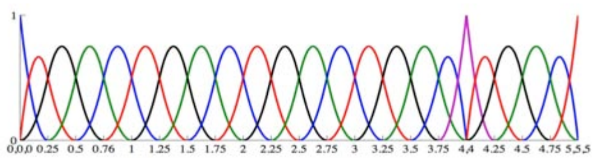
Further *h*-refined Curve



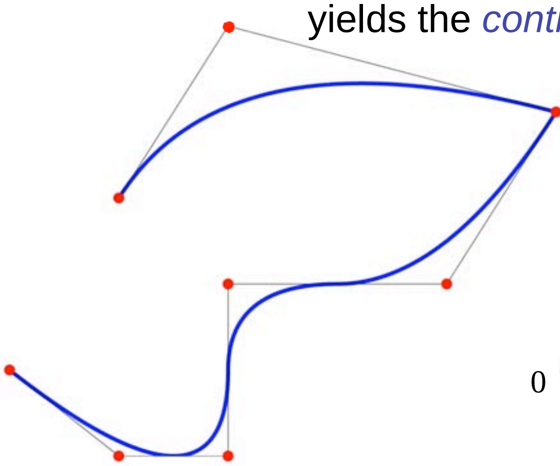


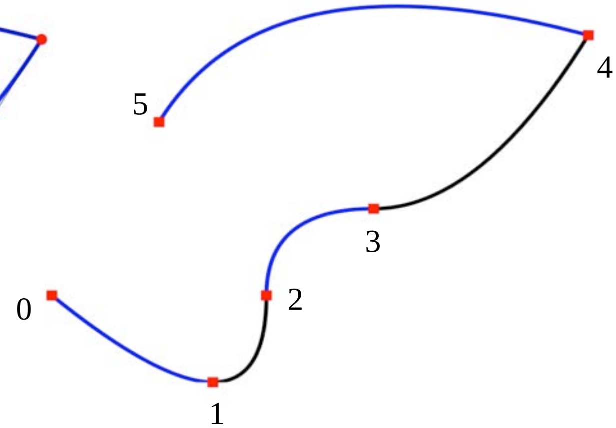
• - control points ■ - knots

Quadratic basis



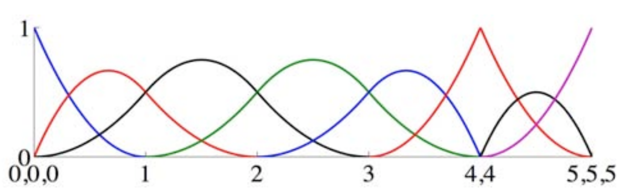
Linear interpolation of control points yields the *control polygon*



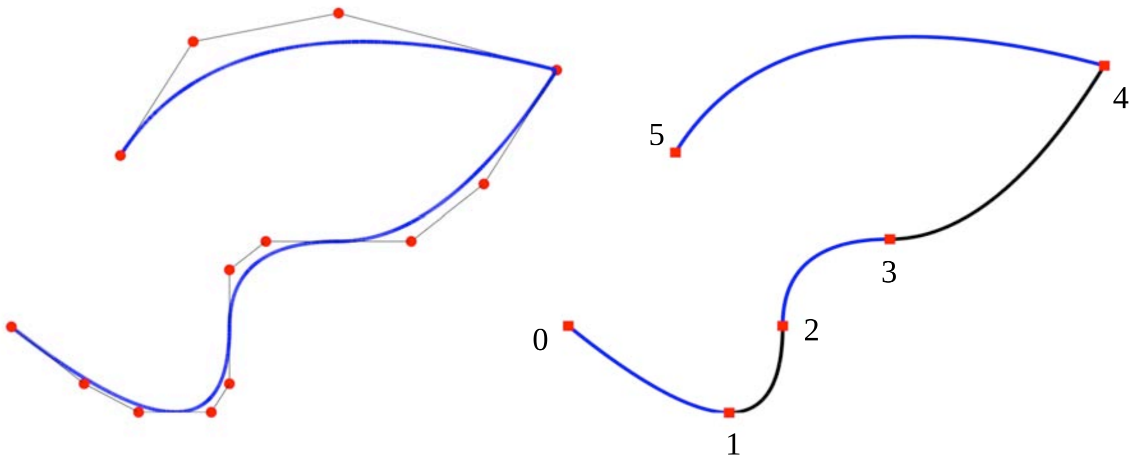


• - control points ■ - knots

Quadratic basis

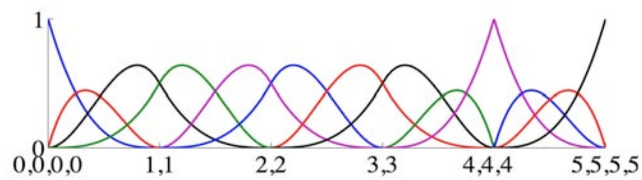


Cubic *p*-refined Curve

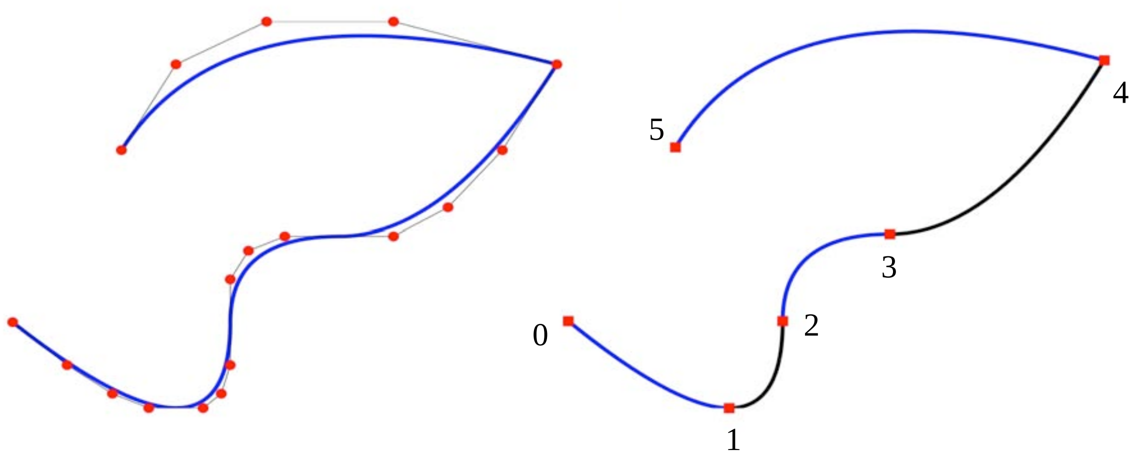


● - control points ■ - knots

Cubic basis

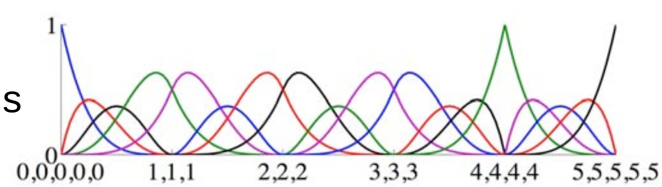


Quartic *p*-refined Curve



● - control points ■ - knots

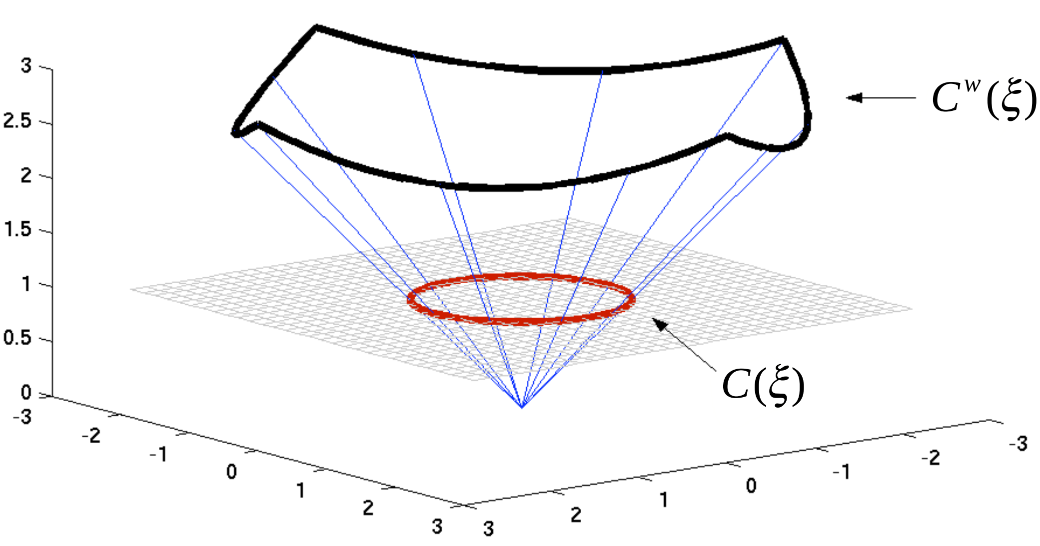
Quartic basis



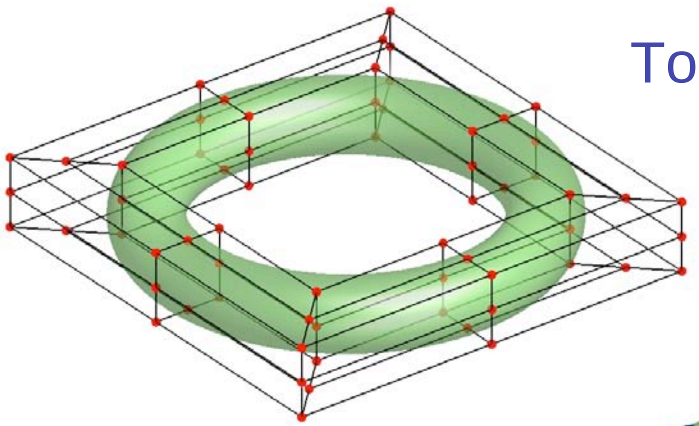
NURBS

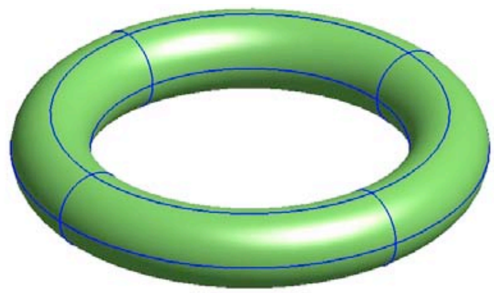
Non-Uniform Rational B-splines

Circle from 3D Piecewise Quadratic Curves

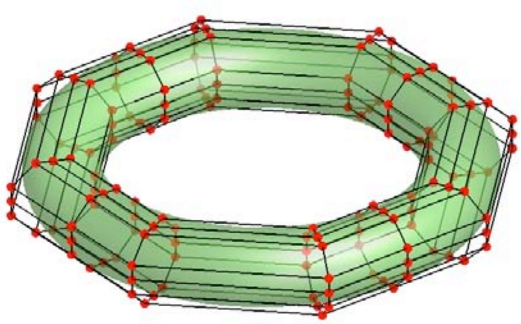


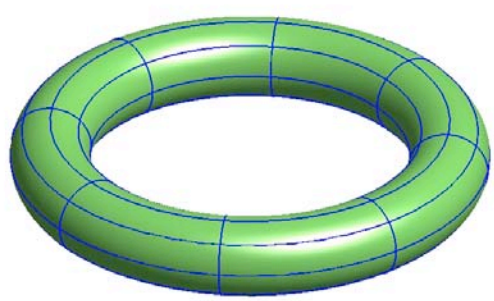
Toroidal Surface


Control net

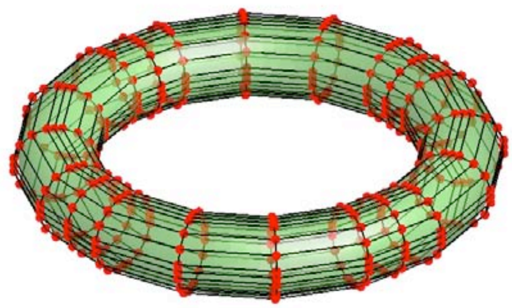

Mesh

h-refined Surface

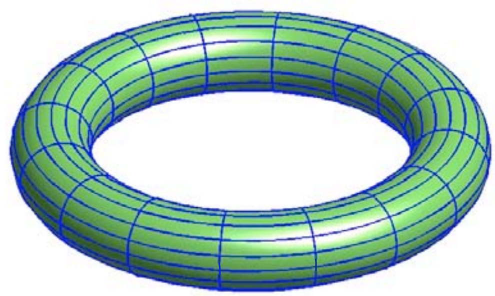

Control net


Mesh

Further *h*-refined Surface

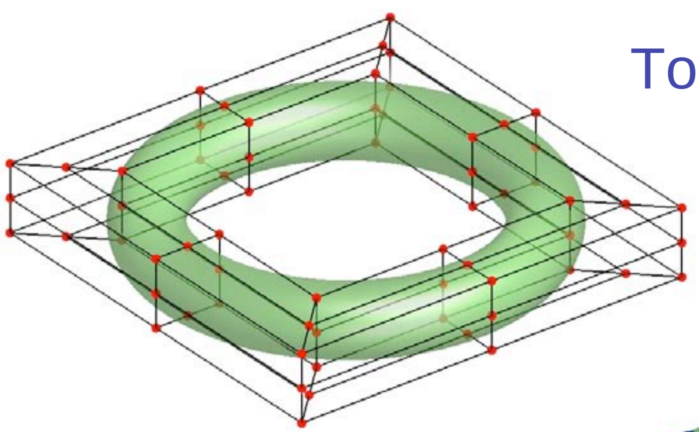


Control net

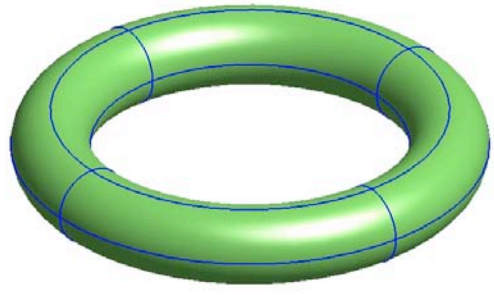


Mesh

Toroidal Surface

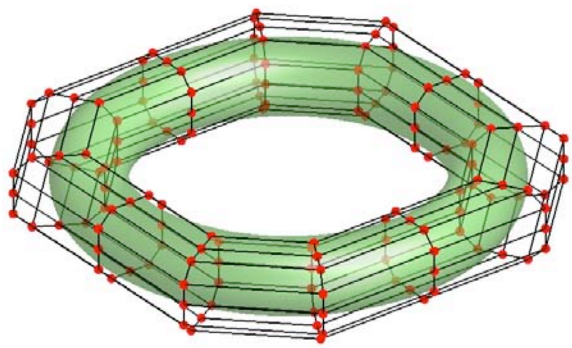


Control net

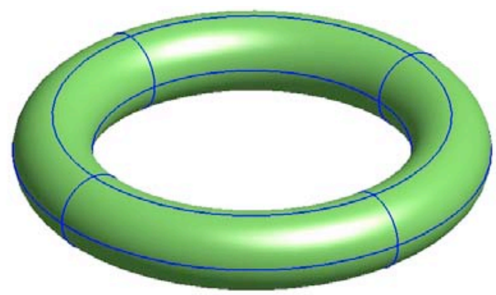


Mesh

Cubic p-refined Surface

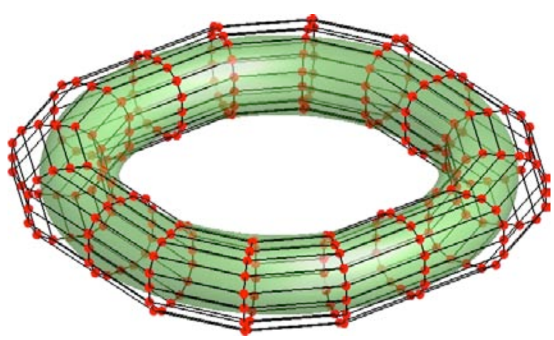


Control net

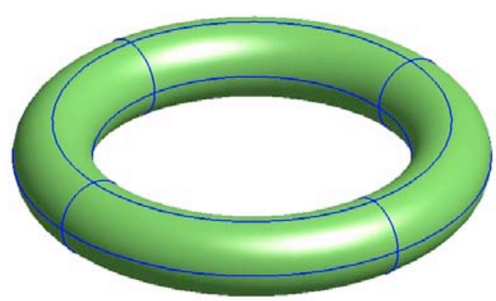


Mesh

Quartic p-refined Surface

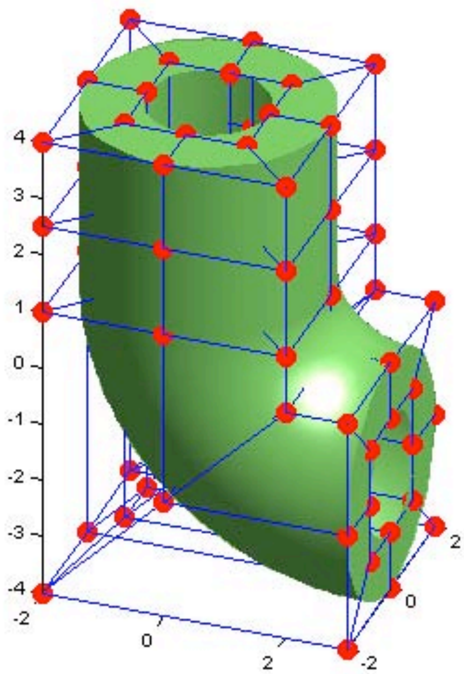


Control net

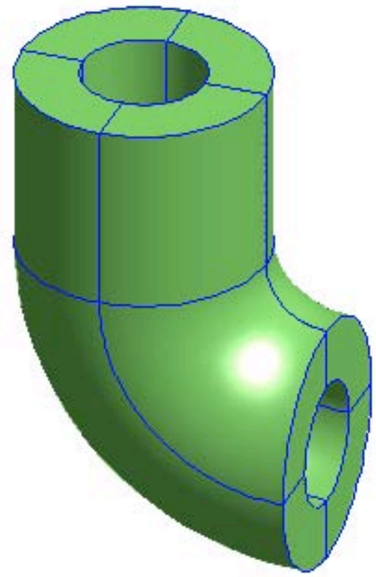


Mesh

Control Net

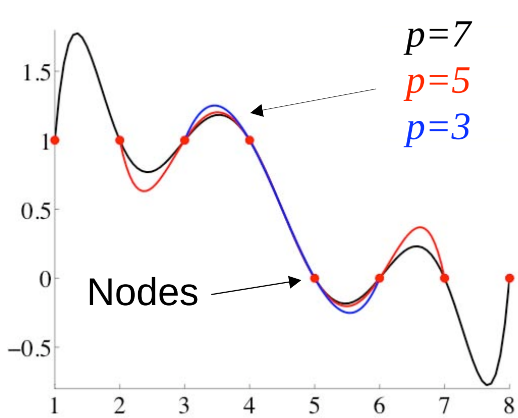


Mesh

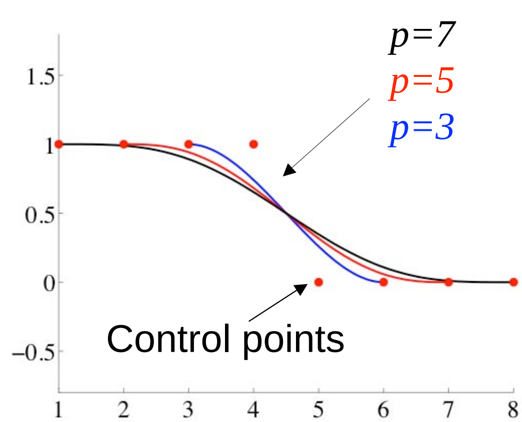


Variation Diminishing Property

Lagrange polynomials



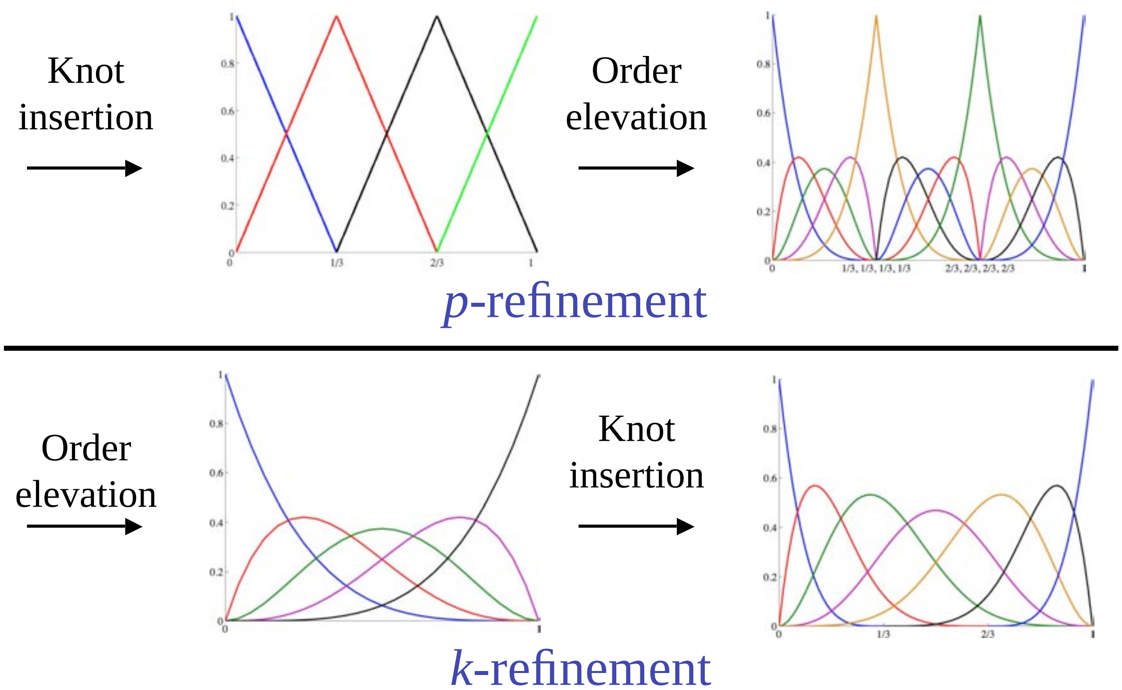
NURBS



Finite Element Analysis and Isogeometric Analysis

▪ Compact support
▪ Partition of unity
▪ Affine covariance
▪ Isoparametric concept
▪ Patch tests satisfied

Three Quartic Elements

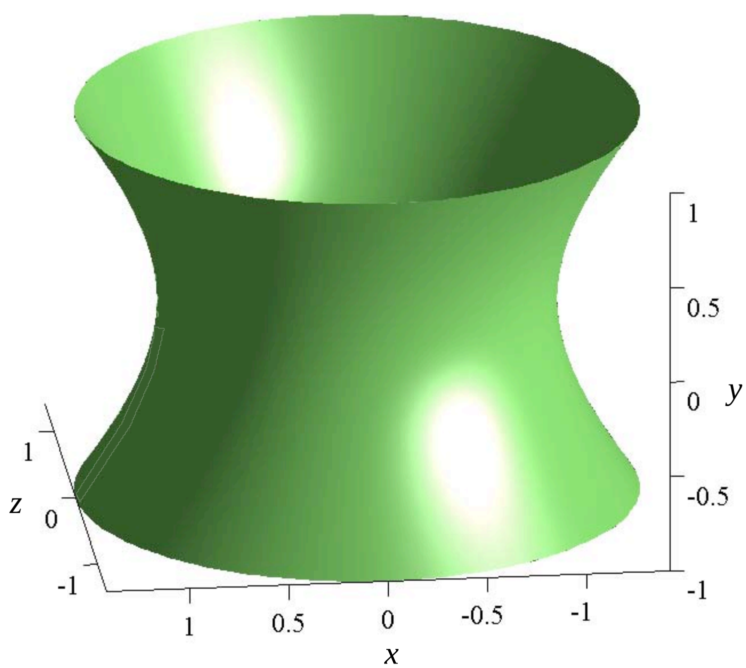


Structural Analysis

- Isoparametric NURBS elements exactly represent all *rigid body motions* and *constant strain states*

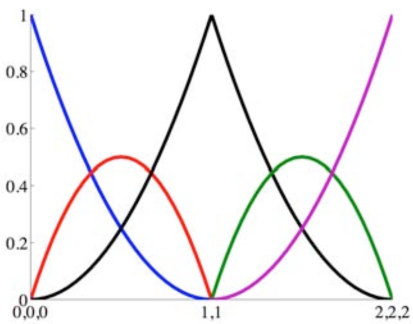
Hyperboloidal Shell

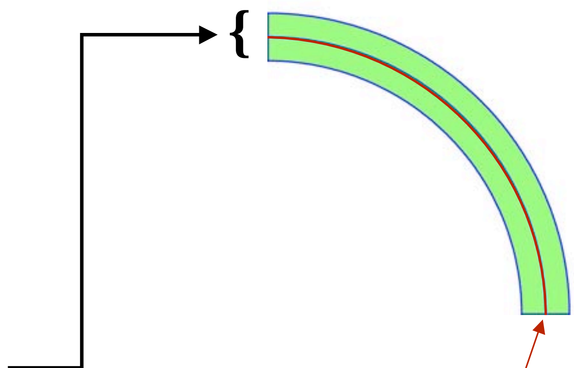
- Mid-surface:
  - $x^2 + z^2 - y^2 = 1$
  - $-1 \leq y \leq 1$
- $R/t = 10^3$
- Fixed at the top and bottom
- Loading:
  - $p = p_0 \cos 2\theta$



Thickness Discretization

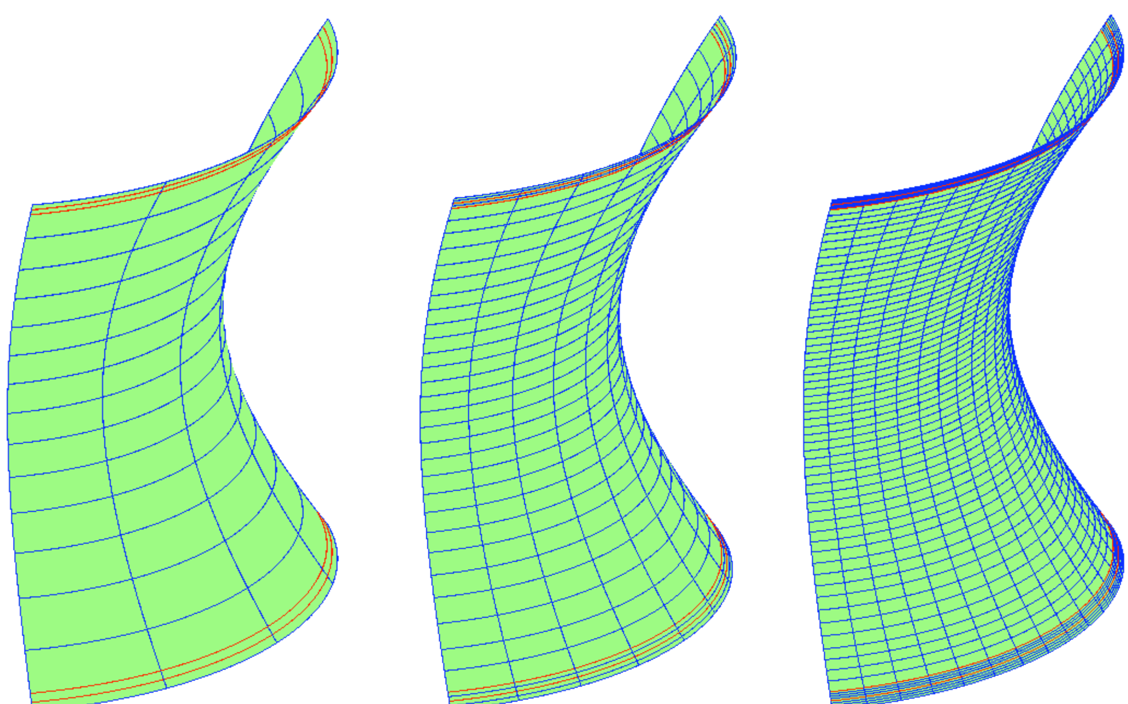
Rational quadratic basis functions through the thickness





The NURBS surface defining the mid-surface is geometrically exact

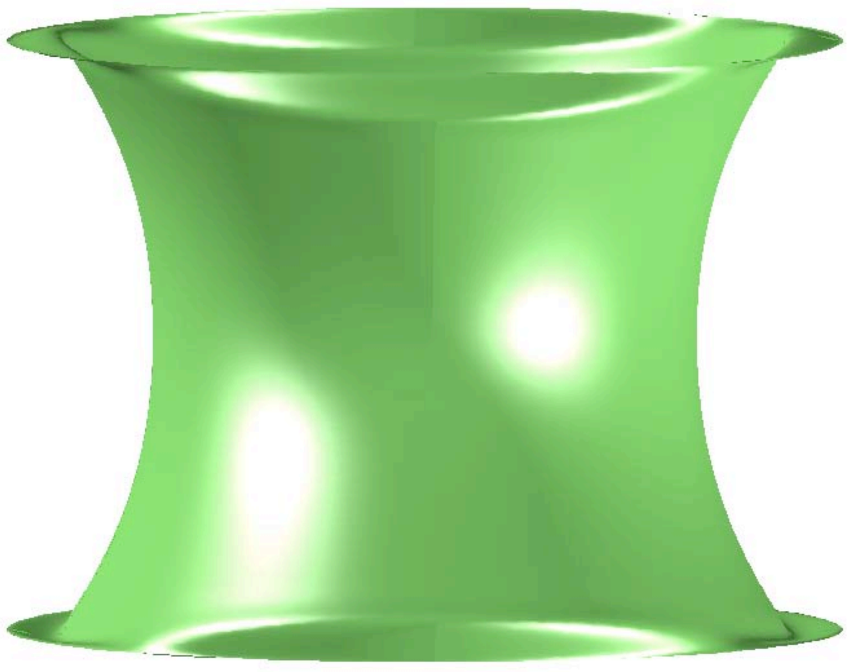
Surface Discretization



Mesh 1 Mesh 2 Mesh 3

View 1

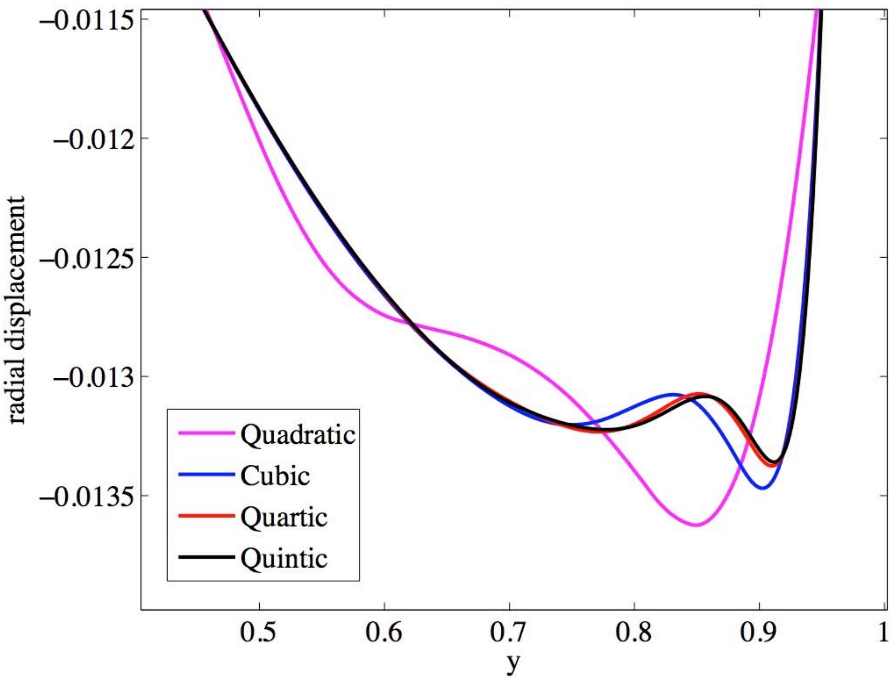
(displacement amplification factor of 10)



View 2

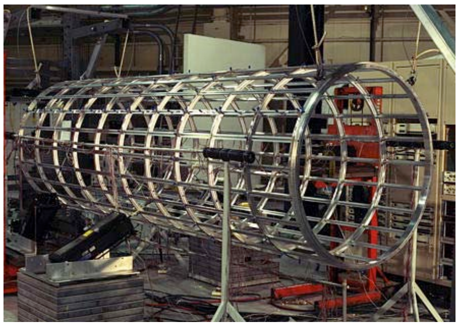
(displacement amplification factor of 10)

Detail of Radial Displacement at Compression Lobe (Mesh 3)

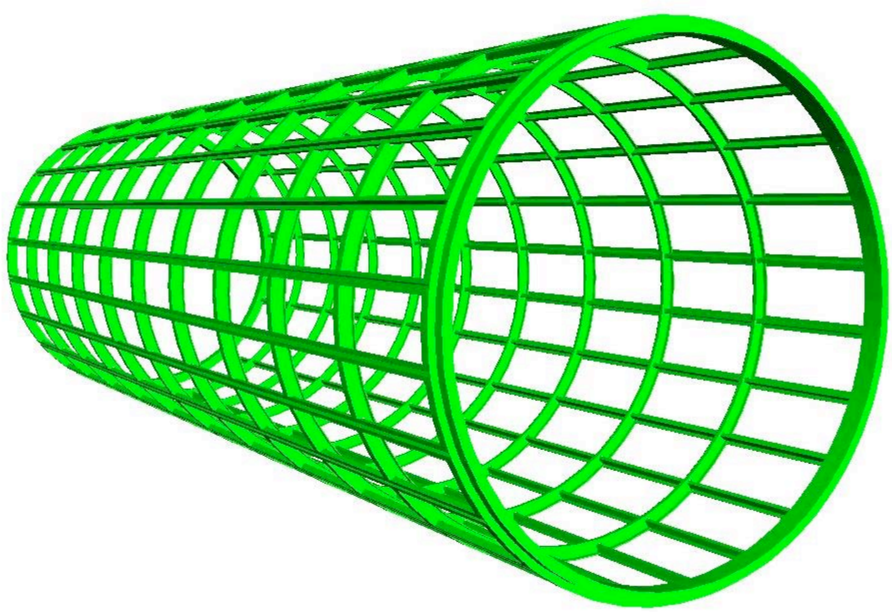


Vibration Analysis

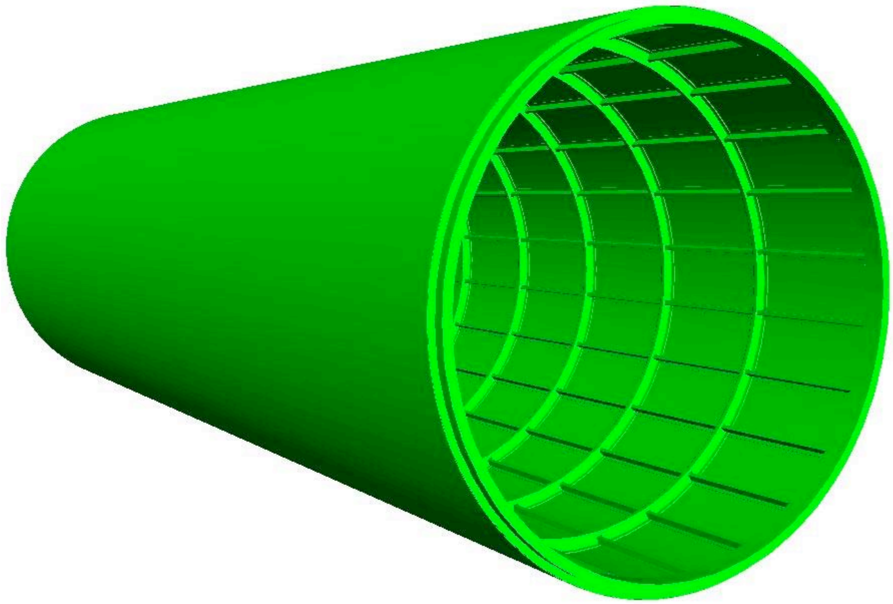
NASA Aluminum Testbed Cylinder (ATC)



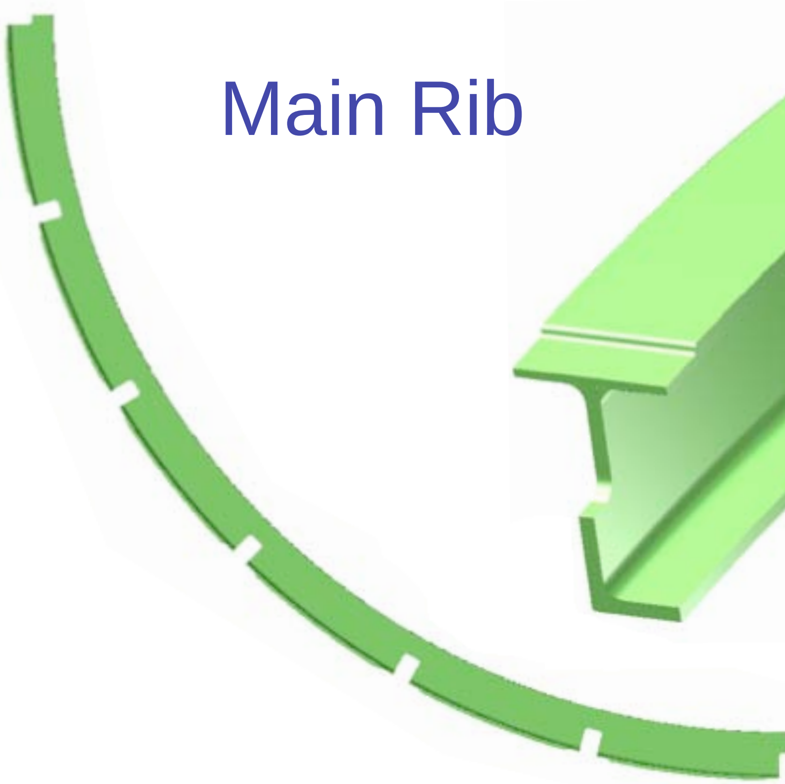
NASA ATC Frame

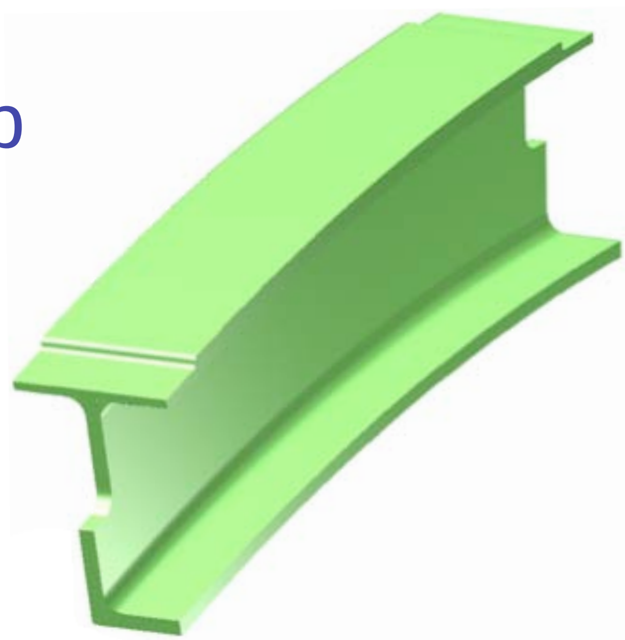


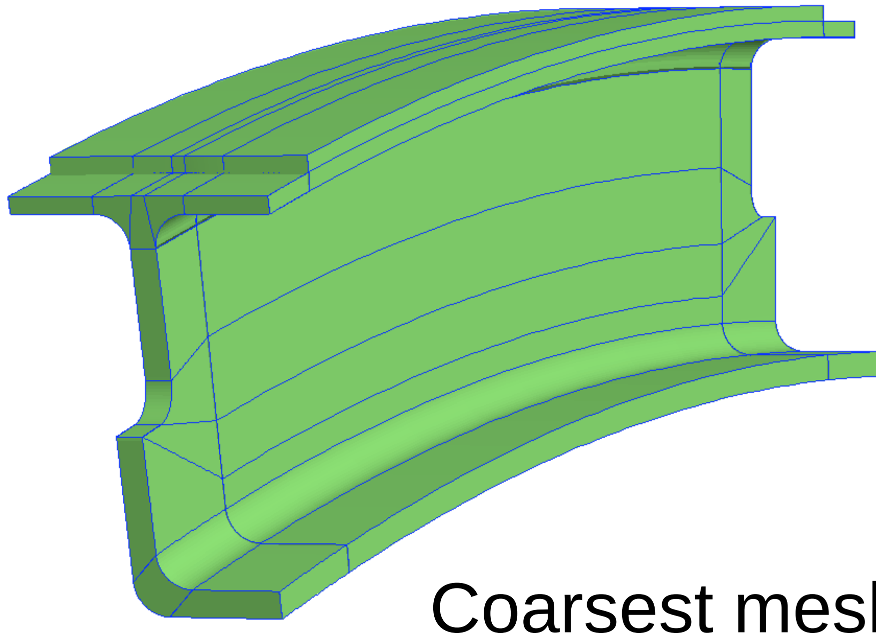
NASA ATC Frame and Skin



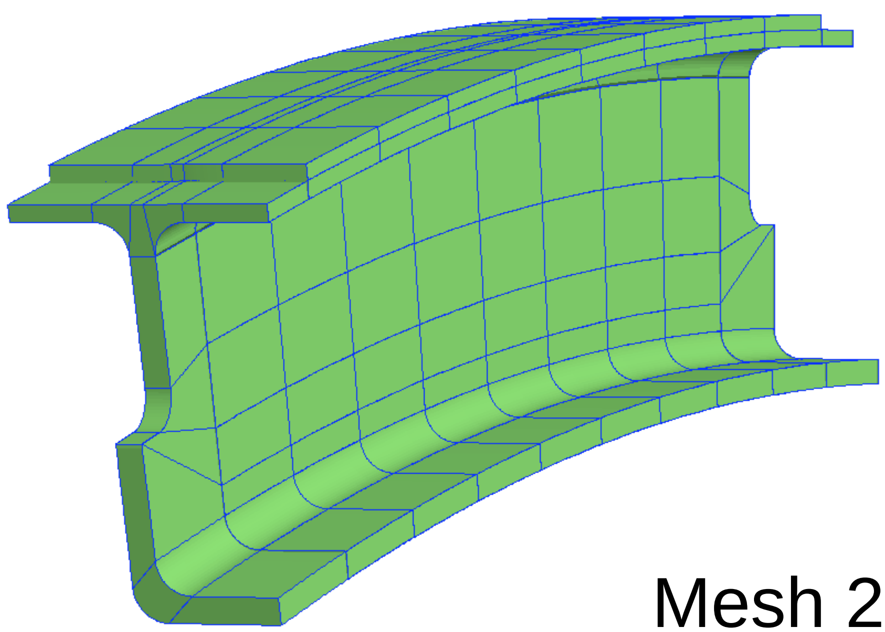
Main Rib



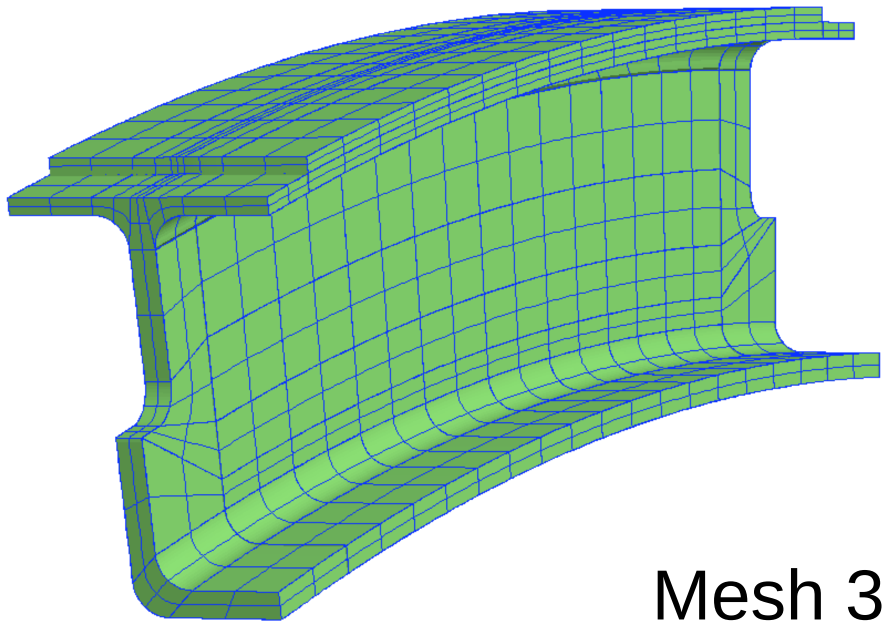




Coarsest mesh
15° segment of main rib

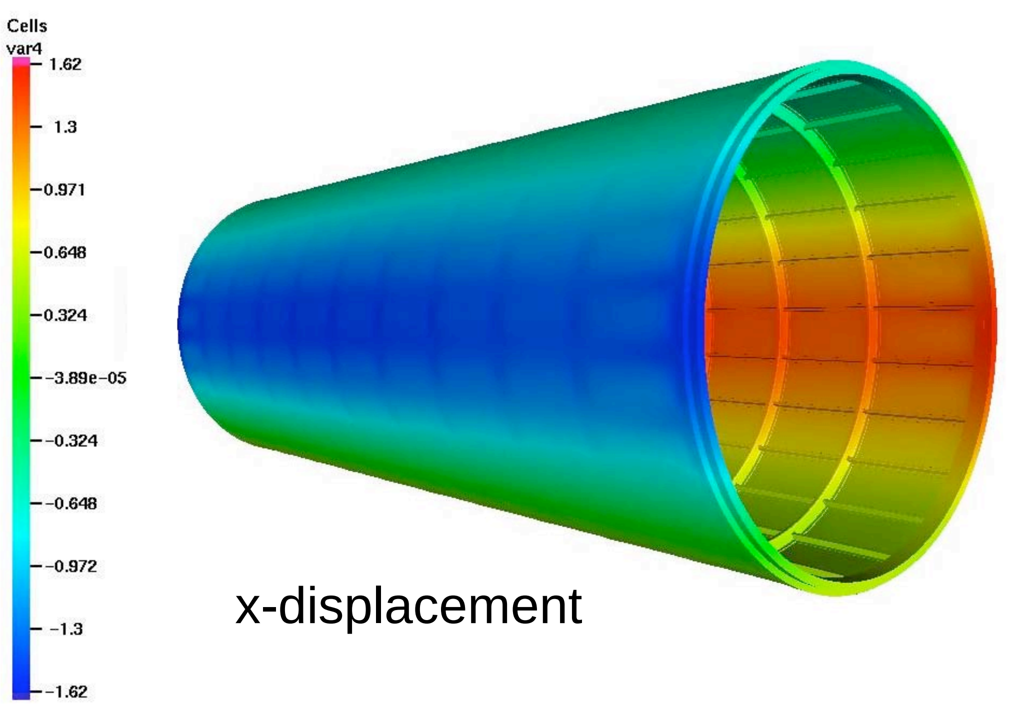


Mesh 2



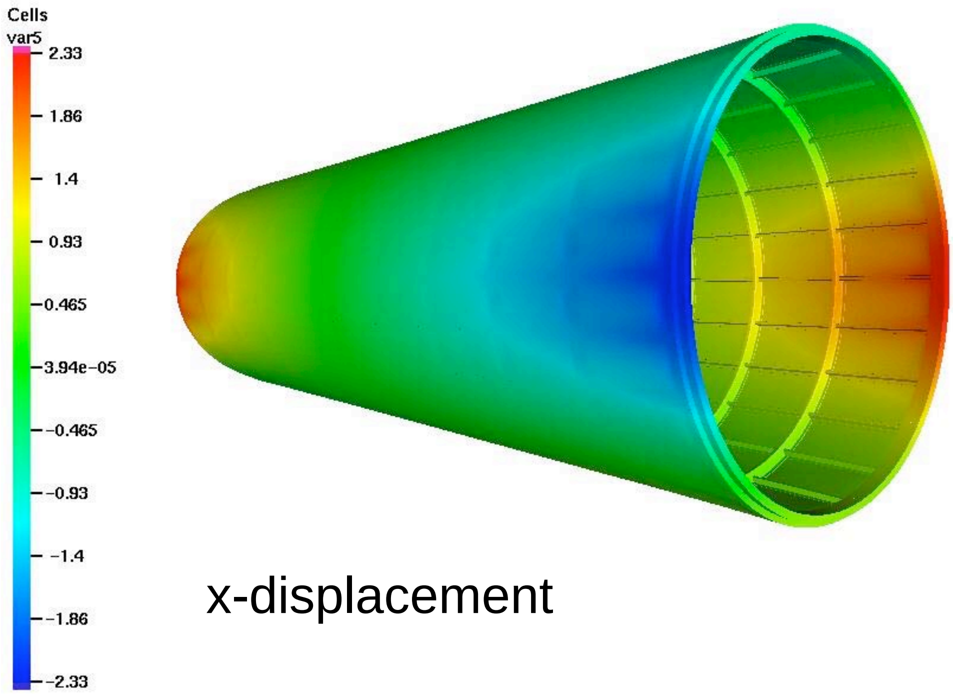
Mesh 3

First Rayleigh Mode



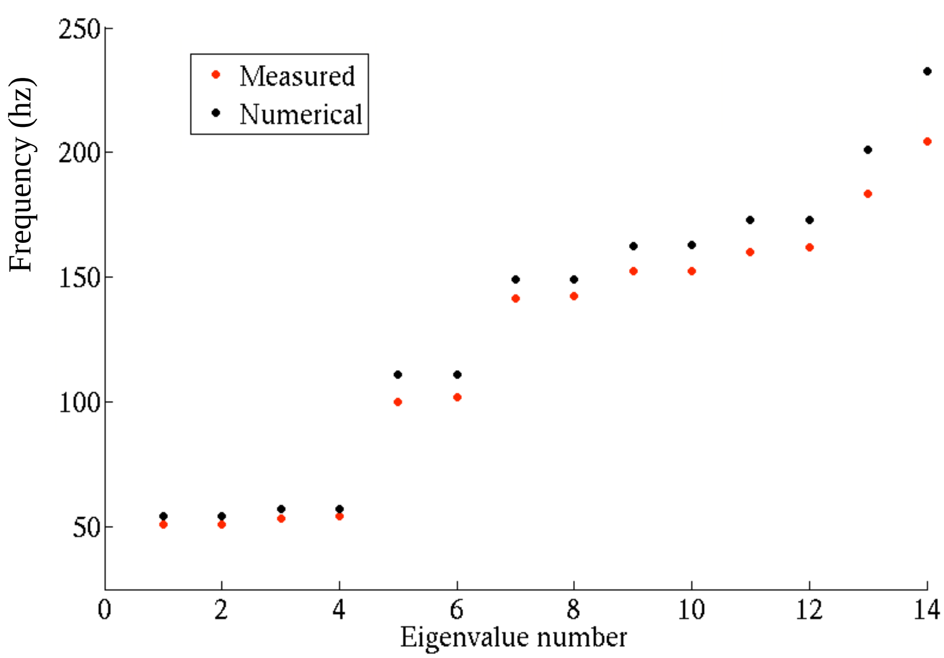
x-displacement

First Love Mode



x-displacement

ATC Frame and Skin



Vibration of a Finite Elastic Rod with Fixed Ends

Problem:

$$\begin{cases} u_{,xx} + \omega^2 u = 0 \quad \text{for} \quad x \in (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

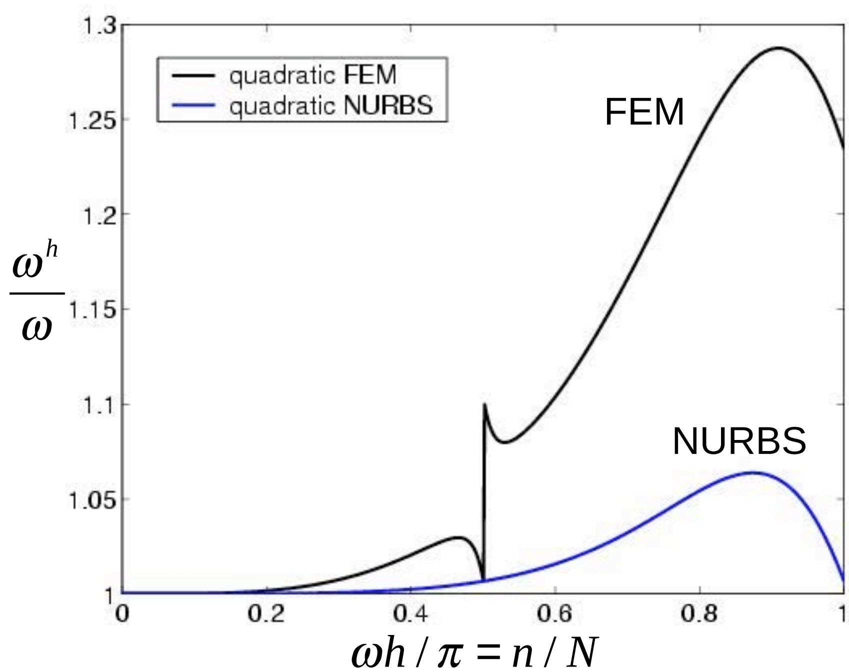
Natural frequencies:

$$\omega_n = n\pi, \quad \text{with} \quad n = 1, 2, 3, \ldots$$

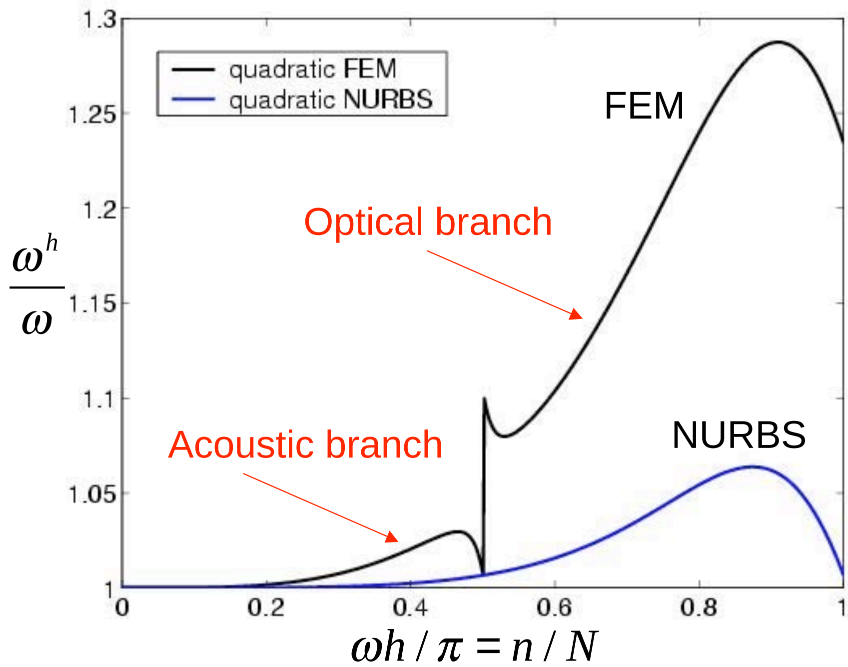
Frequency errors:

$$\omega_n^h / \omega_n$$

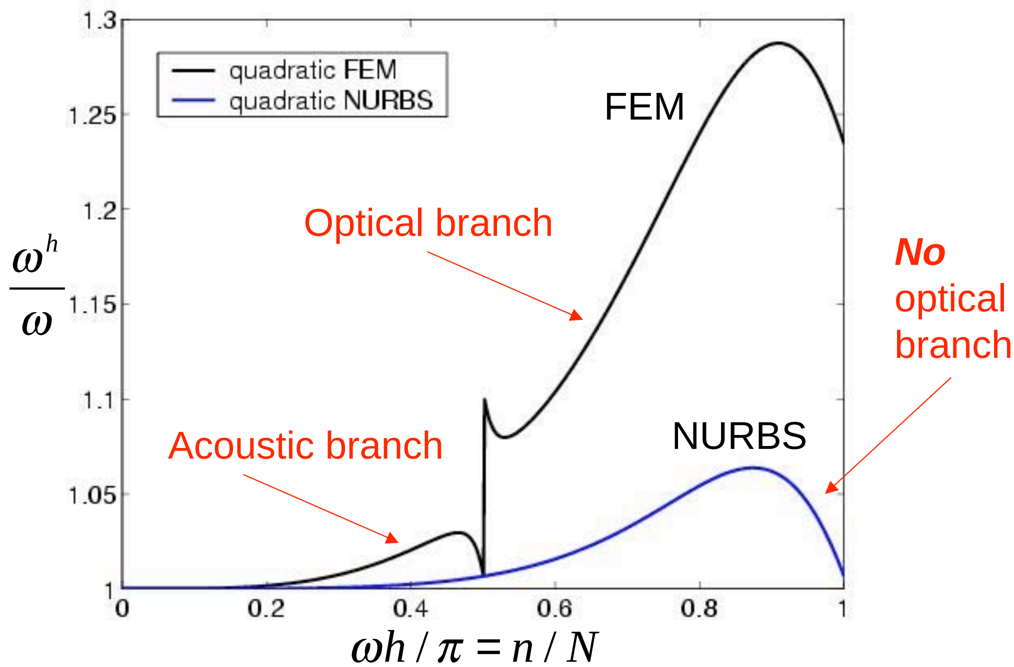
Comparison of FEM (*p*-refinement) and NURBS (*k*-refinement) Frequency Errors



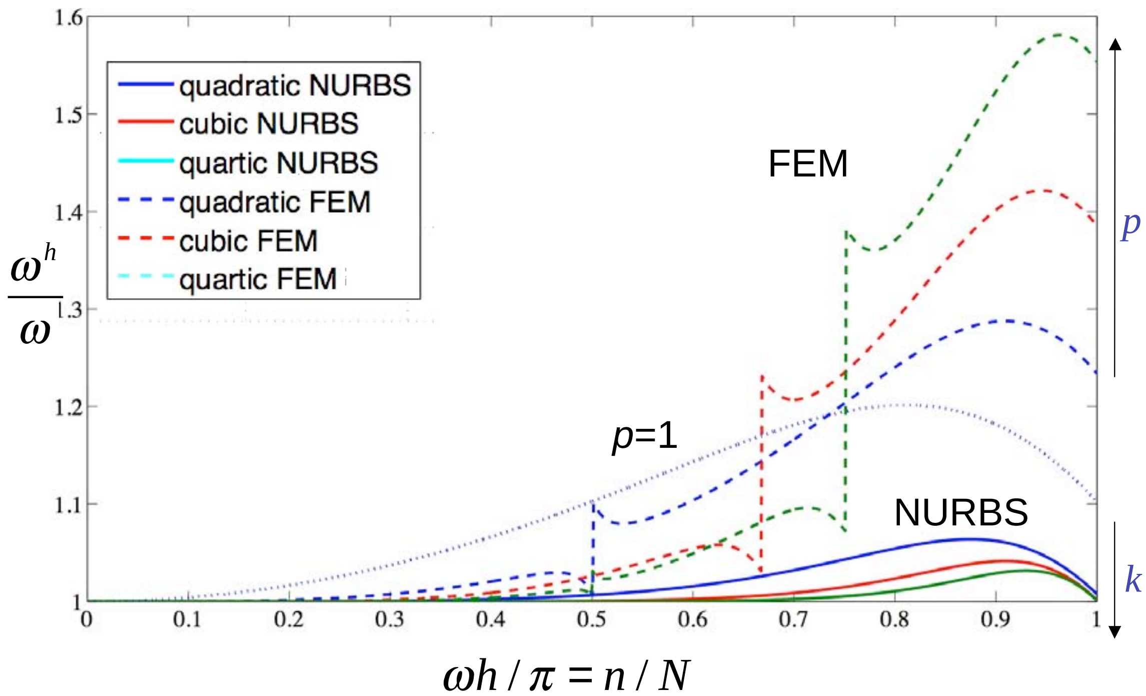
Comparison of FEM (*p*-refinement) and NURBS (*k*-refinement) Frequency Errors



Comparison of FEM (*p*-refinement) and NURBS (*k*-refinement) Frequency Errors



Comparison of FEM (*p*-refinement) and NURBS (*k*-refinement) Frequency Errors



Nonlinear Solids

Incompressibility: $\bar{\boldsymbol{B}}$ and $\bar{\boldsymbol{F}}$ methods

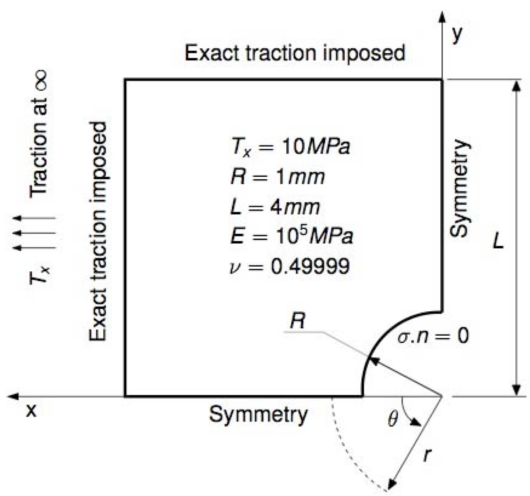
Linear (sum)	Nonlinear (product)
$\boldsymbol{B}$	$\boldsymbol{F} = \partial\phi / \partial\boldsymbol{X}$
$\boldsymbol{B}^{\text{dil}}$	$\boldsymbol{F}^{\text{dil}} = J^{1/3}\boldsymbol{I}$ $J = \det\boldsymbol{F} = \det\boldsymbol{F}^{\text{dil}}$
$\boldsymbol{B}^{\text{dev}} = \boldsymbol{B} - \boldsymbol{B}^{\text{dil}}$	$\boldsymbol{F}^{\text{dev}} = (\boldsymbol{F}^{\text{dil}})^{-1}\boldsymbol{F}$ $= \boldsymbol{F}(\boldsymbol{F}^{\text{dil}})^{-1}$
$\bar{\boldsymbol{B}}^{\text{dil}}$ (improved)	$\bar{\boldsymbol{F}}^{\text{dil}} = (\overline{J^{1/3}})\boldsymbol{I}$ (improved) (e.g., interpolated from reduced quadrature pts.)
$\bar{\boldsymbol{B}} = \boldsymbol{B}^{\text{dev}} + \bar{\boldsymbol{B}}^{\text{dil}}$	$\bar{\boldsymbol{F}} = \boldsymbol{F}^{\text{dev}}\bar{\boldsymbol{F}}^{\text{dil}}$ $= \boldsymbol{F}(\boldsymbol{F}^{\text{dil}})^{-1}\underline{\bar{\boldsymbol{F}}^{\text{dil}}}$ $= \boldsymbol{F}J^{-1/3}\boldsymbol{I}(\overline{J^{1/3}})\boldsymbol{I}$ $= \left(\overline{J^{1/3}} / J^{1/3}\right)\boldsymbol{F}$

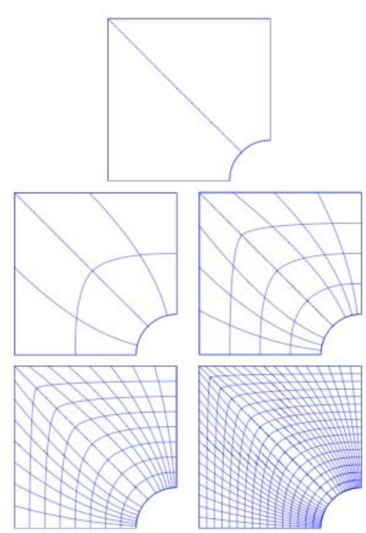
P. Flory 1960's, T. Hughes 1970's, J. Simo 1980's, D.R.J. Owen 2000's

Infinite plate with a circular hole

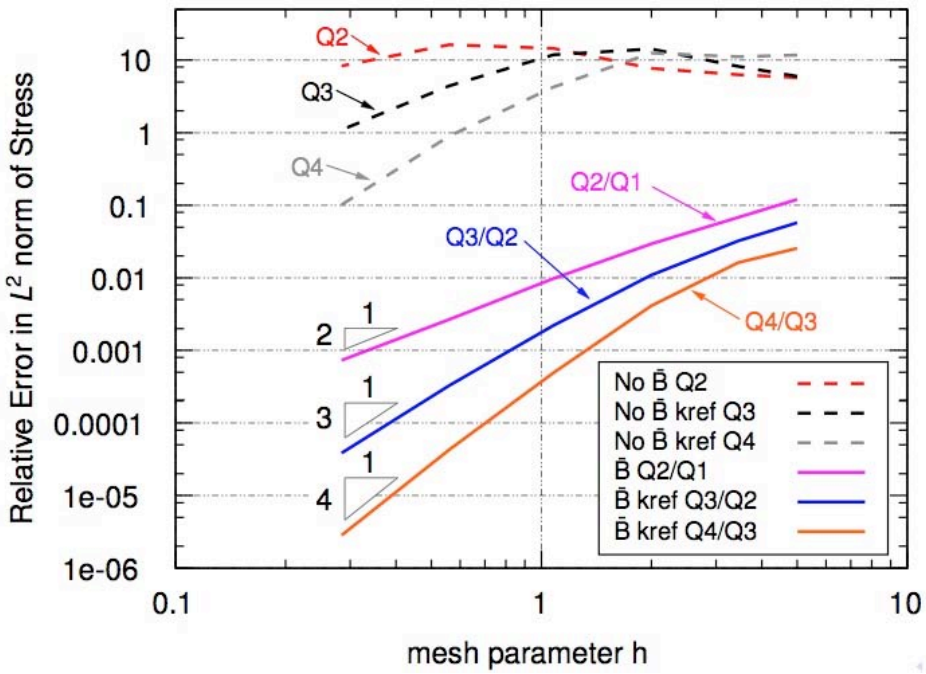
Modeled by a finite quarter plate

with exact boundary conditions





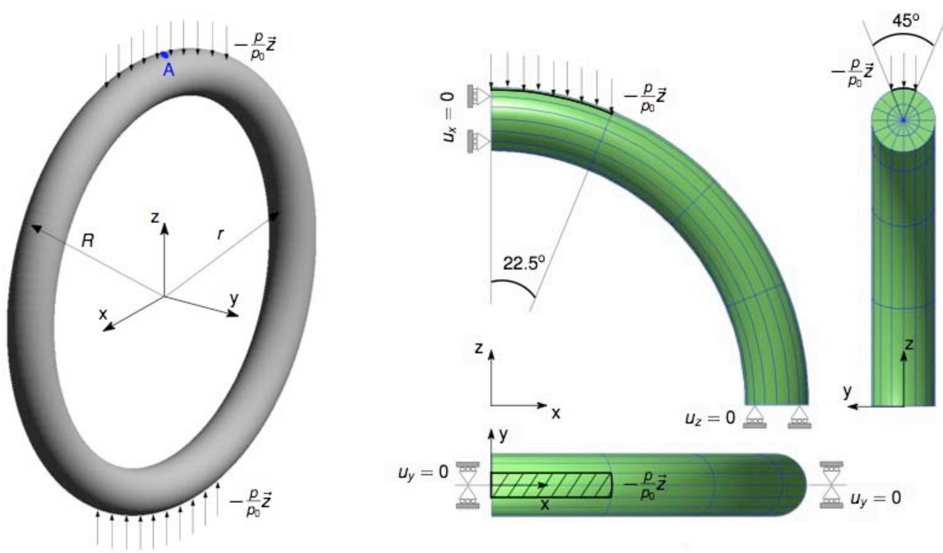
Relative error in the L^2 norm of stress



Torus subjected to a vertical pinching load

The exact geometry is represented by quadratic NURBS

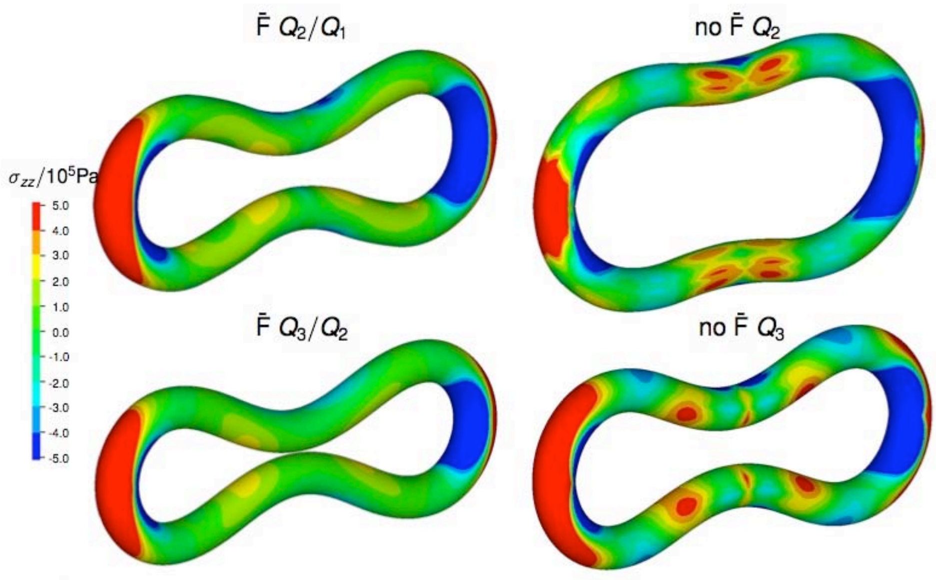
The mesh consists of 4 x 16 x 4 elements

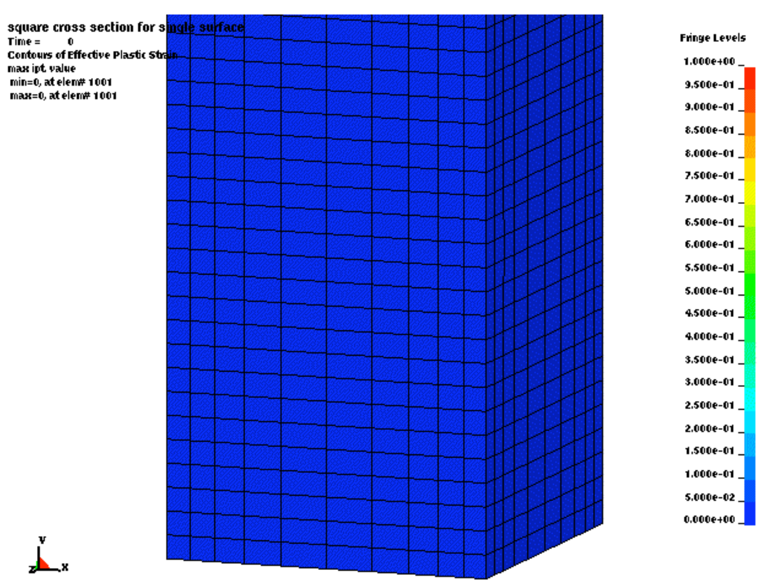


$R = 10$ m $\quad \kappa = 2.8333 \times 10^3$ MPa $\quad \mu = 5.67$ MPa $\quad (\nu = 0.4998)$

$r = 8$ m $\quad p = 0.195$ MPa

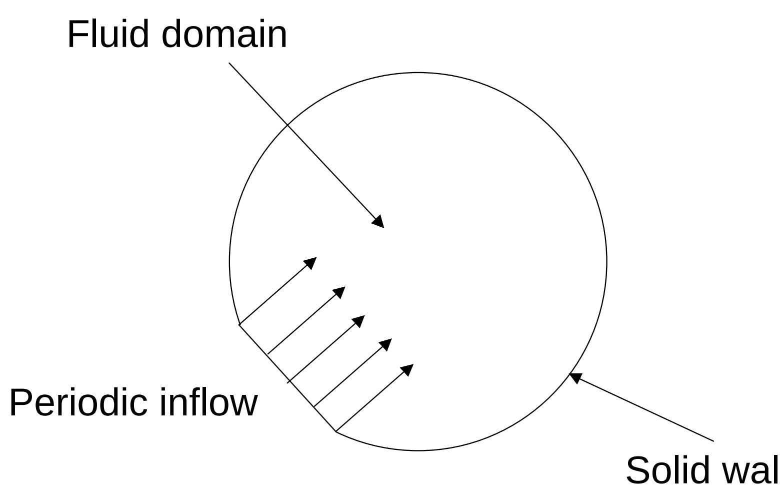
σ_{zz} stress with and without $\bar{F}$





Fluids and Fluid-Structure Interaction

Balloon Containing an Incompressible Fluid

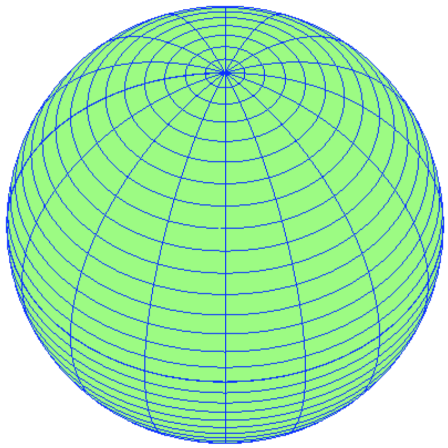


(Re = 4×10^5)

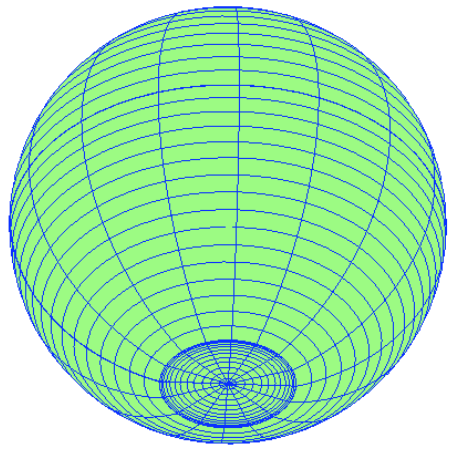
From Wall ’06, Tezduyar ’07

Balloon Containing an Incompressible Fluid

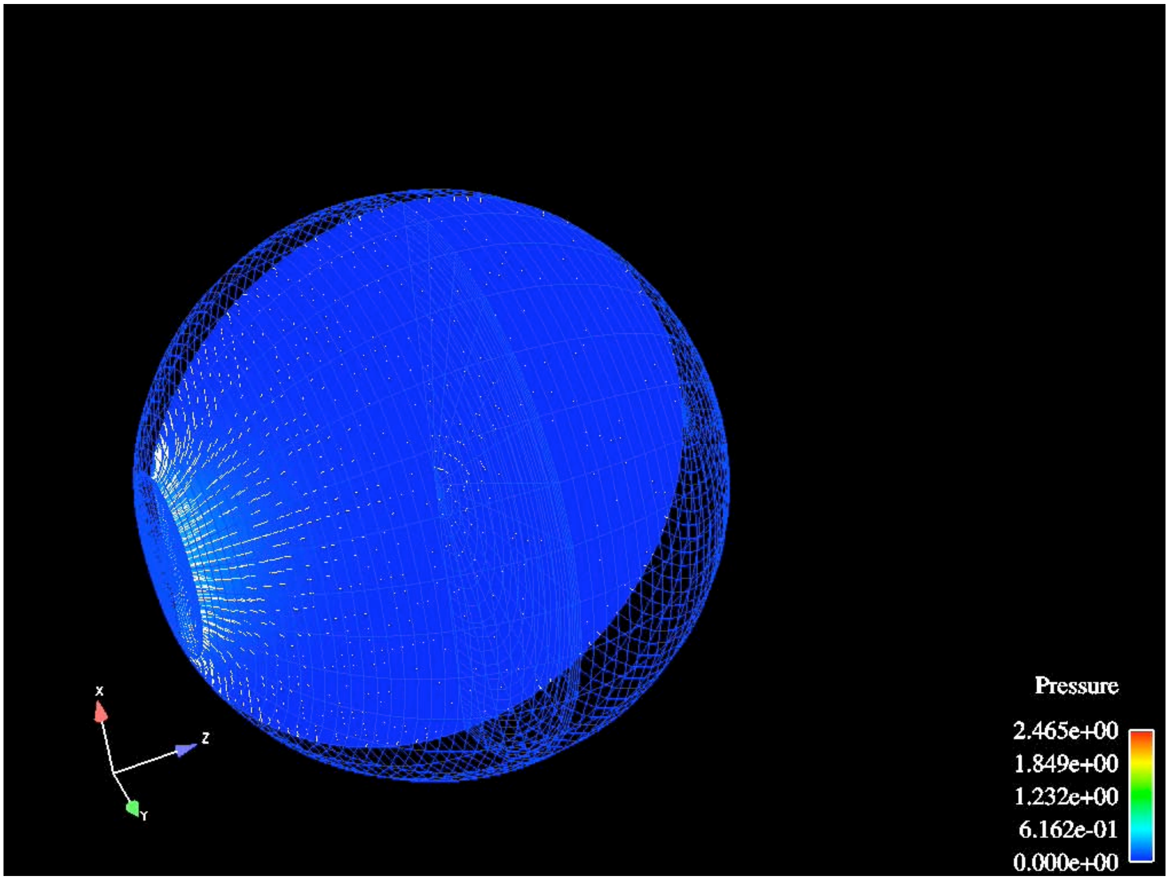
- Quadratic NURBS for both solid and fluid
- Boundary layer meshing



(a) Top view

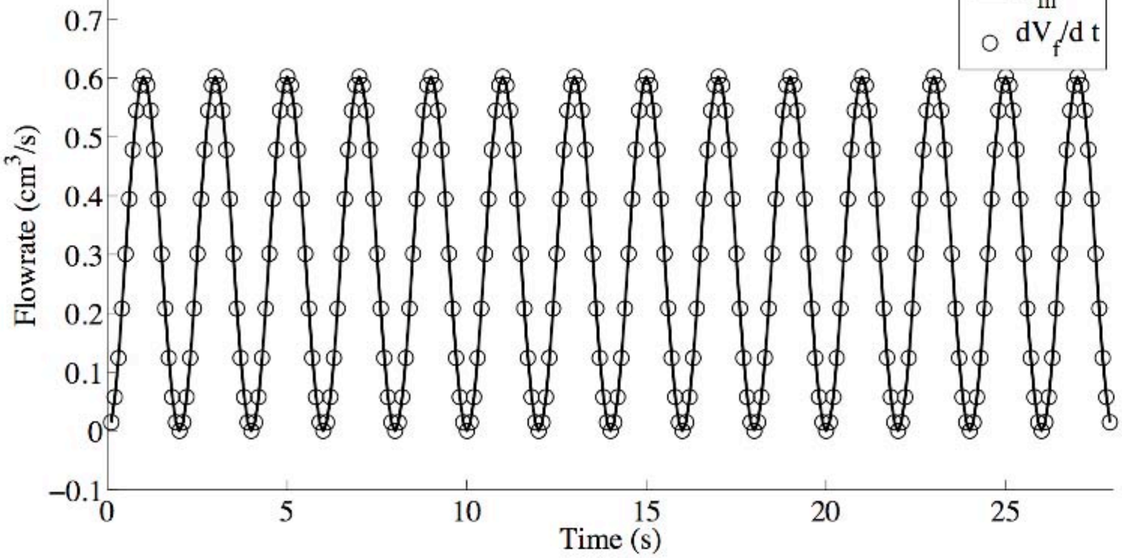


(b) Bottom view

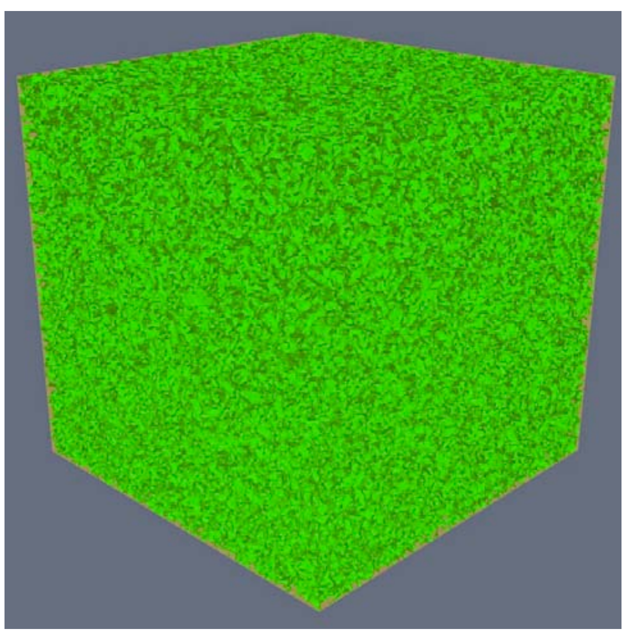


Balloon Containing an Incompressible Fluid

Mass conservation: $\frac{d}{dt}V_f(t) = Q_{in}\left(t\right)$



Phase Field Modeling: Cahn-Hilliard Equation

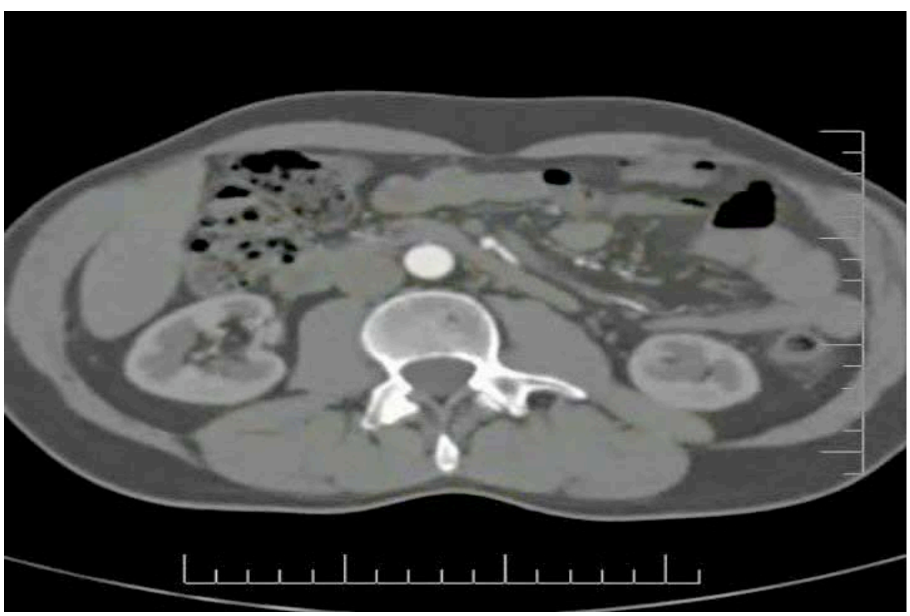


C^1 Quadratic NURBS

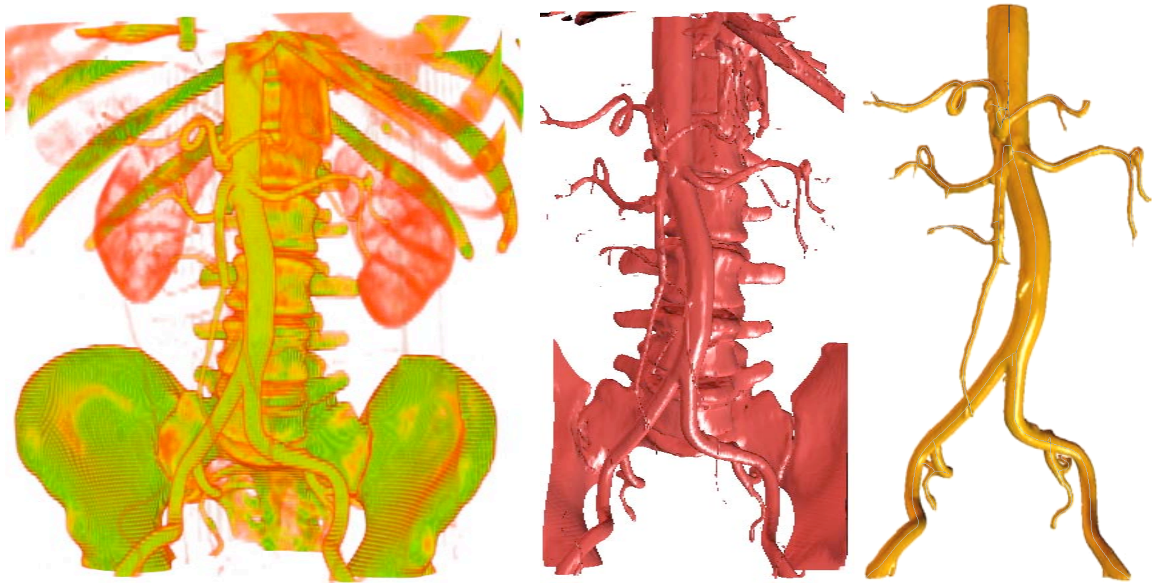
Cardiovascular Research

- Patient-specific mathematical models of major arteries and the heart
- Cardiovascular Modeling Toolkit
  - Abdominal aorta
  - LVADs: Left Ventricular Assist Devices (R. Moser)
  - Aneurysms
  - Vulnerable plaques and drug delivery systems
  - Heart

Medical Imaging: Computed Tomography (CT)

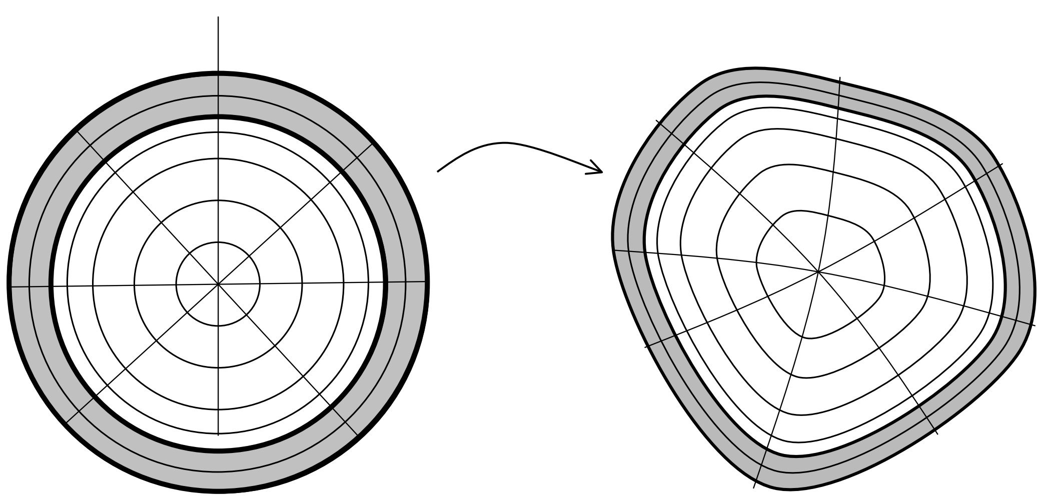


Abdominal Aorta

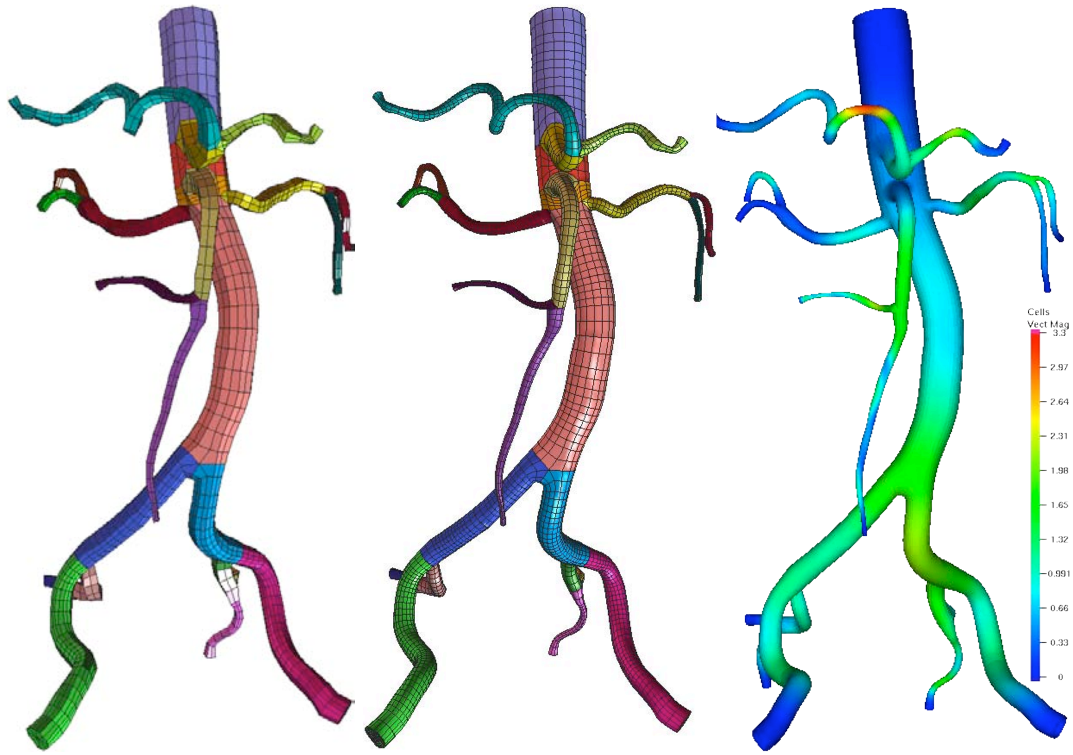


(a) Volume rendering (b) Isocontouring (c) Surface model & path

Mapping onto a patient-specific arterial cross-section

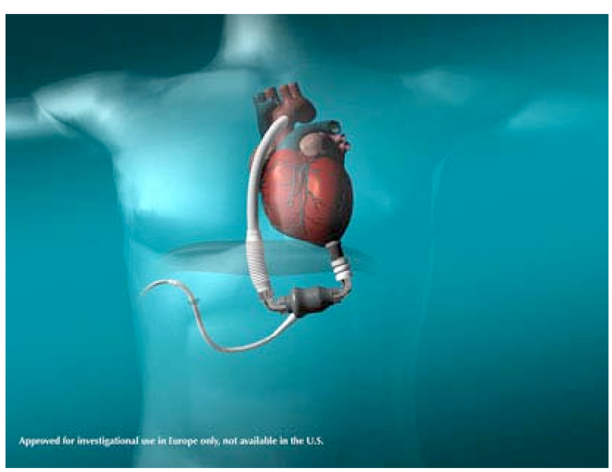


Abdominal Aorta

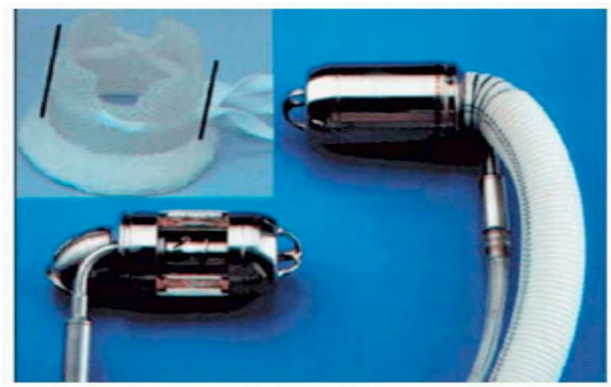


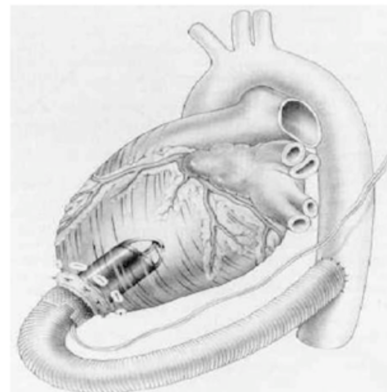
(d) Control mesh (e) Solid NURBS (f) Simulation results

Left Ventricular Assist Device (LVAD) with *Ascending* Aortic Distal Anastomosis

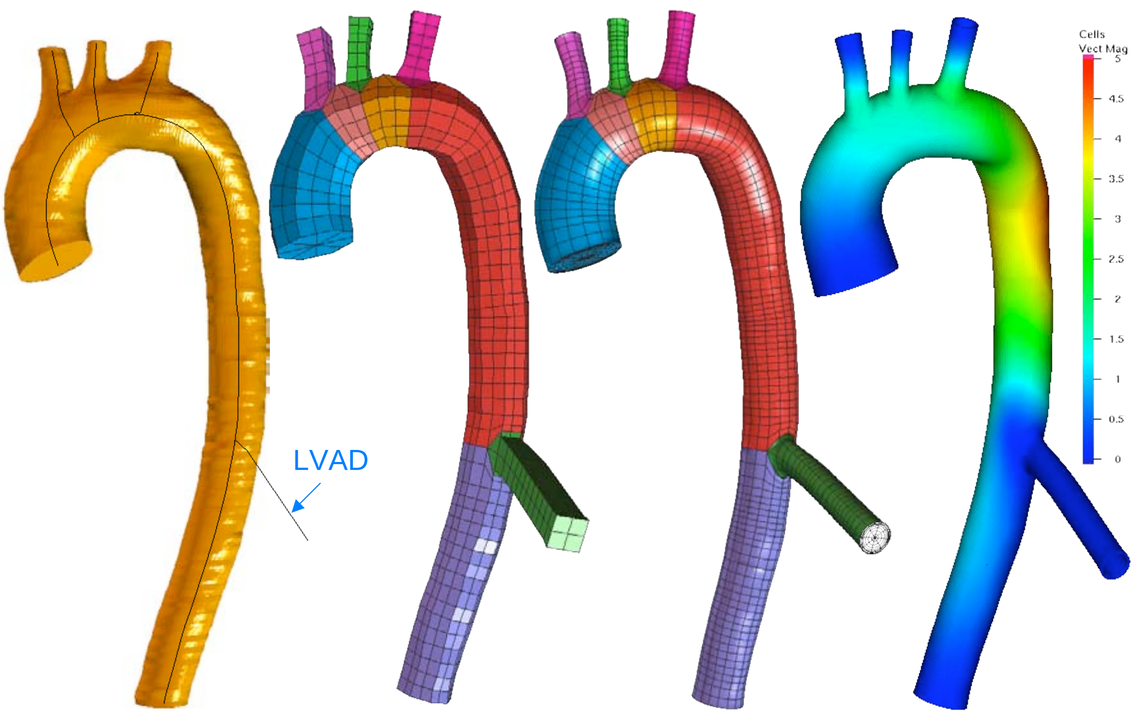


Jarvik 2000 and Schematic of *Descending* Aortic Distal Anastomosis



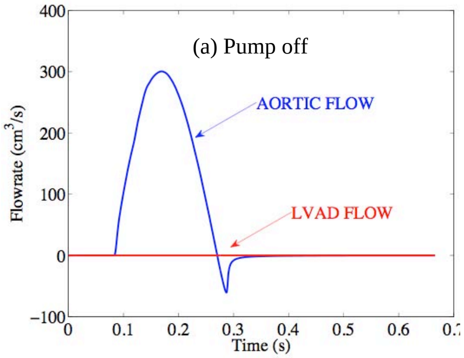


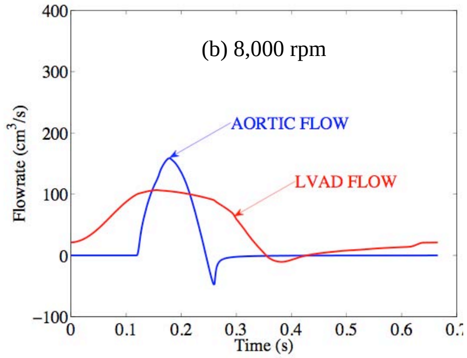
Thoracic Aorta

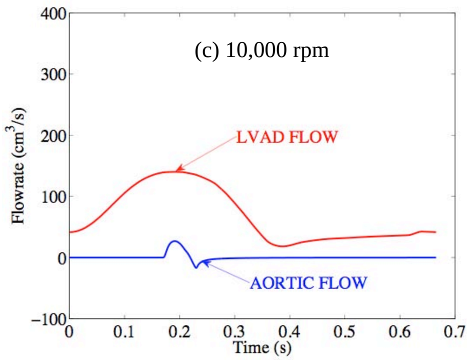


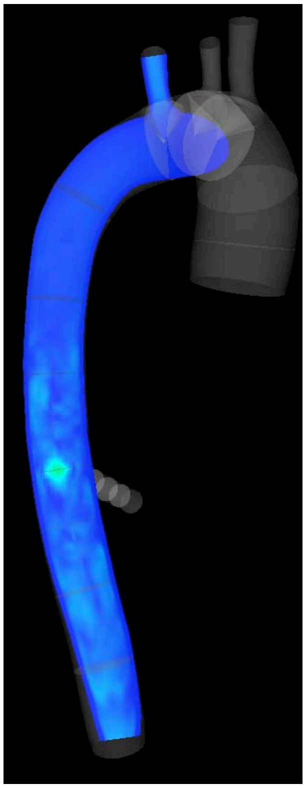
(a) Surface model and path (b) Control mesh (c) Solid NURBS (d) Simulation results

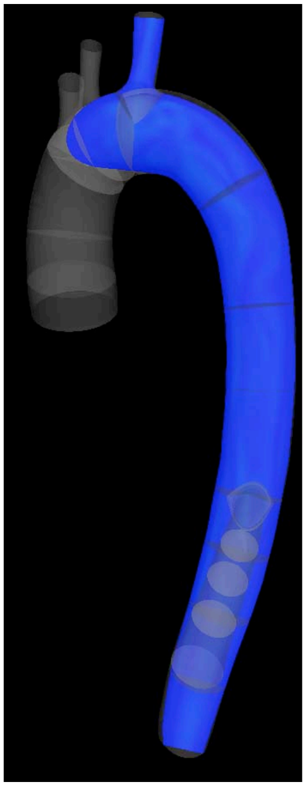
Data from a lumped-parameter model of the CV system with assist

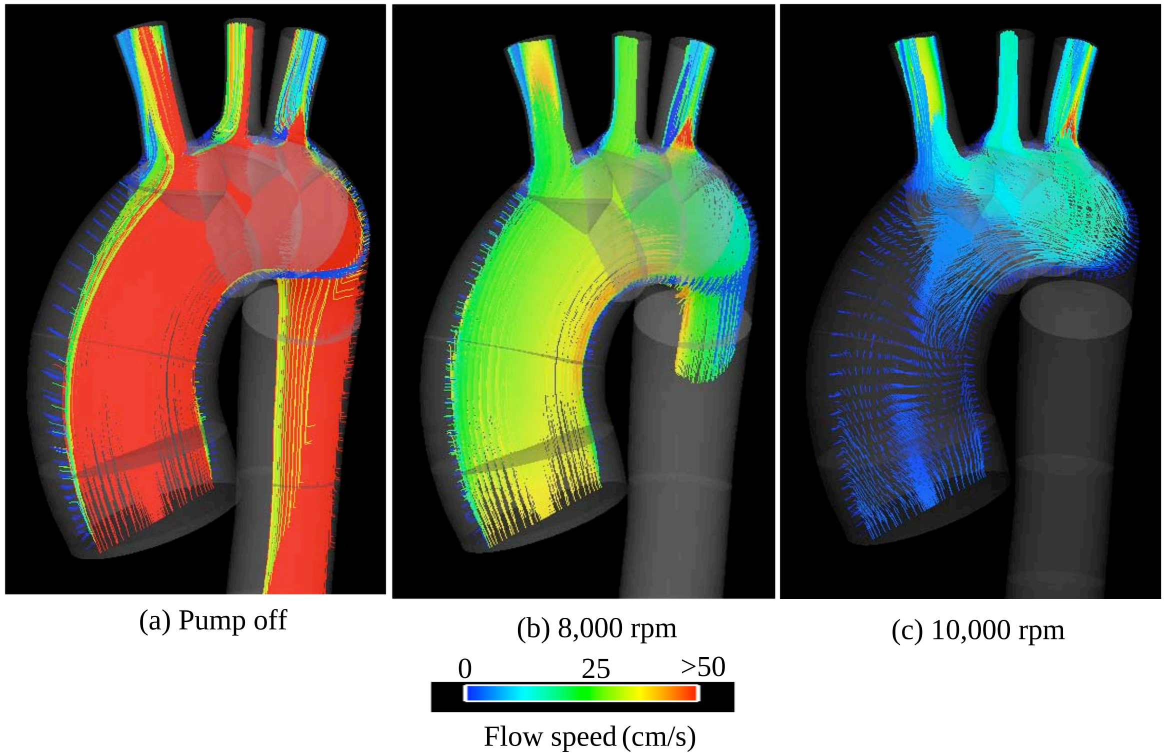


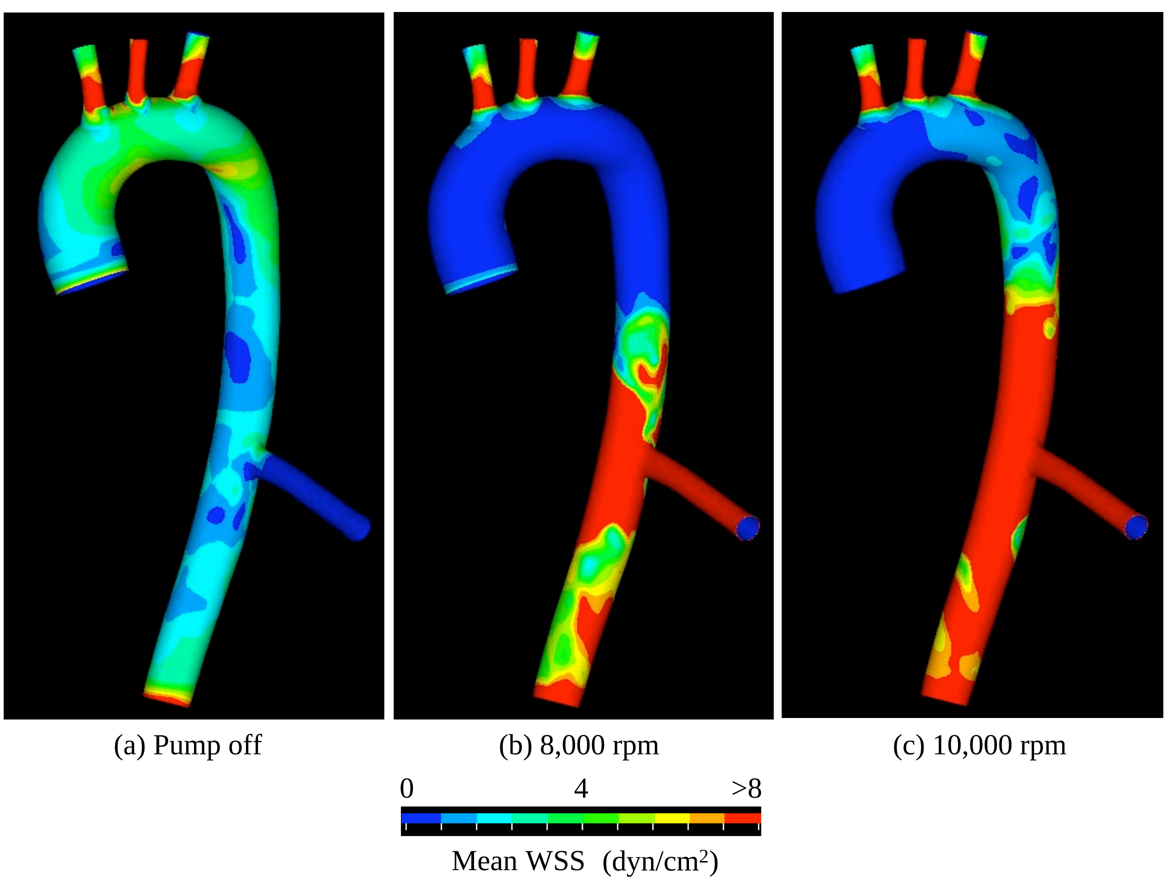









(a) Pump off
(b) 8,000 rpm
(c) 10,000 rpm
0
25
>50
Flow speed (cm/s)


(a) Pump off
(b) 8,000 rpm
(c) 10,000 rpm
0
4
>8
Mean WSS (dyn/cm2)

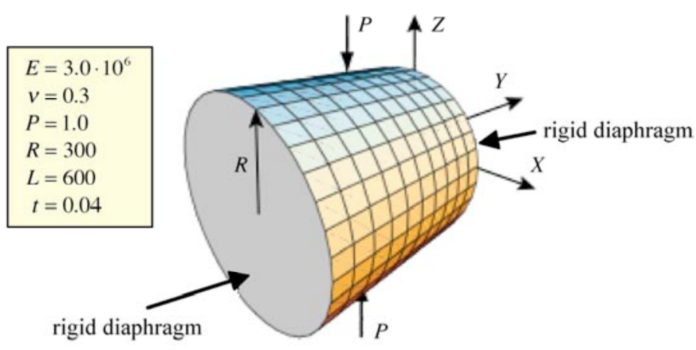
Design to Analysis

- Idea:
  Extract surface geometry file from CAD modeling software and use it directly in FEA software

- Goal:
  Bypass mesh generation

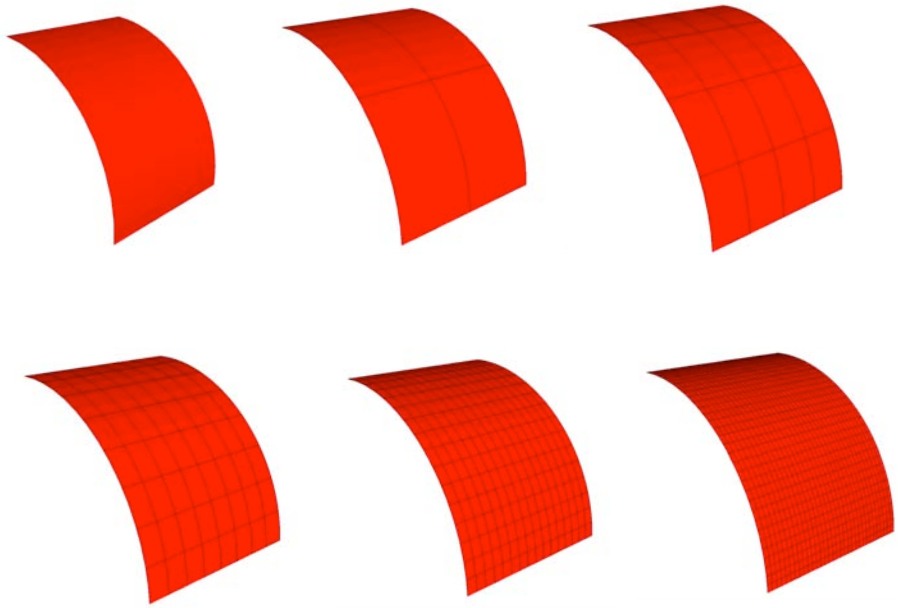
- Test cases:
  Import NURBS surface files directly into LS Dyna for Reissner-Mindlin shell theory analysis

Pinched Cylinder: Problem Definition



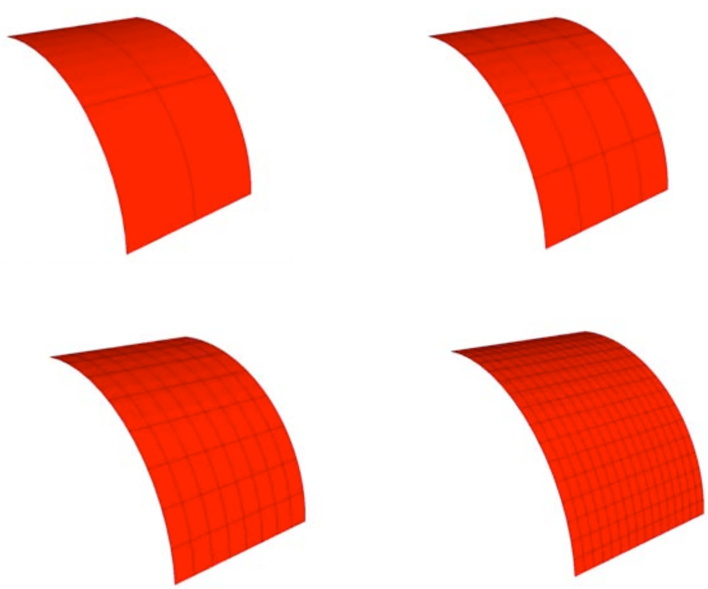
Pinched Cylinder

Quadratic NURBS surface models



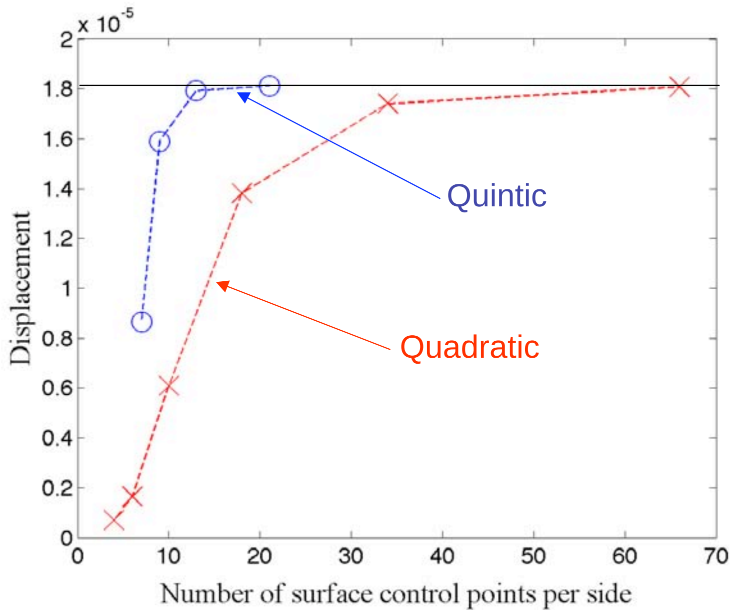
Pinched Cylinder

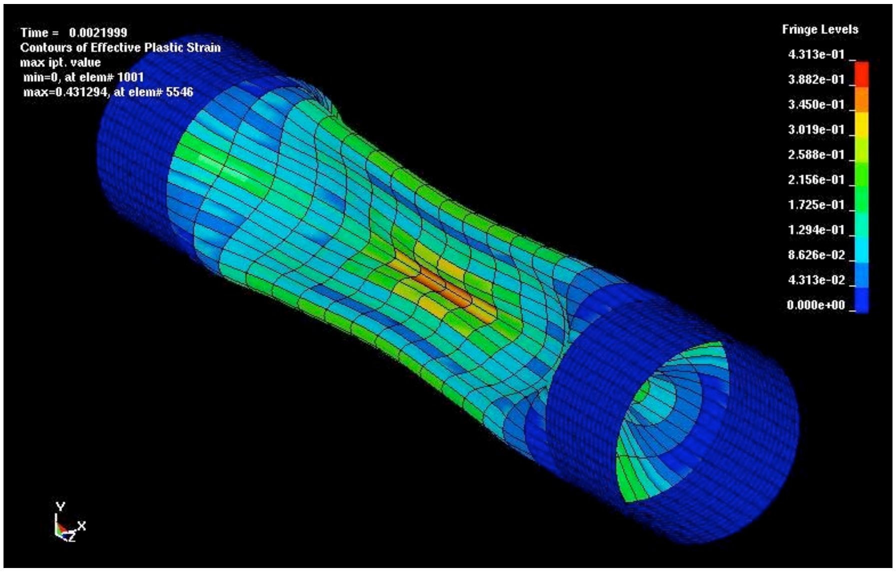
Quintic NURBS surface models

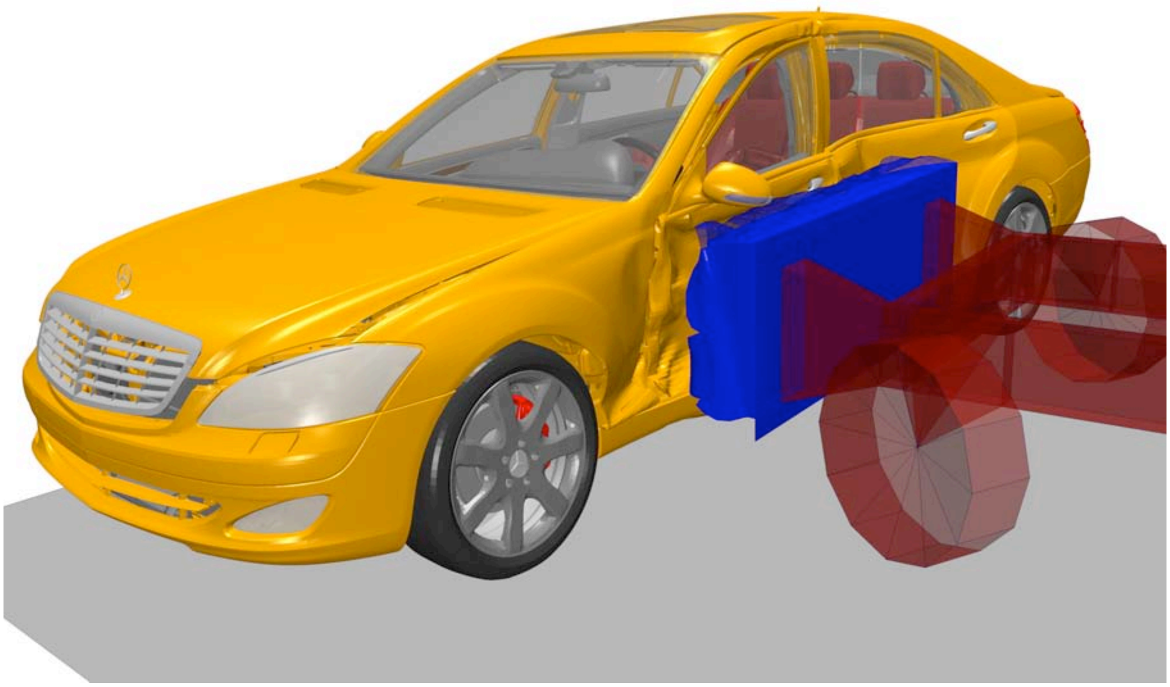


Pinched Cylinder

Convergence of NURBS surface models in LS Dyna



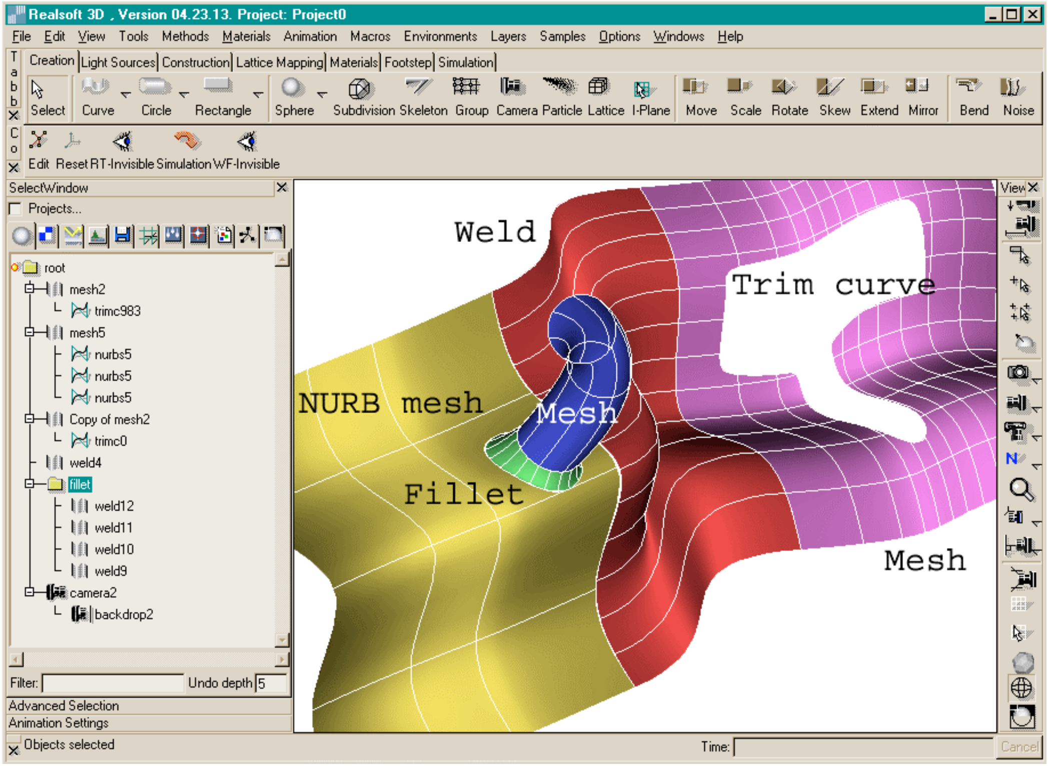




S. Kolling, Mercedez Benz

Problems with NURBS-based Engineering Design

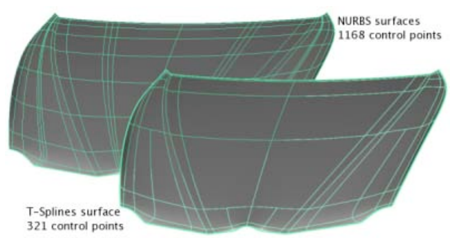
– Water-tight merging of patches
– Trimmed surfaces

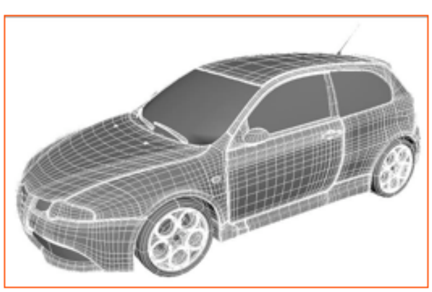

Realsoft 3D , Version 04.23.13. Project: Project0
Weld
Trim curve
NURB mesh
Mesh
Fillet
Mesh

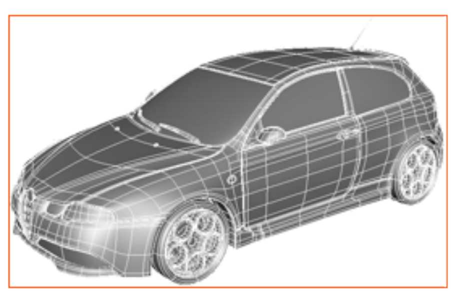
T-splines

Unstructured NURBS Mesh

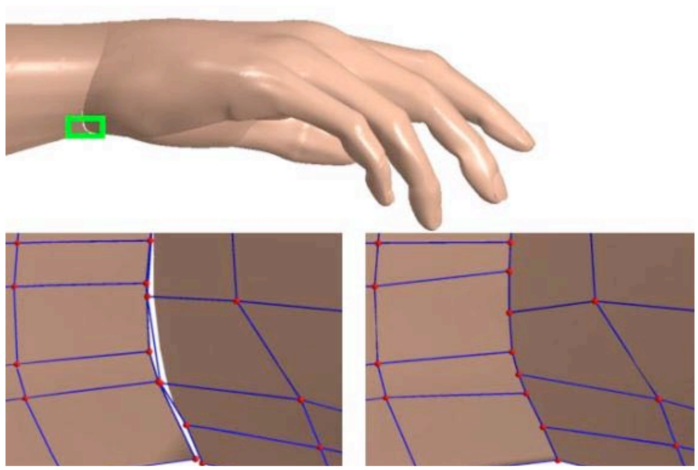
Reduced Number of Control Points



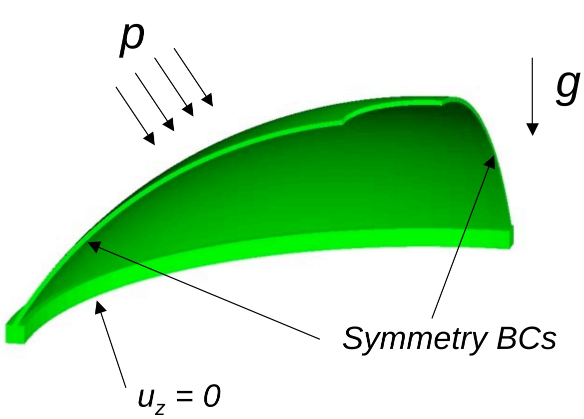




Water tight merging of patches



Hemispherical Shell with Stiffener



Computational domain

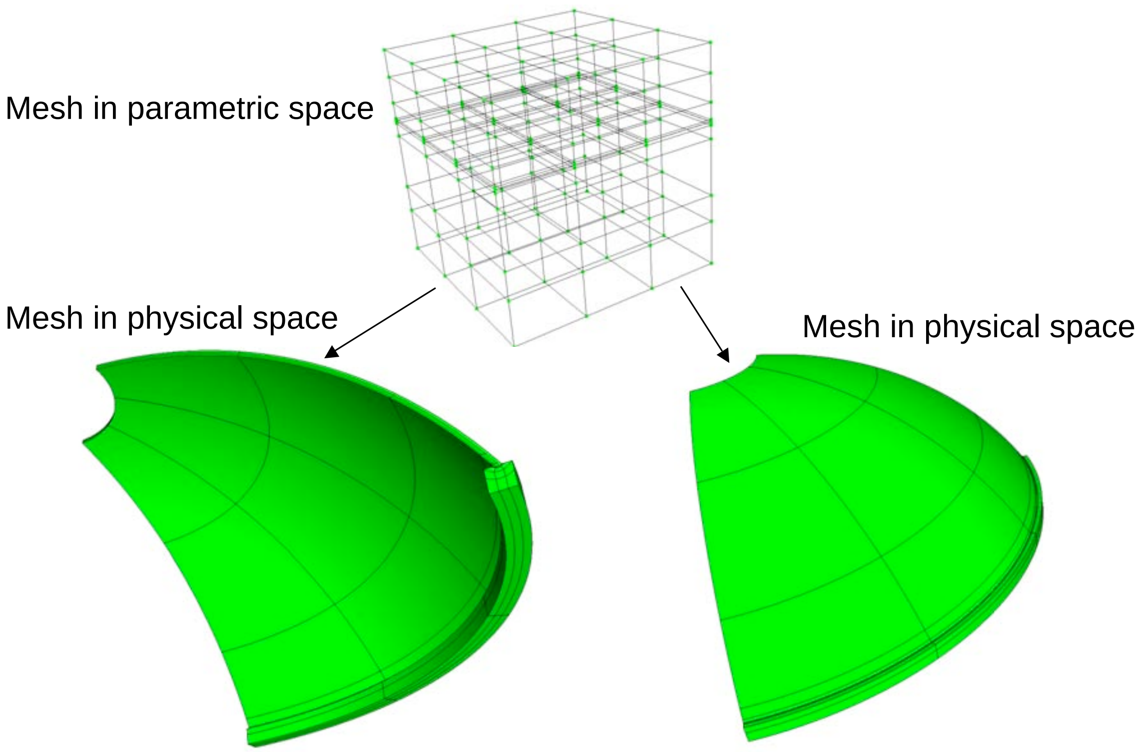
Loading:

- Gravity
- External pressure

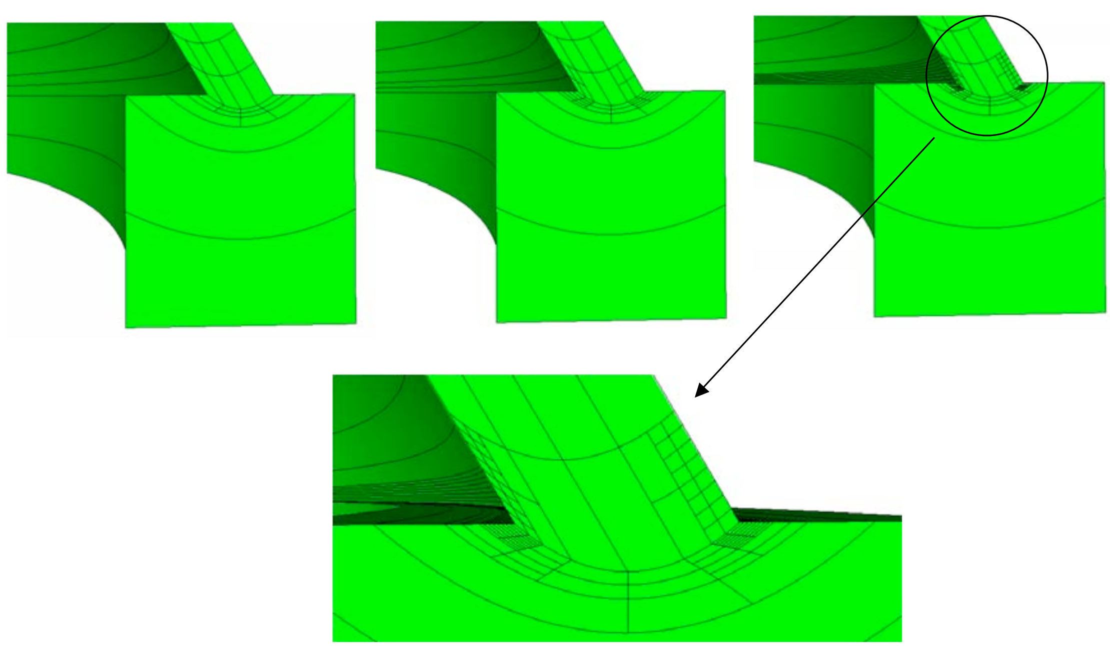
Actual domain

(from E. Rank *et al.*)

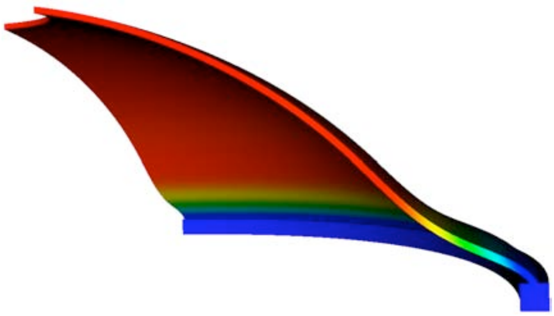
Hemispherical Shell with Stiffener



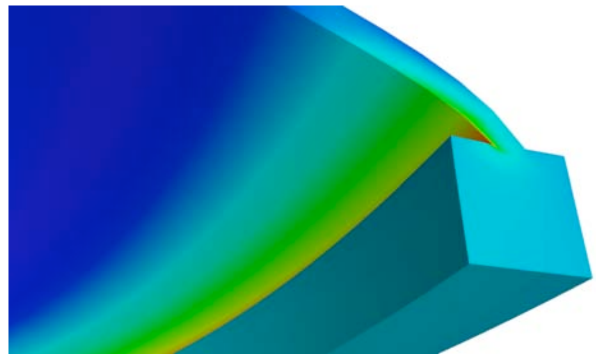
Locally Refined Meshes



Hemispherical Shell with Stiffener



Vertical displacement (smooth)



Von Mises stress (singular)

Conclusions

- Isogeometric Analysis is a powerful generalization of FEA
  - Precise and efficient geometric modeling
  - Mesh refinement is simplified
  - Smooth basis functions with compact support
  - Numerical calculations are very encouraging
  - Higher-order accuracy *and* robustness
  - It may play a fundamental role in *unifying* design and analysis

Philosophy:

Geometry is the foundation of analysis

Computational geometry is the future of computational analysis

Book in progress:

Isogeometric Analysis: Toward Unification of CAD and FEA