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Multiphysics Simulation using Direct Coupled-Field Element Technology

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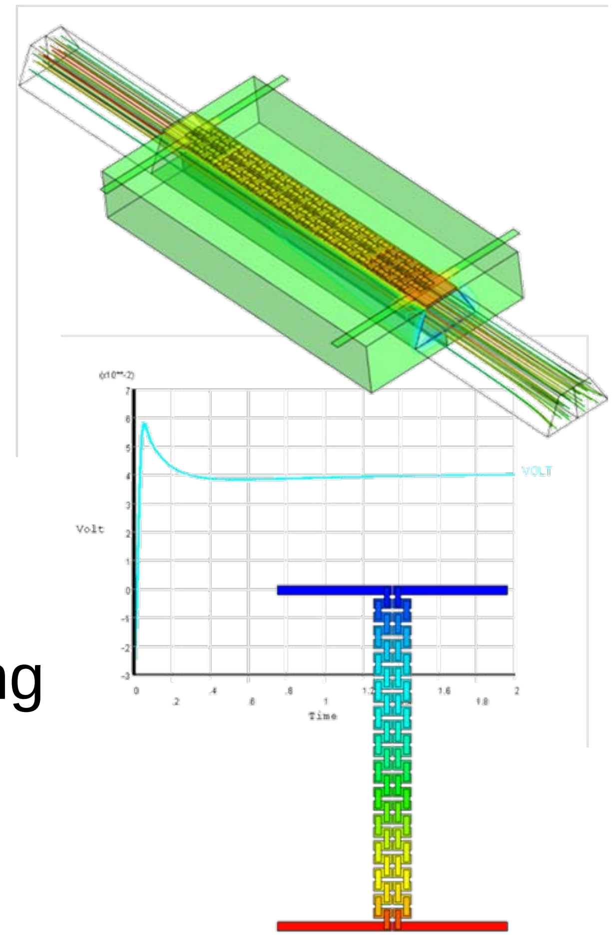
Agenda

- Multiphysics Overview
- Direct Coupled-Field Elements
- Applications
  - Piezoelectricity
  - Thermal-Electric Coupling
  - Electroelasticity
  - Structural-Thermal-Electric Coupling
- Conclusions
- Questions and Answers

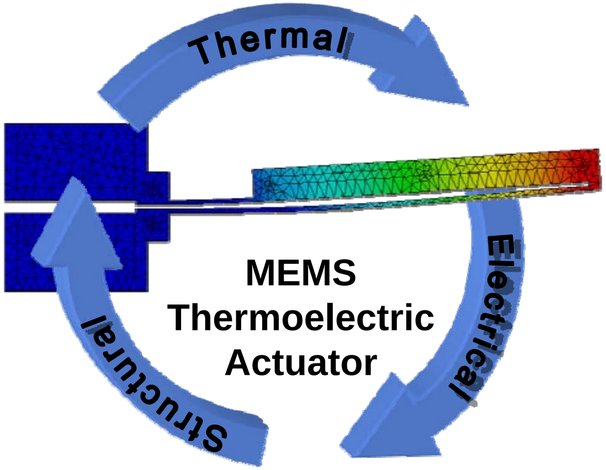


Thermoelectric Generator

Multiphysics - Why Consider It?

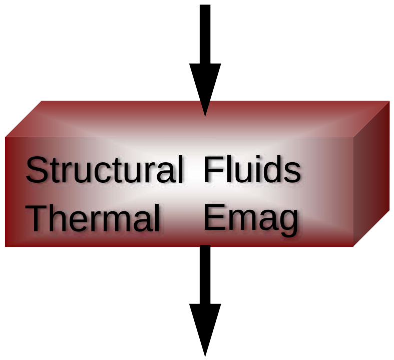
- You can't afford to ignore coupled physics when:
  - The real world is a multiphysics world
  - Your design depends on coupled physical phenomena
  - You are faced with small error margins
  - Physical testing is too costly

Innovative companies are moving beyond single physics analysis



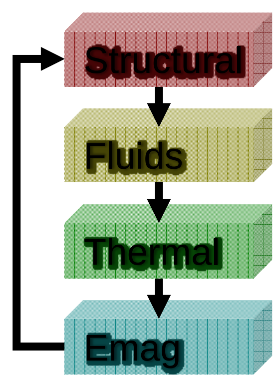
Methods of Coupling Physics

- Direct Coupling
  - A single analysis employing a coupled-field element containing all the necessary DOFs to solve the coupled-field problem.



Direct Coupling
- Element-level coupling
- Highly coupled physics
- Single model & mesh

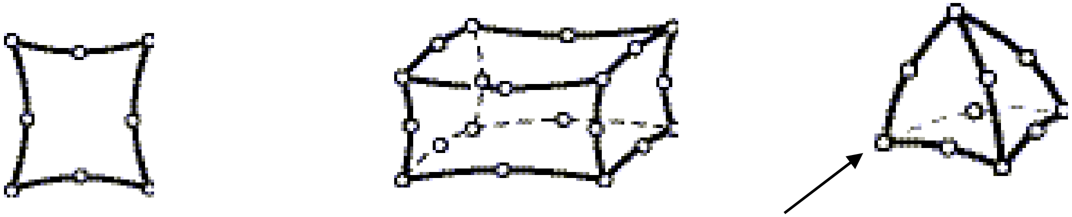
- Load Transfer
  - Two or more analysis are coupled by applying results from one analysis as loads in another analysis.



Load Transfer
- Sequential solution
- Separate model & mesh
- Separation of expertise

Direct Coupled-Field Element

- A finite element that couples the effects of interrelated physics within the element matrices or load vectors, which contain all necessary terms required for the coupled physics solution.
- Strong (matrix) or weak (sequential) coupling
- Many industry applications



DOF: UX, UY, UZ, TEMP, VOLT, (AZ)

Strong (Matrix) Coupling

- Finite element matrix equation:

$$\begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \{X_1\} \\ \{X_2\} \end{Bmatrix} = \begin{Bmatrix} \{F_1\} \\ \{F_2\} \end{Bmatrix}$$

- The coupling is accounted for by the off-diagonal matrices $[K_{12}]$ and $[K_{21}]$.
- Provides for a coupled solution in one iteration for linear problems
- Examples: Piezoelectricity, Seebeck effect, thermoelasticity

Weak (Sequential) Coupling

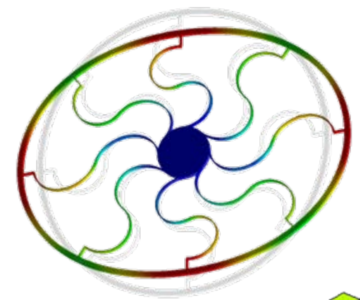
- Finite element matrix equation:

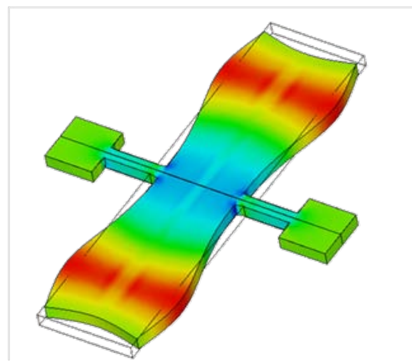
$$\begin{bmatrix} [K_{11}(\{X_1\},\{X_2\})] & [0] \\ [0] & [K_{22}(\{X_1\},\{X_2\})] \end{bmatrix} \begin{Bmatrix} \{X_1\} \\ \{X_2\} \end{Bmatrix} = \begin{Bmatrix} \{F_1(\{X_1\},\{X_2\})\} \\ \{F_2(\{X_1\},\{X_2\})\} \end{Bmatrix}$$

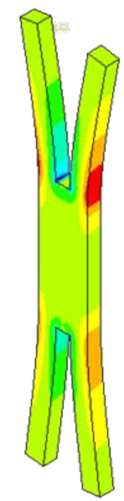
- The coupled effect is accounted for in the dependency of $[K_{11}]$ and $\{F_1\}$ on $\{X_2\}$ as well as $[K_{22}]$ and $\{F_2\}$ on $\{X_1\}$.
- At least two iterations are required to achieve a coupled response.
- Examples: Joule heating, electric forces, piezoresistivity

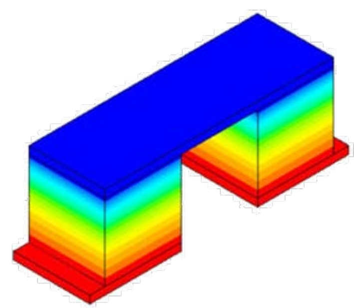
Applications for Coupled-Field Elements

- Piezoelectric
  - Transducers, resonators, sensors and actuators, vibration control
- Piezoresistive
  - Pressure sensors, strain gauges
- Thermal-electric
  - Wires, busbars, Peltier coolers
- Thermoelastic damping
  - MEMS resonators
- Electroelastic
  - Actuators, artificial muscle
- Coriolis Effect
  - Quartz angular velocity sensors



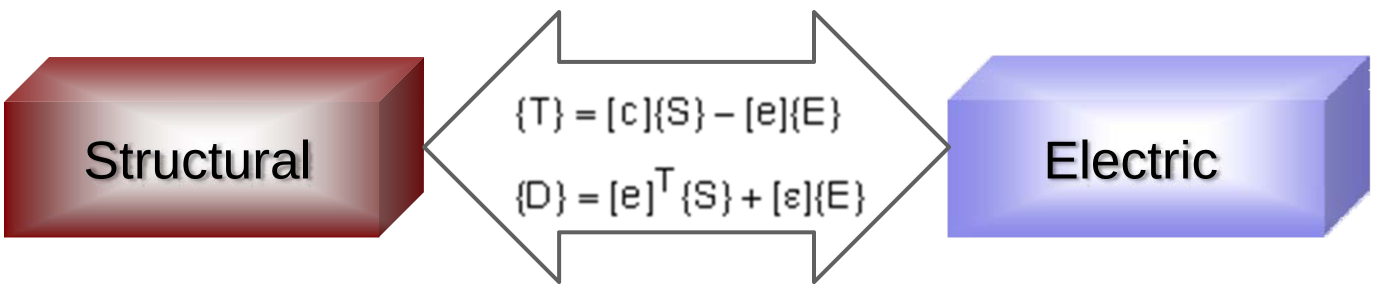






Piezoelectric Analysis

- In a piezoelectric analysis, the structural and electrostatic fields are coupled by the piezoelectric constants [e]:



[e] - piezoelectric matrix

{T} - stress vector
{S} - elastic strain vector
[c] - elastic stiffness

{D} - electric flux density
{E} - electric field intensity
[ε] - dielectric permittivity

Piezoelectric Finite Element Equations

- Strong (Matrix) Coupling

$$\begin{bmatrix} K_{UU} & K_{UV} \\ K_{UV}^{T} & -K_{VV} \end{bmatrix} \begin{Bmatrix} U \\ V \end{Bmatrix} + \begin{bmatrix} C_{UU} & 0 \\ 0 & -C_{VV} \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{V} \end{Bmatrix} + \begin{bmatrix} M_{UU} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{V} \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix}$$

- U and V are strongly coupled piezoelectric matrices $[K_{UV}]$ and $[K_{UV}]^T$
- Symmetric Matrices
- Static, Harmonic, Transient

Element matrices:

K_{UU} - structural stiffness

K_{VV} - dielectric permittivity

K_{UV} - piezoelectric coupling

C_{UU} - structural damping

C_{VV} - dielectric dissipation

M_{UU} - mass

Piezoelectric Fan Example

- Piezoelectric Fan
  - Spot cooling
  - Low magnetic permeability
  - Driven by piezoelectric bimorph

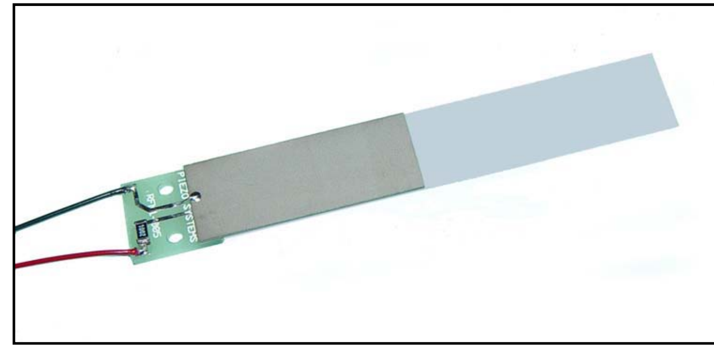
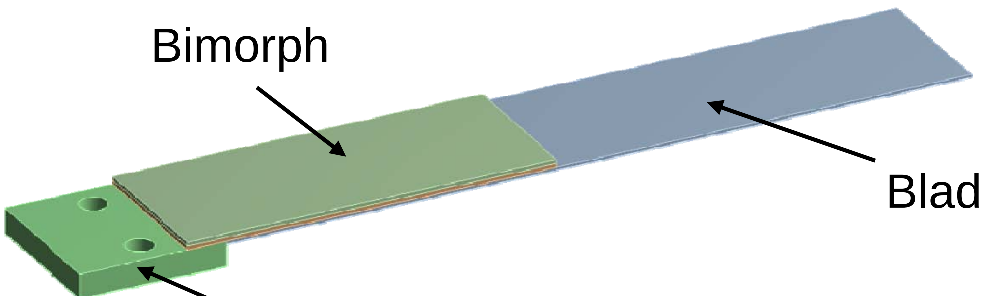
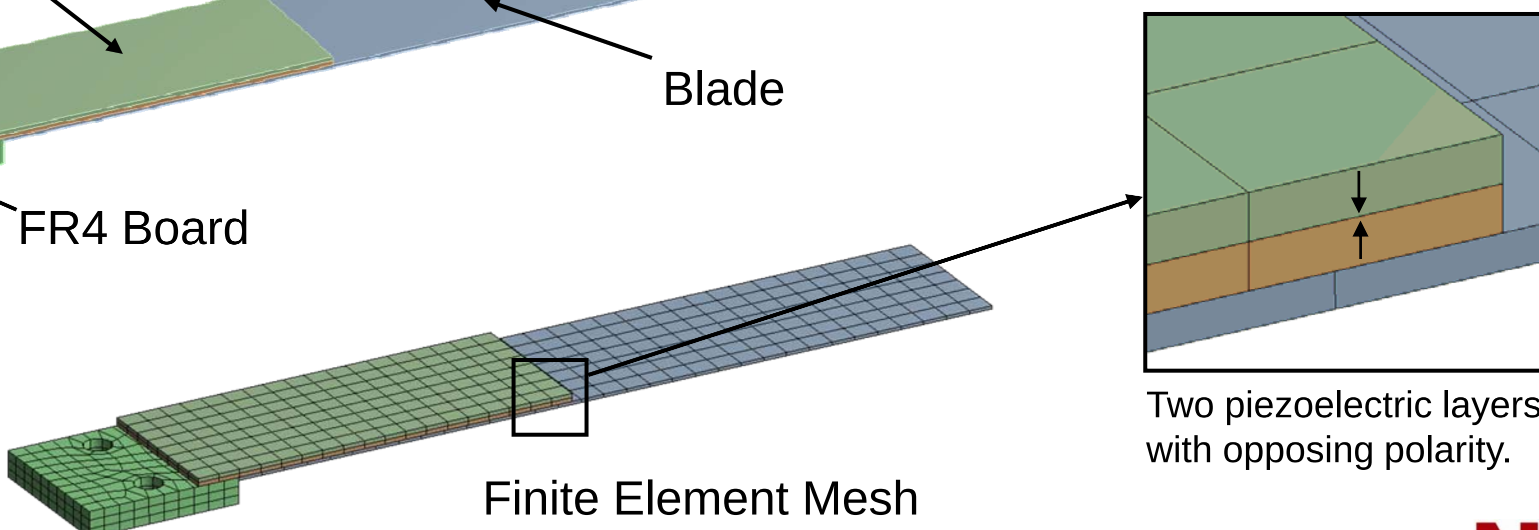


Image courtesy of Piezo Systems, Inc.



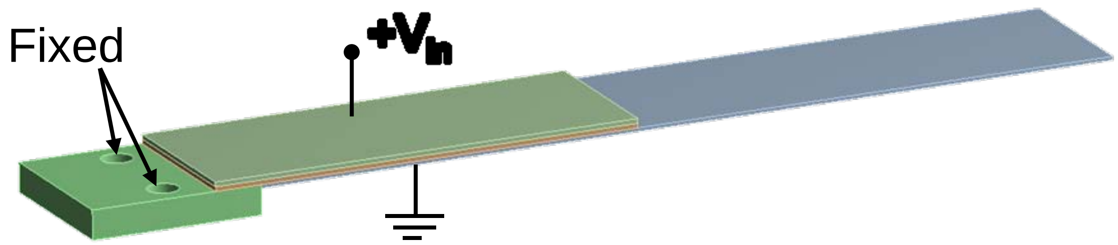


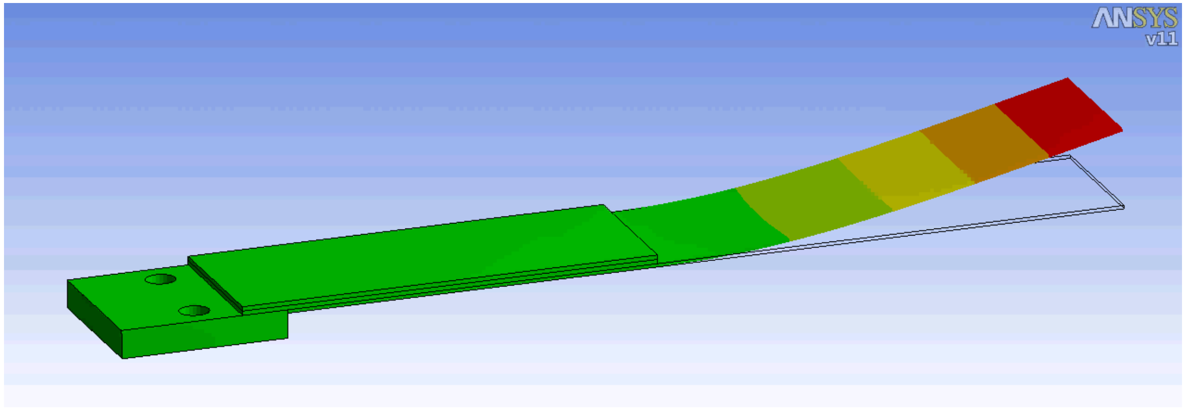
Finite Element Mesh

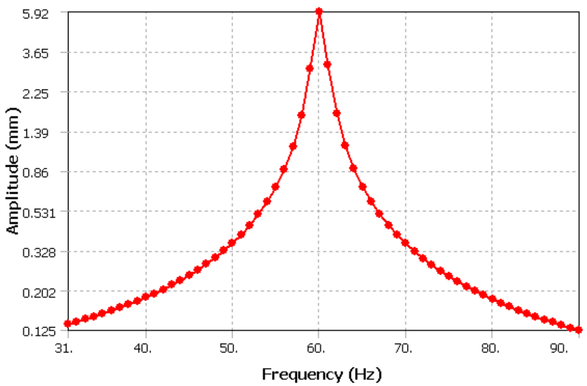
Two piezoelectric layers with opposing polarity.

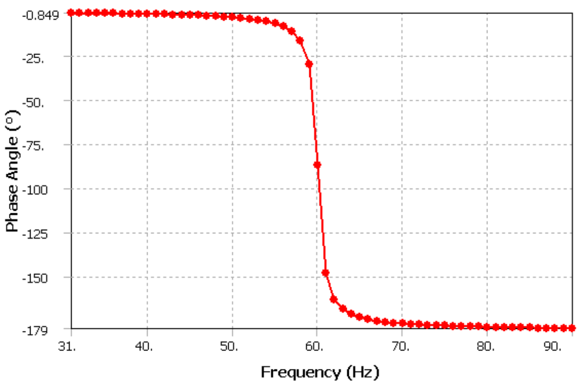
Piezoelectric Fan Example

- Results
  - Fan driven at resonance
  - 120 Volts at 60 Hz



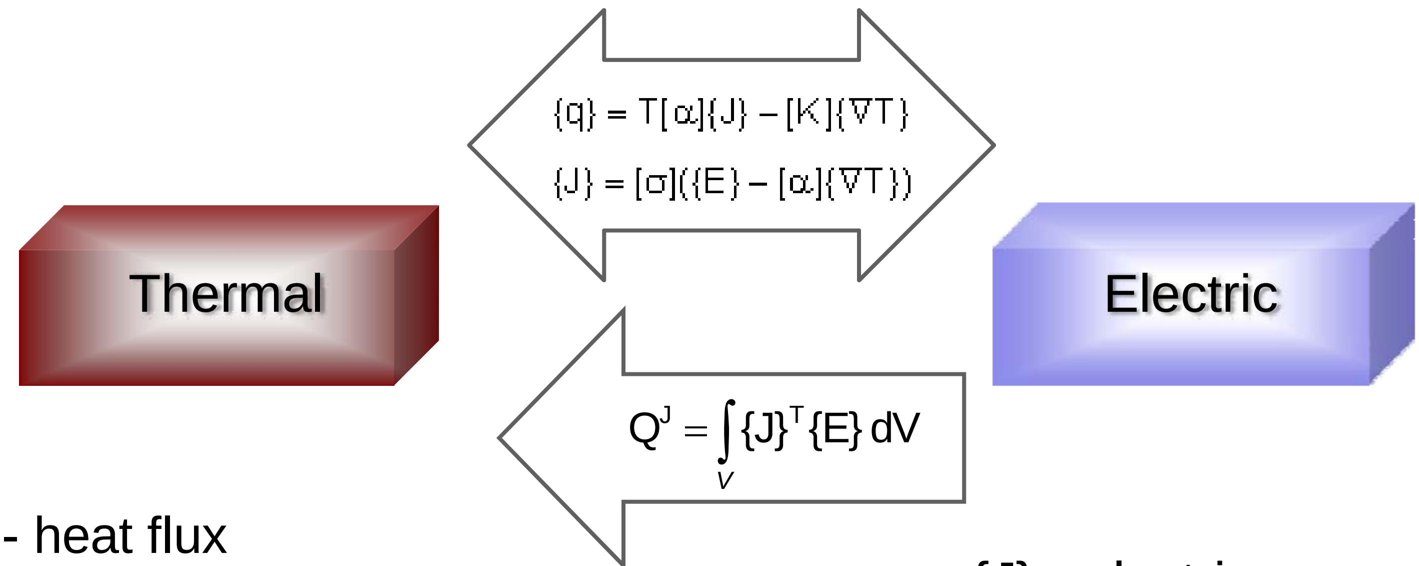






Thermal-Electric Analysis

- In a thermoelectric analysis, the thermal and electric fields are coupled by the Joule heat Q^J and Seebeck coefficients $[\alpha]$:



$\{q\}$ - heat flux

T - temperature (absolute)

$[K]$ - thermal conductivity

$[\alpha]$ - Seebeck coefficients

$\{J\}$ - electric current density

$\{E\}$ - electric field intensity

$[\sigma]$ - electrical conductivity

Thermal-Electric Finite Element Equation

- Matrix and/or Load Vector Coupling

$$\begin{bmatrix} K_{TT} & 0 \\ K_{VT} & K_{VV} \end{bmatrix} \begin{Bmatrix} T \\ V \end{Bmatrix} + \begin{bmatrix} C_{TT} & 0 \\ 0 & C_{VV} \end{bmatrix} \begin{Bmatrix} \dot{T} \\ \dot{V} \end{Bmatrix} = \begin{Bmatrix} Q+Q^P+Q^J \\ I \end{Bmatrix}$$

- T an V Coupling
  - Matrix
    - Seebeck K_{VT}
  - Load Vector
    - Joule Q^J Load Vector
    - Peltier Q^P Load Vector
- Symmetric - Joule Heating
- Unsymmetric - Seebeck

Element matrices:

K_{TT} - thermal conductivity

K_{VV} - electric conductivity

K_{VT} - Seebeck coupling

C_{TT} - specific heat damping

C_{VV} - dielectric damping

Thermoelectric Cooler Example

- Thermoelectric Cooler
  - Thermal-electric coupling
    - n-type and p-type semiconductors

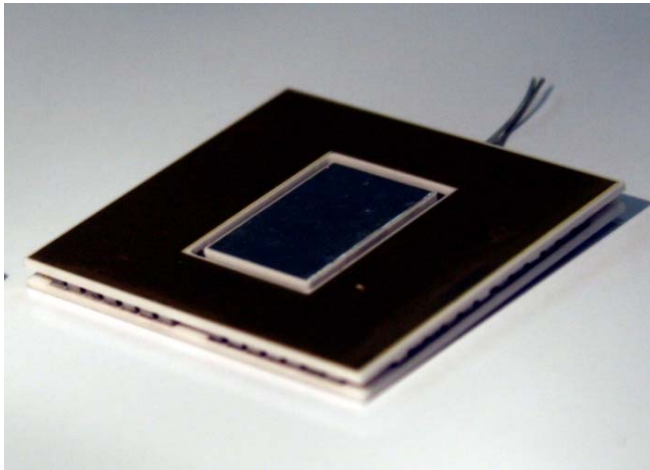
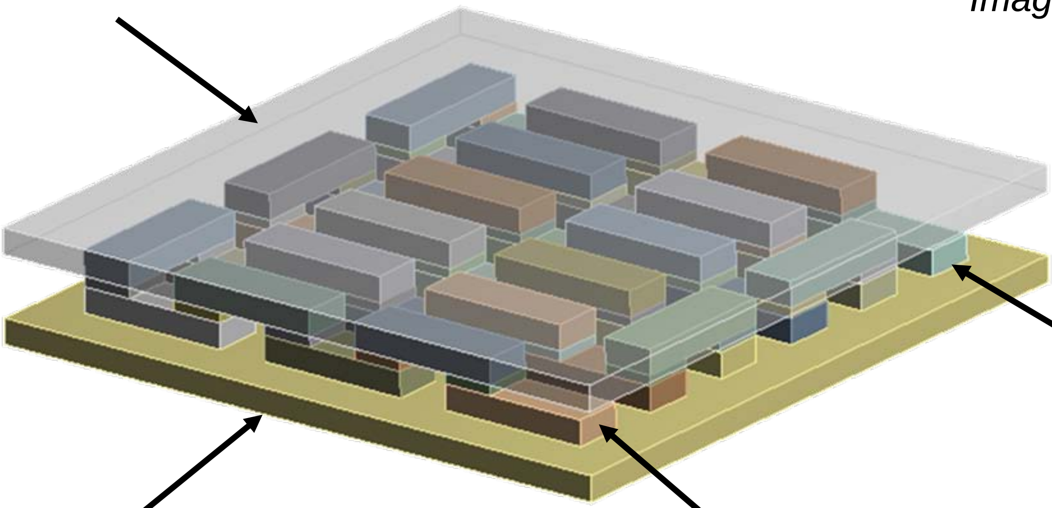
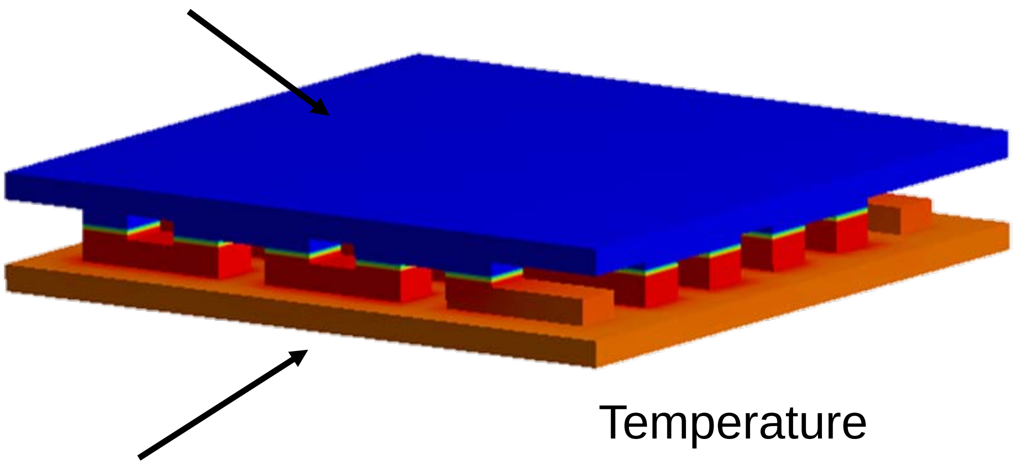


Image courtesy of Marlow Industries

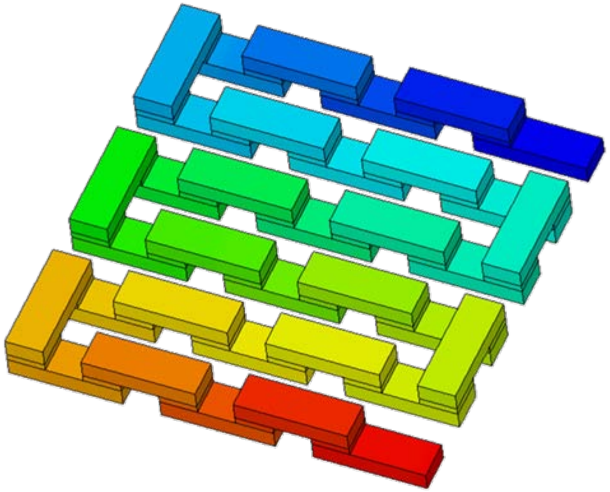


Thermoelectric Cooler Example

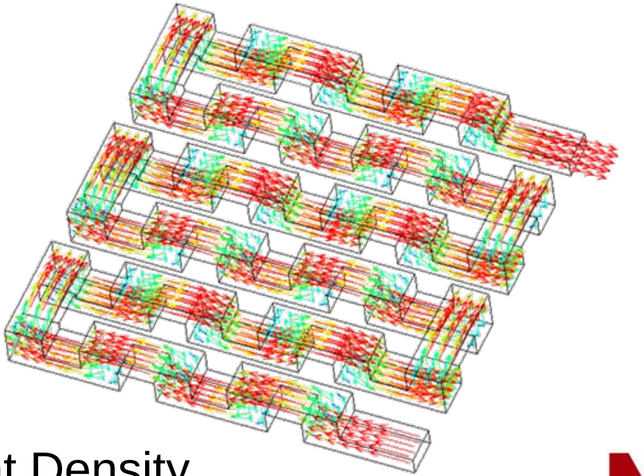
- Thermoelectric Results
  - Thermal gradient in Peltier blocks



Temperature



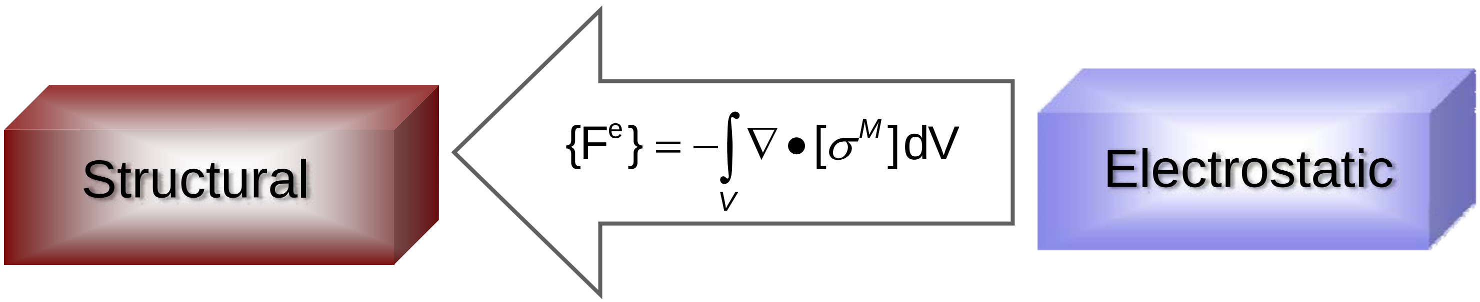
Electric Potential



Current Density

Electroelastic Analysis

- In an electroelastic analysis, structural and electrostatic fields are coupled by an electric force $\{F^e\}$:



$$\{F^e\} = -\int_V \nabla \bullet [\sigma^M] dV$$

$$[\sigma^M] \text{ - Maxwell stress tensor} = \frac{1}{2}\left(\{E\}\{D\}^T + \{D\}\{E\}^T - \{D\}^T\{E\}\{I\}\right)$$

$\{D\}$ - electric flux density

$\{E\}$ - electric field intensity

$\{I\}$ - identity matrix

Electroelastic Finite Element Formulation

- Load Vector (Weak) Coupling

$$\begin{bmatrix} K_{UU} & 0 \\ 0 & K_{VV} \end{bmatrix} \begin{Bmatrix} U \\ V \end{Bmatrix} + \begin{bmatrix} C_{UU} & 0 \\ 0 & C_{VV} \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{V} \end{Bmatrix} + \begin{bmatrix} M_{UU} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{V} \end{Bmatrix} = \begin{Bmatrix} F+F^{e} \\ Q \end{Bmatrix}$$

- U and V are couple through the electric force load vector $\{F^{e}\}$
- Iterative Solution
- Symmetric Matrix

Element matrices:

K_{UU} - structural stiffness

K_{VV} - dielectric permittivity

C_{UU} - structural damping

C_{VV} - dielectric dissipation

M_{UU} - mass

Folded Dielectric Elastomer Actuator

- Linear Actuator
  - Applied voltage between the compliant electrodes
  - Compresses based on electrostatic pressure

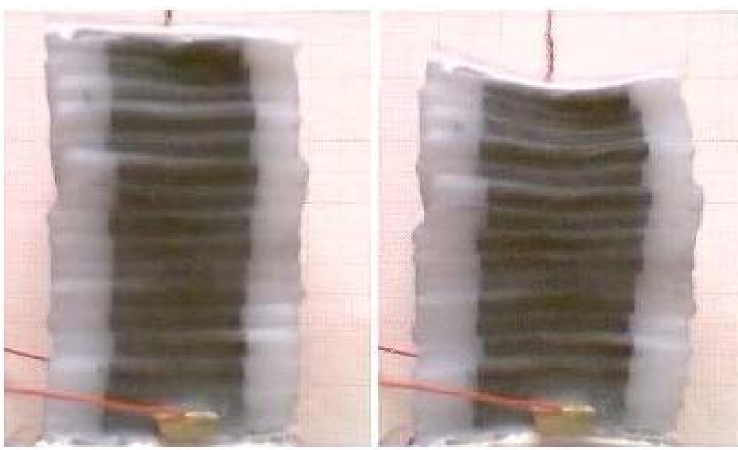
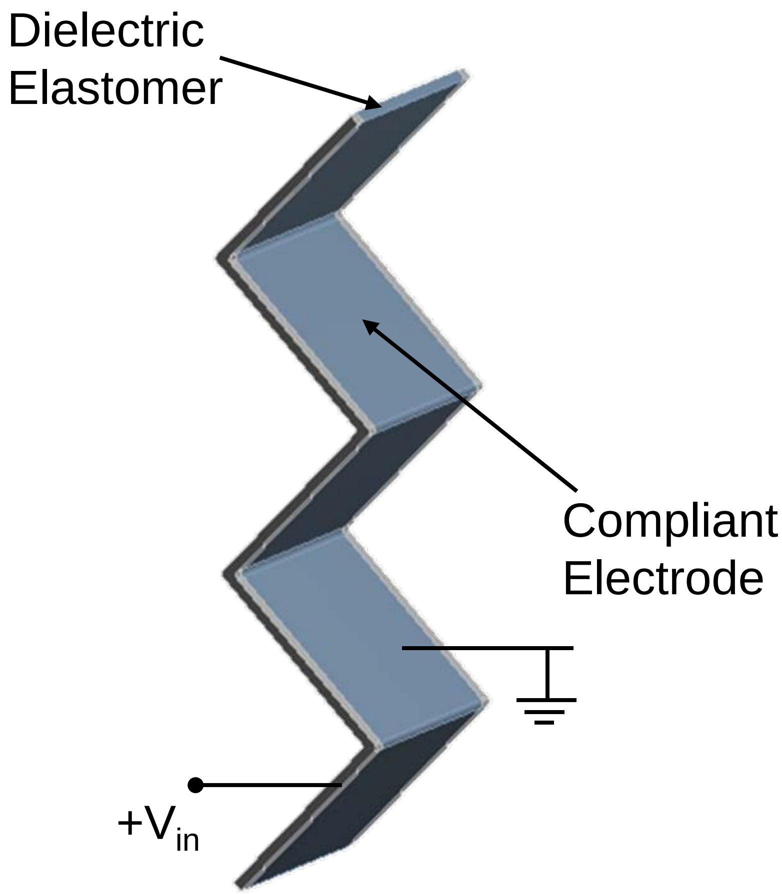


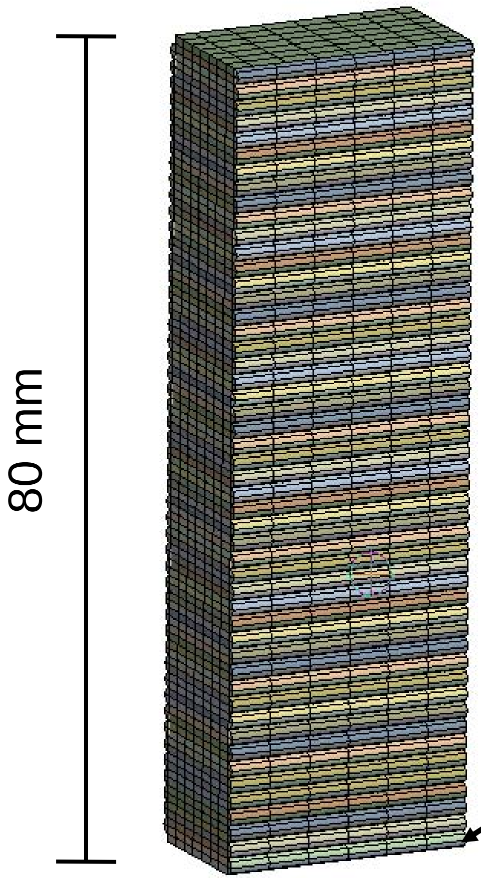
Image courtesy of Federico Carpi, University of Pisa



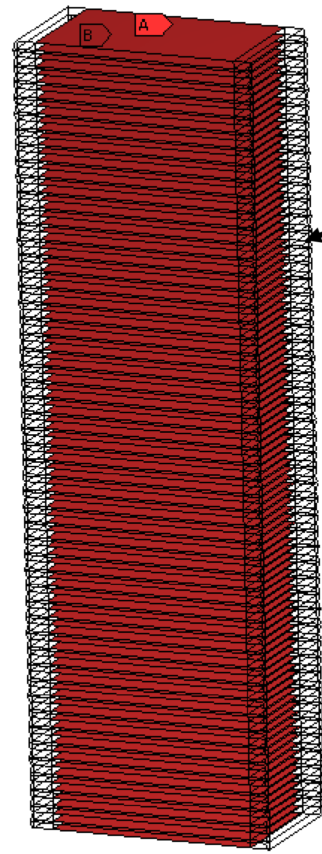
Folded Sheet of Dielectric Elastomer

Folded Dielectric Elastomer Actuator

- Finite Element Model



Mechanically Fixed Base



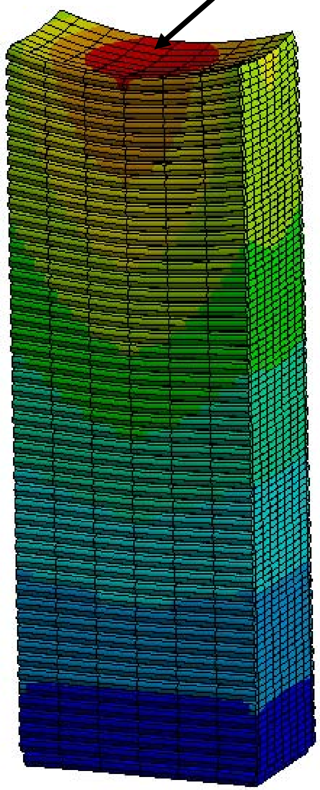
Electrodes - 8500 Volts

Actuator is 80 mm with 100 0.8 mm polymer layers.

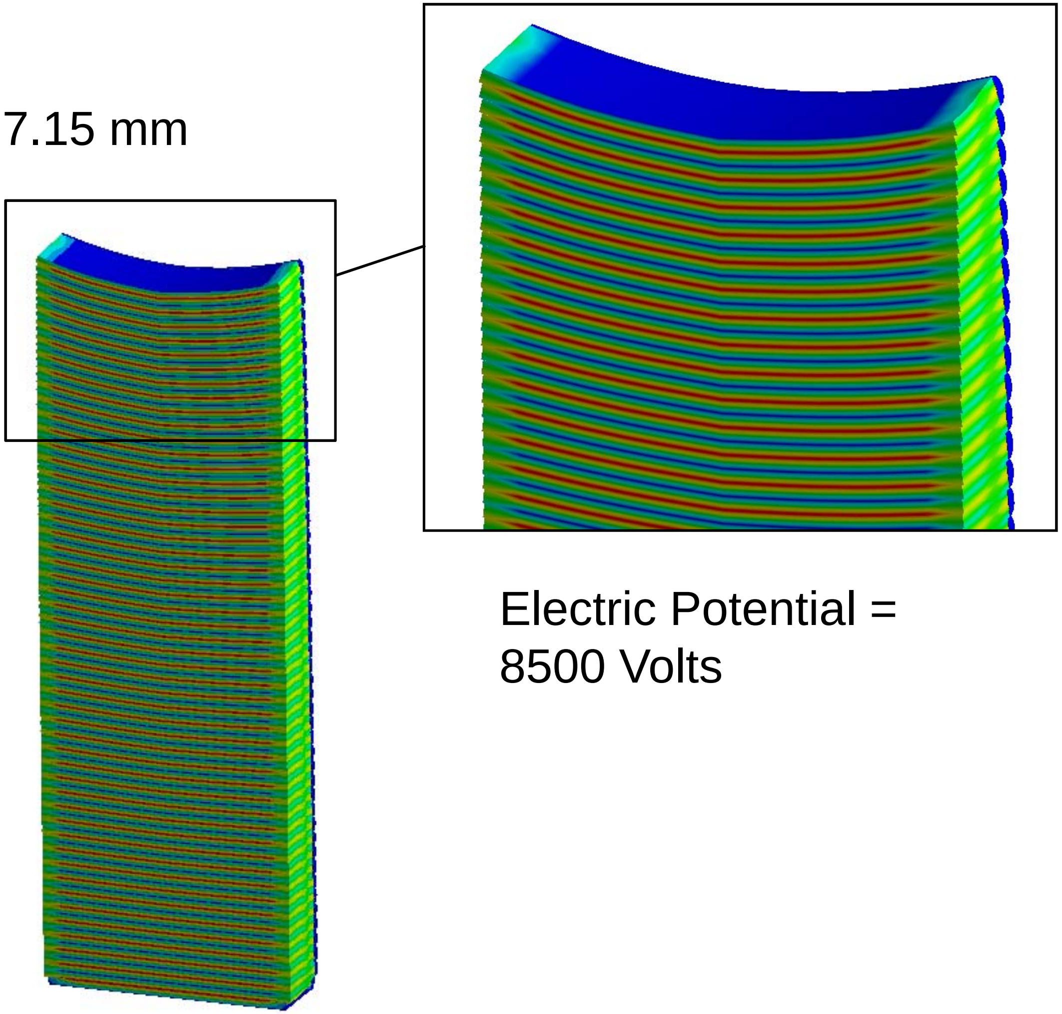
Folded Dielectric Elastomer Actuator

- Results

Deflection = 7.15 mm



Deformation

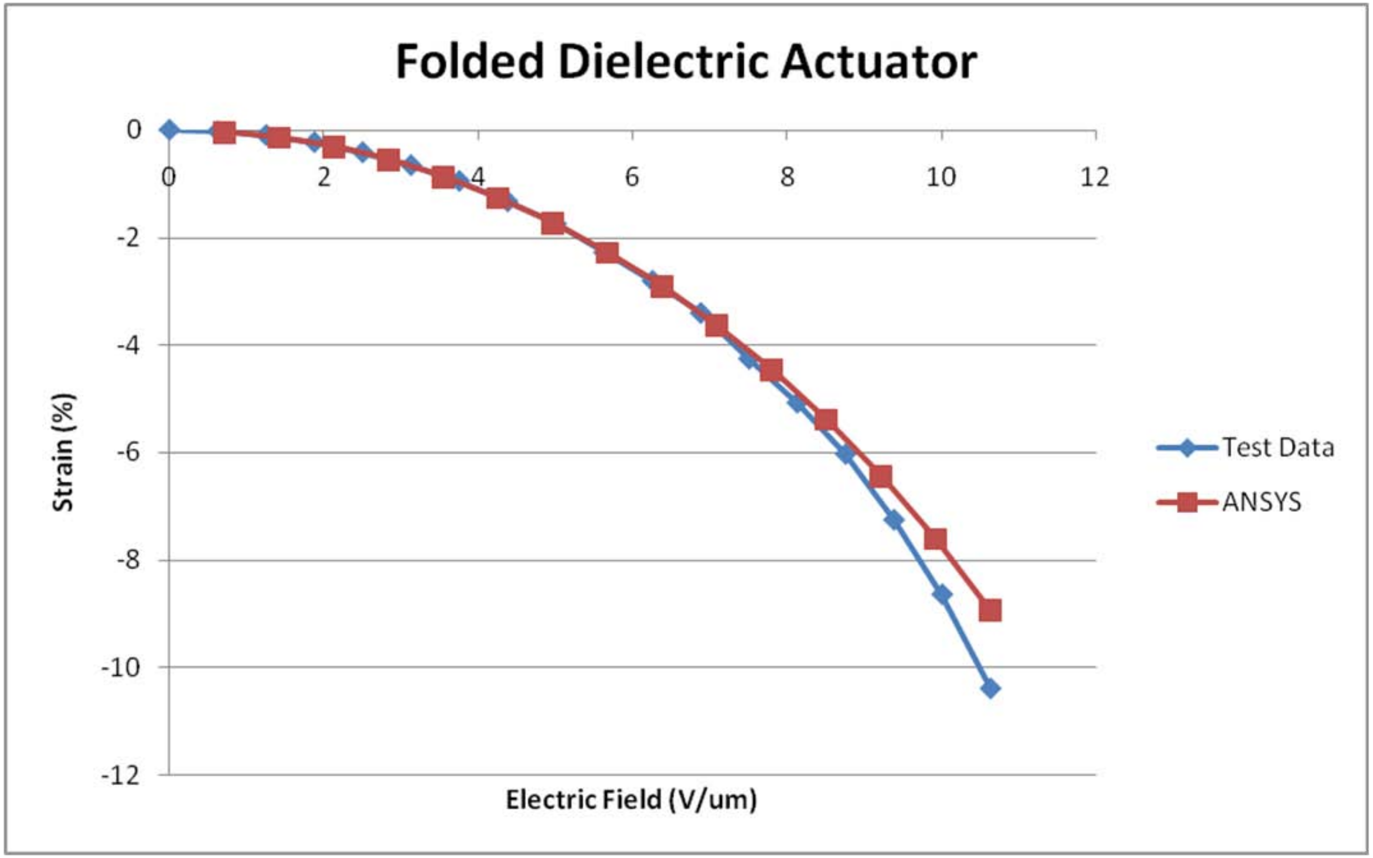


Electric Potential

Electric Potential =
8500 Volts

Folded Dielectric Elastomer Actuator

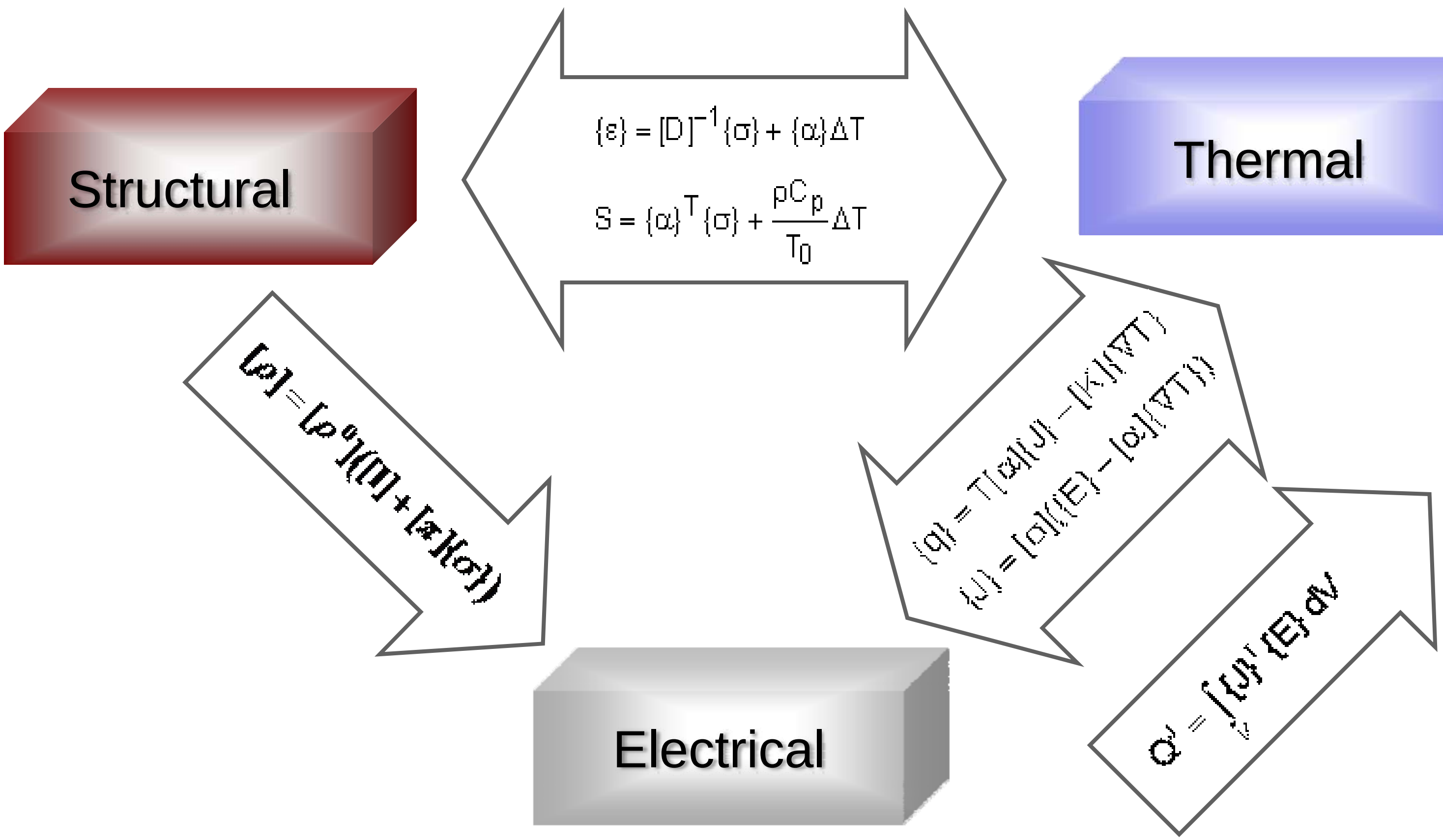
- Comparison with Test Data



Excellent correlation to the test data to about 7% strain. Model is stiffer than actual device between 7% and 10% strain.

Structural-Thermal-Electric Coupling

- A structural-thermal-electric analysis including thermal expansion, piezocaloric, Joule heating, Seebeck/Peltier and piezoresistive effects:



Finite Element Formulation

- Matrix/Load Vector Coupling

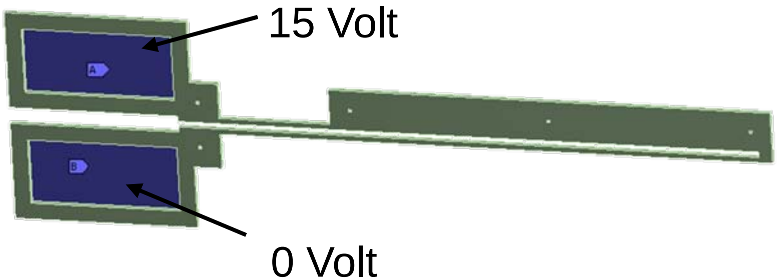
$$\begin{bmatrix} K_{UU} & K_{UT} & 0 \\ 0 & K_{TT} & 0 \\ 0 & K_{VT} & K_{VV}(U) \end{bmatrix} \begin{Bmatrix} U \\ T \\ V \end{Bmatrix} + \begin{bmatrix} C_{UU} & 0 & 0 \\ -T_0 K^T{}_{UT} & C_{TT} & 0 \\ 0 & 0 & C_{VV} \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{T} \\ \dot{V} \end{Bmatrix} + \begin{bmatrix} M_{UU} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{T} \\ \ddot{V} \end{Bmatrix} = \begin{Bmatrix} F \\ Q+Q^J+Q^P \\ I \end{Bmatrix}$$

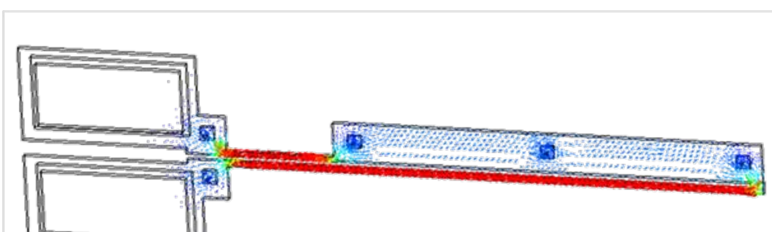
- U and T are strongly coupled by thermoelastic coupling matrices K_{UT} and C_{TU}
- T and V are strongly coupled by Seebeck coefficient matrix K_{VT} and weakly coupled by Peltier and Joule load vectors Q^J Q^P
- U and V are weakly coupled by the piezoresistive coefficients $K_{VV}(U)$
- Unsymmetric Matrix

Element matrices:
K_{UU} - structural stiffness
K_{TT} - thermal conductivity
K_{UT} - thermoelastic coupling
K_{VV} - electric conductivity
K_{VT} - Seebeck coupling
C_{UU} - structural damping
C_{TT} - specific heat damping
C_{VV} - dielectric damping
M_{UU} - mass
T_0 - absolute reference temperature

Thermal-Electric Actuator Example

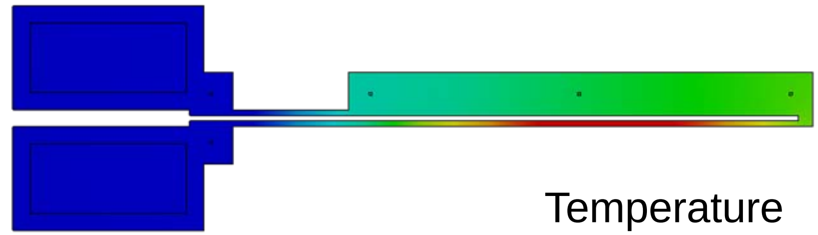
- Thermal-Electric Actuator



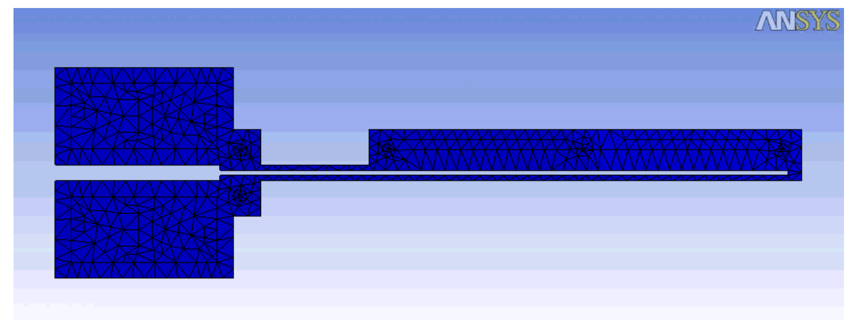


Current Density

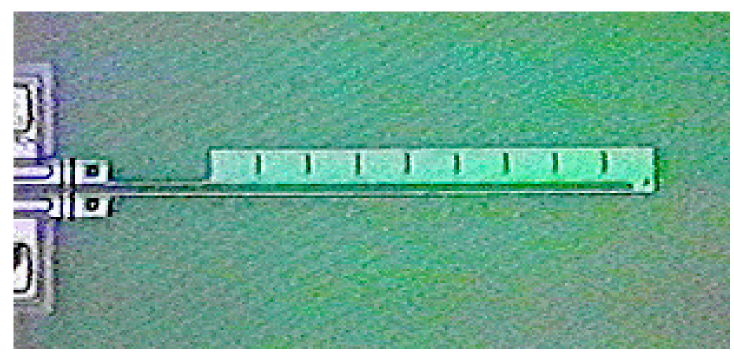
Deflection is induced by the differential thermal expansion between the thin and wide arms.



Temperature



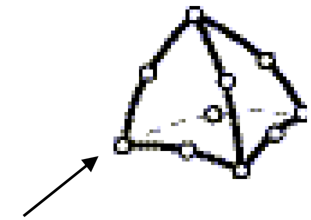
Deflection



SEM image courtesy of Victor Bright, University of Colorado at Boulder

Conclusions

- Direct Coupled Field Elements
  - Advantages
    - Ease of use
      - One element type
      - One finite element model
      - One set of results
    - Robust for nonlinear coupled-field solutions
    - Nonlinear geometric effects
  - Disadvantages
    - Large model sizes
    - Can create unsymmetric matrices



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