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Advances in Element Technology: Solid-Shell Element

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Outline

- Motivation
- Formulations
- Composite application
- Future development

Limitations in shell elements

- Nonlinear MPCs or transitional elements are required for connecting shell and solid elements
- Difficulties in the specialization of general three-dimensional material laws to plane-stress state.
- Definition of contact interaction needs special attention
- Treatment of variable thickness is unclear

In the above areas, use of continuum elements would be more desirable.

Locking in linear solid elements

- The error in the kinematic approximation with linear solid elements becomes apparent in bending dominant problems
- This error is magnified as the thickness to span ratio decreases, which beyond a certain ratio may make the FE model excessively stiff → locking
- Current element technologies, such as the enhanced strains, are shown not sufficient to remedy this numerical locking in linear solid elements

Design of a Solid-Shell (SOLSH190)

- Involves only displacement nodal DOFs and features an eight-node brick connectivity. Thus the transition problem between solid and shell elements can be eliminated.
- Performs well in simulating shell structures with a wide range of thickness (from extremely thin to moderate thick).
- Is compatible with 3D constitutive models and automatically accounts for thickness change.
- Performs well for both flat-plate and curved shells.

Issues to address

- Transverse shear locking that originates from spurious shear stresses in plate bending cases
- Thickness locking due to the spurious thickness normal stress in plate bending cases
- Curvature locking that stems from the incompatible thickness normal strain when the element is skewed in thickness direction
- Membrane locking induced by the spurious in-plane stresses in the in-plane bending cases

Issues to address (cont.)

- Numerical ill-conditioning due to large ratio between transverse and in-plane stiffness
- Mesh distortion
- Geometrical nonlinearity, including finite transverse shear deformation
- Material nonlinearity that also accounts for nonlinear transverse shear constitutive relations

All above issues must be properly handled to produce a truly locking-free Solid-Shell in both linear and nonlinear analyses

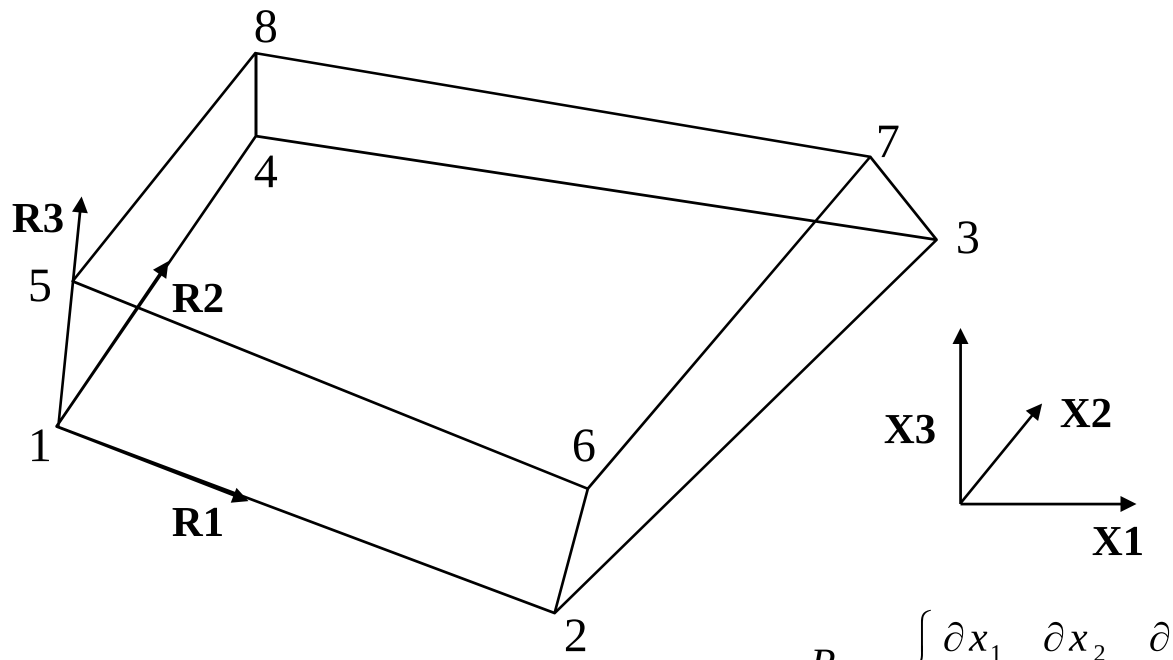
Literature

- R. Hauptmann and K. Schweizerhof (1998) -- early formulation based on the de-coupled strain approximation in natural space
- Ted Belytschko and Jingxiao Xu (2002) -- intensive transverse strain enhancements
- C. A. Felippa (2002) -- handling element distortion
- X. Tan and L. Vu-Quoc (2005) – optimal thickness strain enhancements
- K. Y. Sze and X. H. Liu (2007) -- hybrid stress formulation

Basic SOLSH190 approaches

- Establish kinematic relations for in-plane and normal directions separately with different methods.
  - The in-plane strain components are evaluated with the standard tri-linear shape functions, while the thickness-related components are obtained based on the assumed natural strains.
- Employ de-coupled strain enhancements to further improve the element performance in both out-of-plane and in-plane bending dominant situations.

Solid-Shell Bases



X1, **X2**, and **X3**: bases for the global Cartesian system. Corresponding coordinates: **x1**, **x2**, and **x3**.

R1, **R2**, and **R3**: bases for element natural system. Corresponding coordinates: **r1**, **r2**, **r3**.

$$R_1 = \left\{ \frac{\partial x_1}{\partial r_1}, \frac{\partial x_2}{\partial r_1}, \frac{\partial x_3}{\partial r_1} \right\}$$

$$R_2 = \left\{ \frac{\partial x_1}{\partial r_2}, \frac{\partial x_2}{\partial r_2}, \frac{\partial x_3}{\partial r_2} \right\}$$

$$R_3 = \left\{ \frac{\partial x_1}{\partial r_3}, \frac{\partial x_2}{\partial r_3}, \frac{\partial x_3}{\partial r_3} \right\}$$

Evaluation of Natural Strains

- In-plane components: ε_{11}, ε_{22} and ε_{12}

$$\varepsilon_{11} = \sum_{i=1}^{8} \frac{\partial N_i}{\partial r_1} \left(U^i \cdot R_1 \right)$$

$$\varepsilon_{22} = \sum_{i=1}^{8} \frac{\partial N_i}{\partial r_2} \left(U^i \cdot R_2 \right)$$

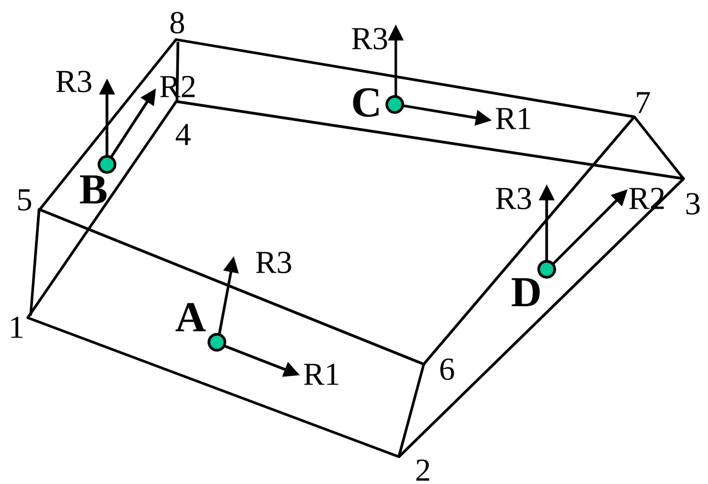
$$\varepsilon_{12} = \sum_{i=1}^{8} \frac{\partial N_i}{\partial r_1} \left(U^i \cdot R_2 \right) + \sum_{i=1}^{8} \frac{\partial N_i}{\partial r_2} \left(U^i \cdot R_1 \right)$$

Where N_i, $i = 1,\ldots, 8$, are eight tri-linear shape functions, and $U^i = \left\{ u_1^i, u_2^i, u_3^i \right\}$ are eight nodal displacement vectors referred in global Cartesian system.

Evaluation of Natural Strains (cont.)

- Assumed natural transverse shear (NTS) strains: ε_{13} and ε_{23}

Problem addressed: transverse shear locking



NTS strains at the centroids of four element faces:

$$\varepsilon_{13}^{A} = \sum_{i=1}^{4} \frac{\partial H_i^A}{\partial r_1}\left(U^{(i)} \cdot R_3^A\right) + \sum_{i=1}^{4} \frac{\partial H_i^A}{\partial r_3}\left(U^{(i)} \cdot R_1^A\right)$$

$$\varepsilon_{13}^{C} = \sum_{i=1}^{4} \frac{\partial H_i^C}{\partial r_1}\left(U^{(i)} \cdot R_3^C\right) + \sum_{i=1}^{4} \frac{\partial H_i^A}{\partial r_3}\left(U^{(i)} \cdot R_1^C\right)$$

$$\varepsilon_{23}^{B} = \sum_{i=1}^{4} \frac{\partial H_i^B}{\partial r_2}\left(U^{(i)} \cdot R_3^B\right) + \sum_{i=1}^{4} \frac{\partial H_i^B}{\partial r_3}\left(U^{(i)} \cdot R_2^B\right)$$

$$\varepsilon_{23}^{D} = \sum_{i=1}^{4} \frac{\partial H_i^D}{\partial r_2}\left(U^{(i)} \cdot R_3^D\right) + \sum_{i=1}^{4} \frac{\partial H_i^D}{\partial r_3}\left(U^{(i)} \cdot R_2^D\right)$$

$$\varepsilon_{13} = \frac{1}{2}\left(1 - r_2\right)\varepsilon_{13}^{A} + \frac{1}{2}\left(1 + r_2\right)\varepsilon_{13}^{C} \qquad \varepsilon_{23} = \frac{1}{2}\left(1 - r_1\right)\varepsilon_{23}^{B} + \frac{1}{2}\left(1 + r_1\right)\varepsilon_{23}^{D}$$

Where H_i, $i = 1,\ldots, 4$, are four bi-linear shape functions.

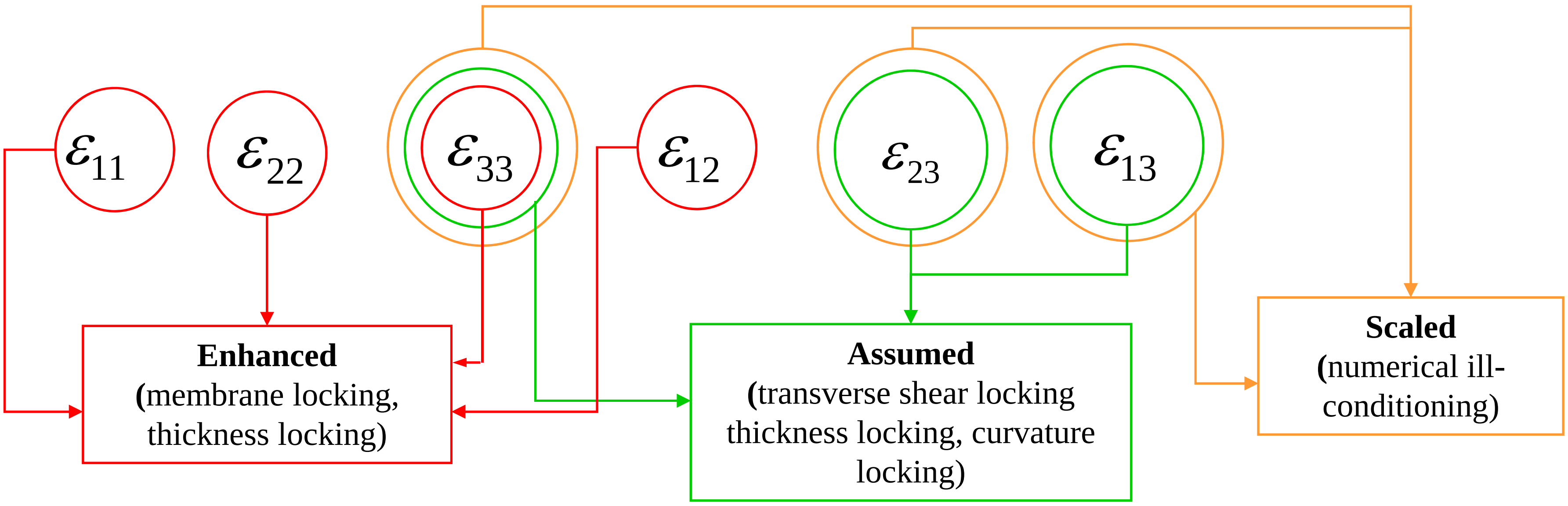
Other SOLSH190 element technologies

- A similar assumed strain approach is taken for formulating the thickness natural stains.
  Problem addressed: curvature locking

- De-coupled strain enhancements are applied separately to in-plane and thickness strain components.
  Problem addressed: membrane and thickness locking

- Transverse normal and shear stiffness are reduced when thickness becomes extremely small. This scaling is justified since in this case transverse strain energy is insignificant.
  Problem addressed: numerical ill-conditioning

Summary of SOLSH190 Formulations

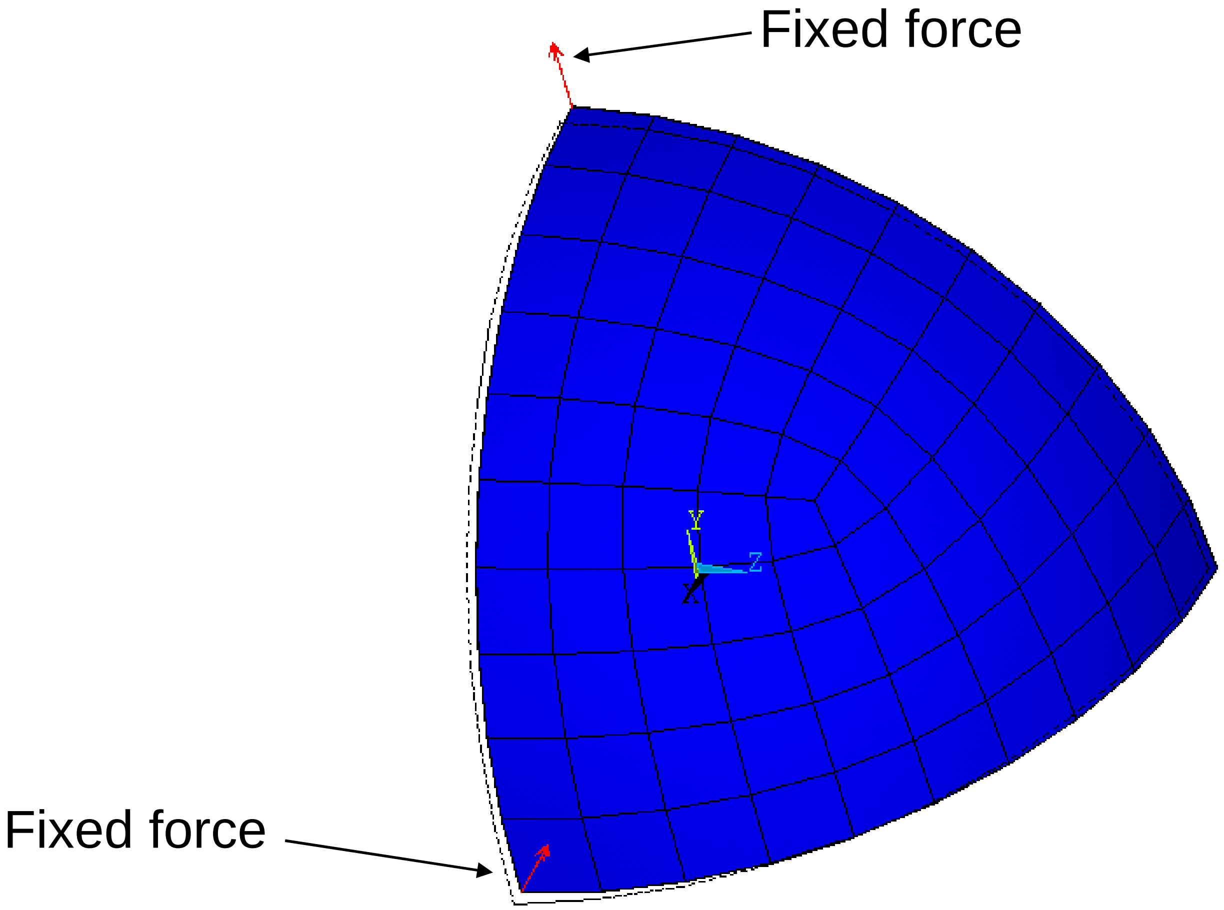


Not discussed:

- Nonlinear extension to account for both material and geometrical nonlinearities
- Handling of distorted geometry

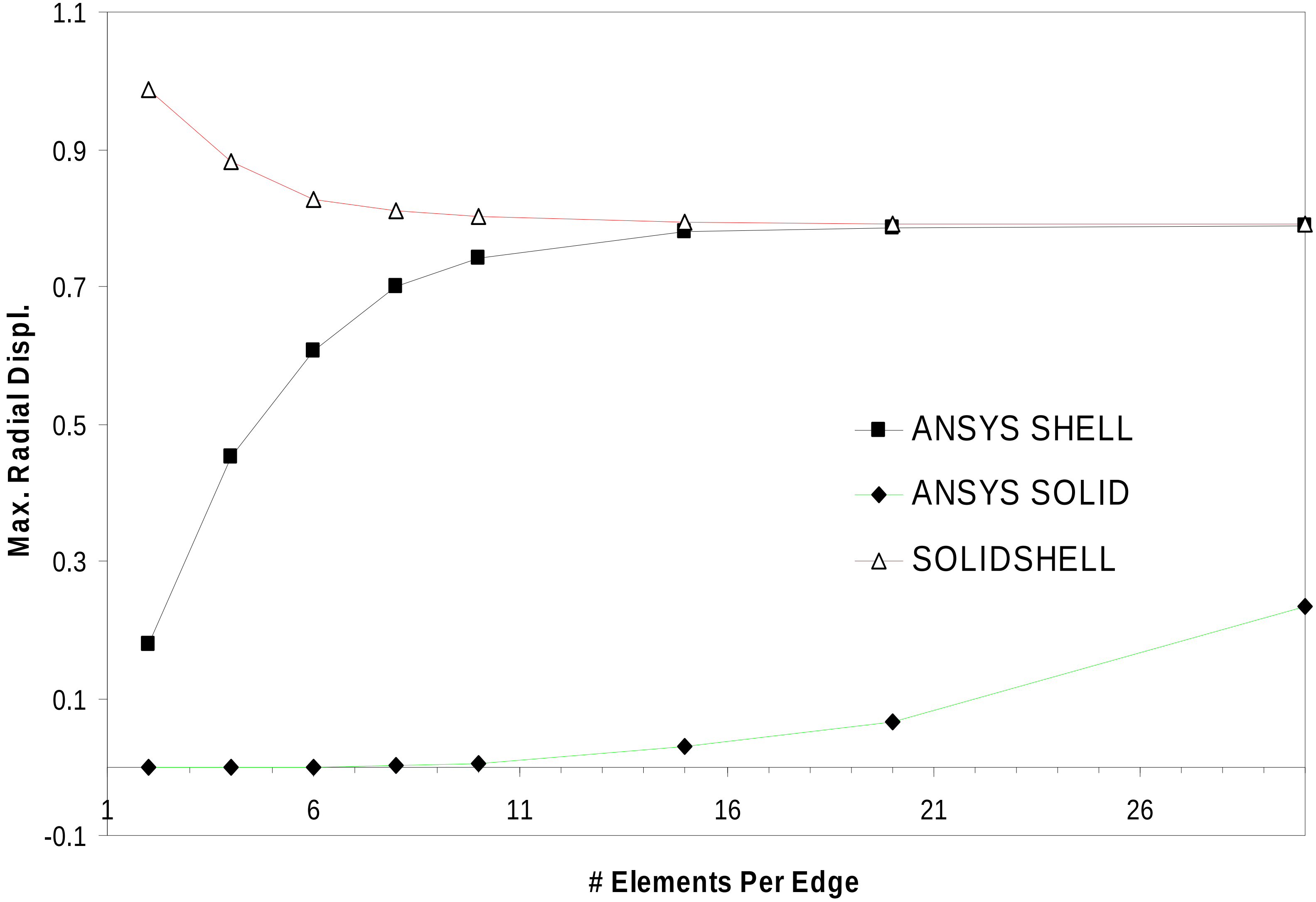
Example: pinched Hemi-sphere Shell

(t/r = 0.001, homogeneous)

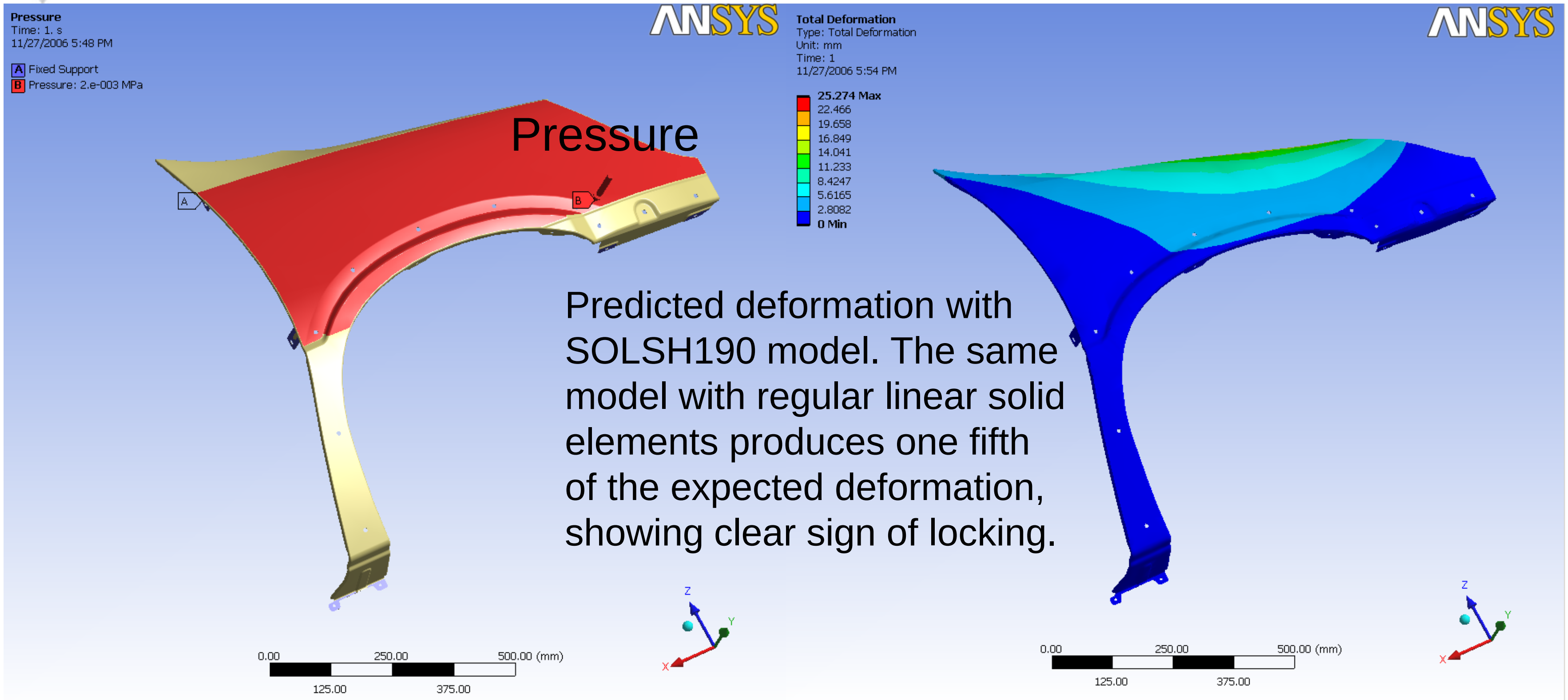


Attributes: membrane, bending, twisting and warping actions

Solution convergence wrt mesh refinement

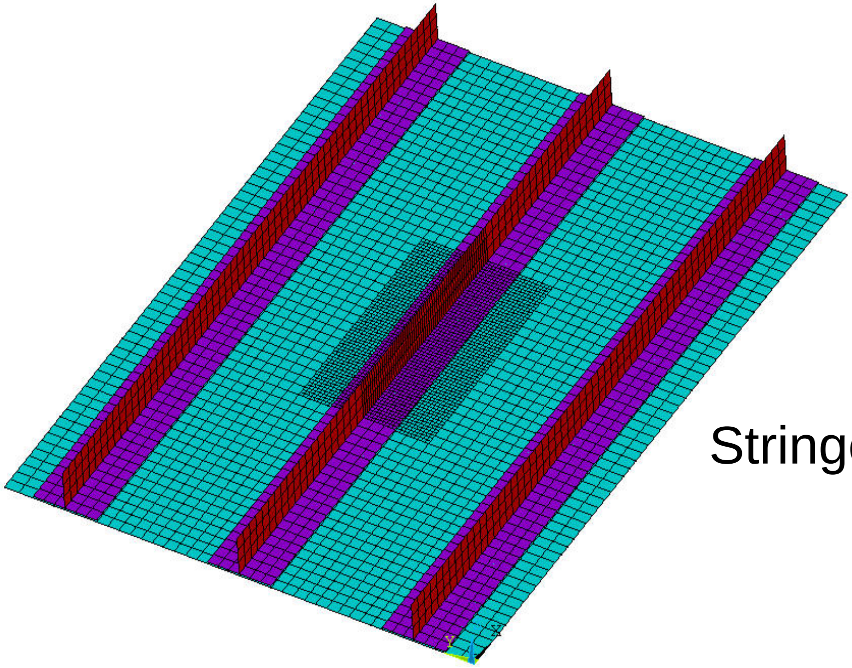


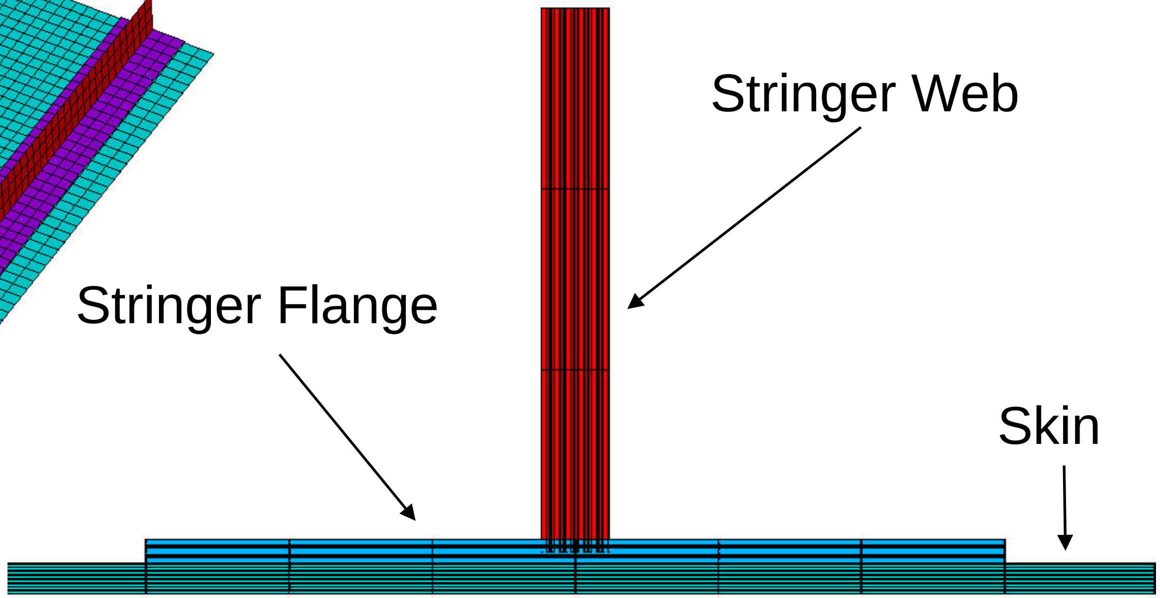
Example: automobile exterior panel



Mid-surface extraction based shell meshing may not be practical for parts with complex geometry and variable thicknesses, or when design parameters still need constant modification.

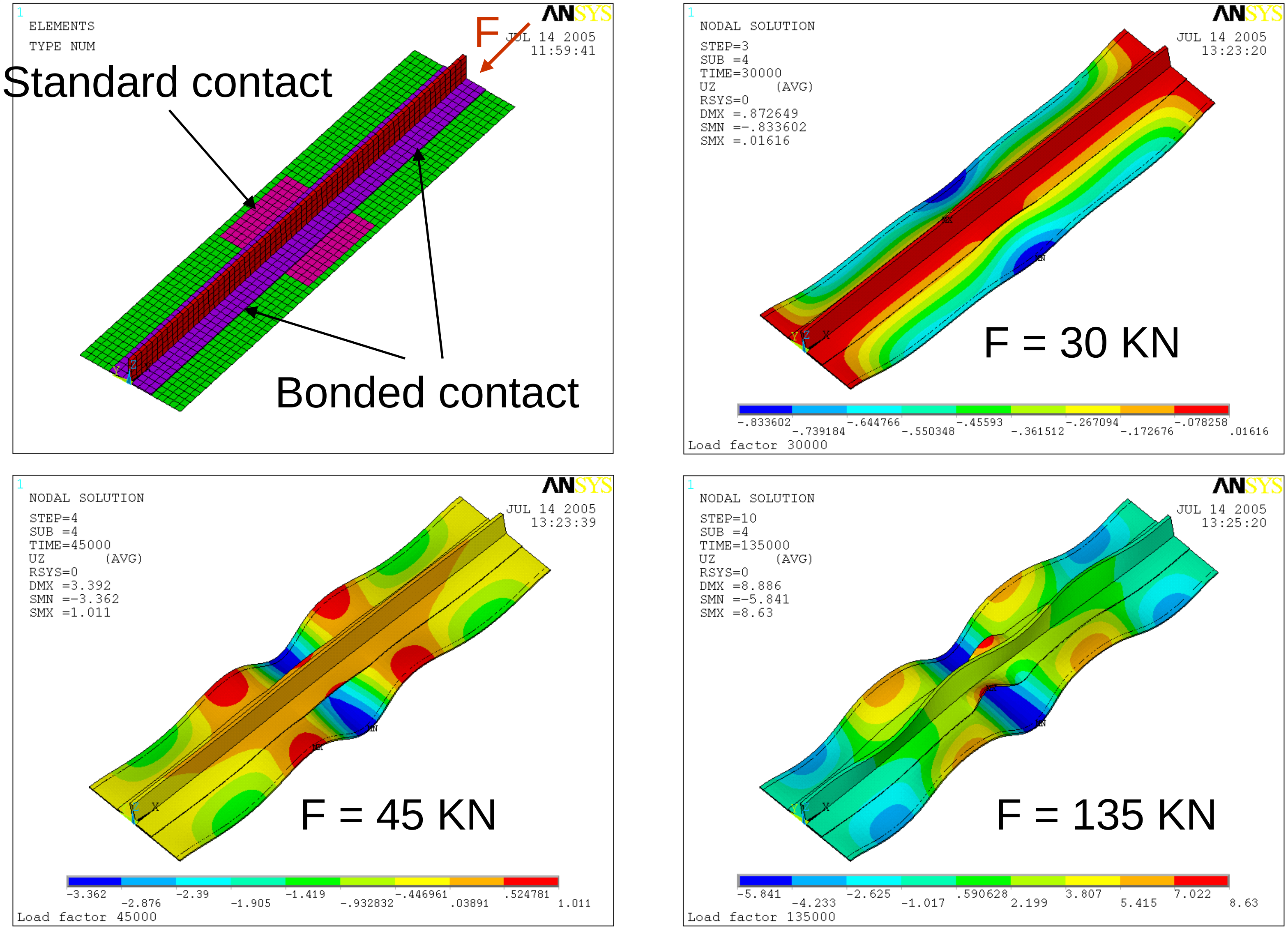
Example: composite stiffened panel





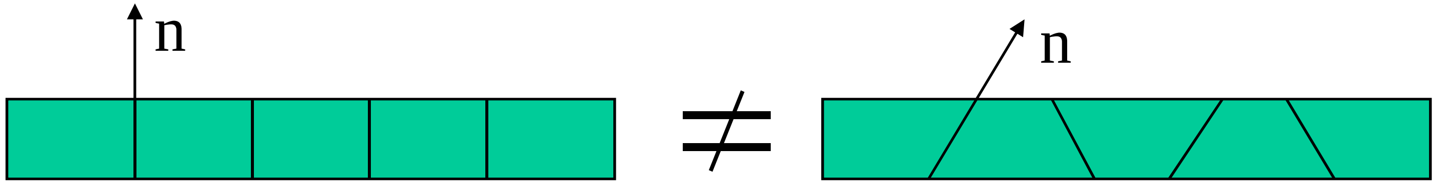
- Difficulties in shell modeling:
  - Need to consider offsets and specify top/bottom contact faces in the contact set-up between the flange and skin
  - The web and flange connection needs special attention

Example: composite stiffened panel



Limitations

- A special rule for forming element connectivity must be followed to define the thickness direction → extra pre-processing work
- Sensitive to through-the-thickness mesh distortion → need for better meshing controls and algorithms



Application to Laminated composites

- Layered composites commonly have low transverse stiffness. Finite and highly non-uniform transverse deformation can occur.
- Locking and other numerical irregularities may occur in thin composite models.
- Material and damage modeling are often based on the full 3D stress states.

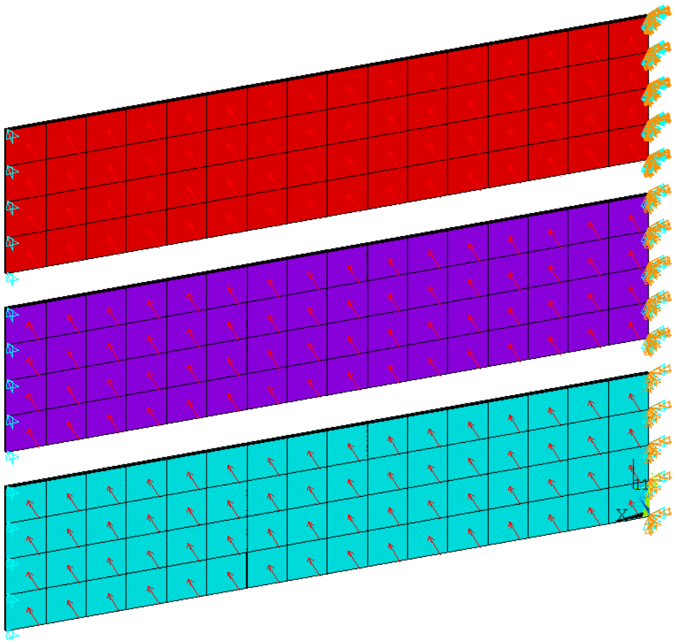
With the locking-free properties and solid element connectivity, SOLSH190 is desirable in layered composite simulations.

Predication of Interlaminar stresses

- SOLSH190 only has linear displacement field through-the-thickness → limited accuracy in transverse strains.
- The shear adjustment technique cannot be applied due to the difficulty in determining the top/bottom traction conditions.
- Generally, use of more elements in the thickness direction (or stacking) is required to obtain accurate interlaminar stress distributions.

Example: thin laminated plate

- Unsymmetric 5-ply laminate (steel, epoxy, and rubber plies)
- Both pressure and thermal body loads applied
- FE models compared:
  - Stacked SOLSH190 plate construction (5 elements thick)
  - SOLSH190 element plate construction (1 element thick)
  - SHELL element plate construction

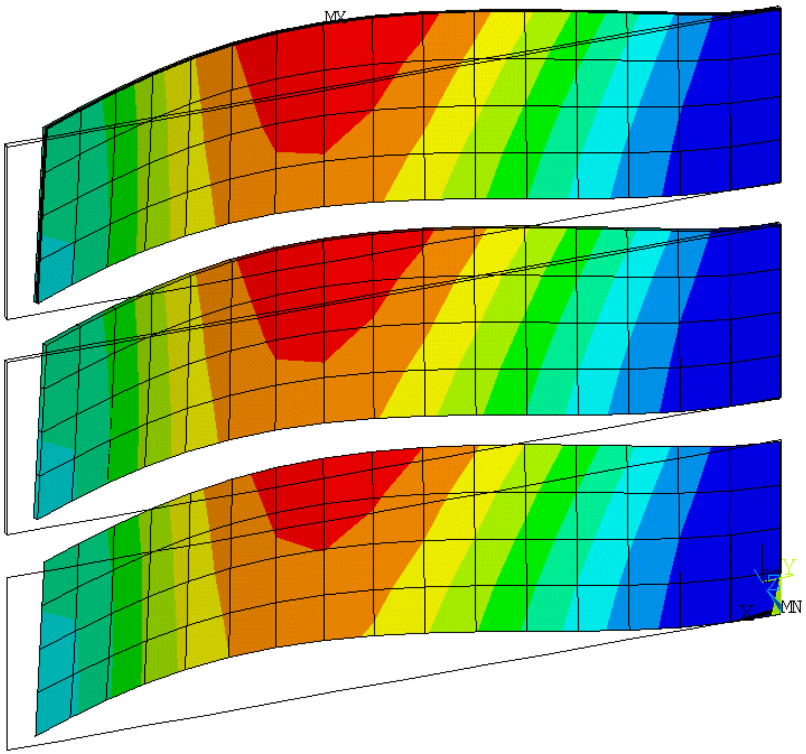


Stacked SOLSH190 model

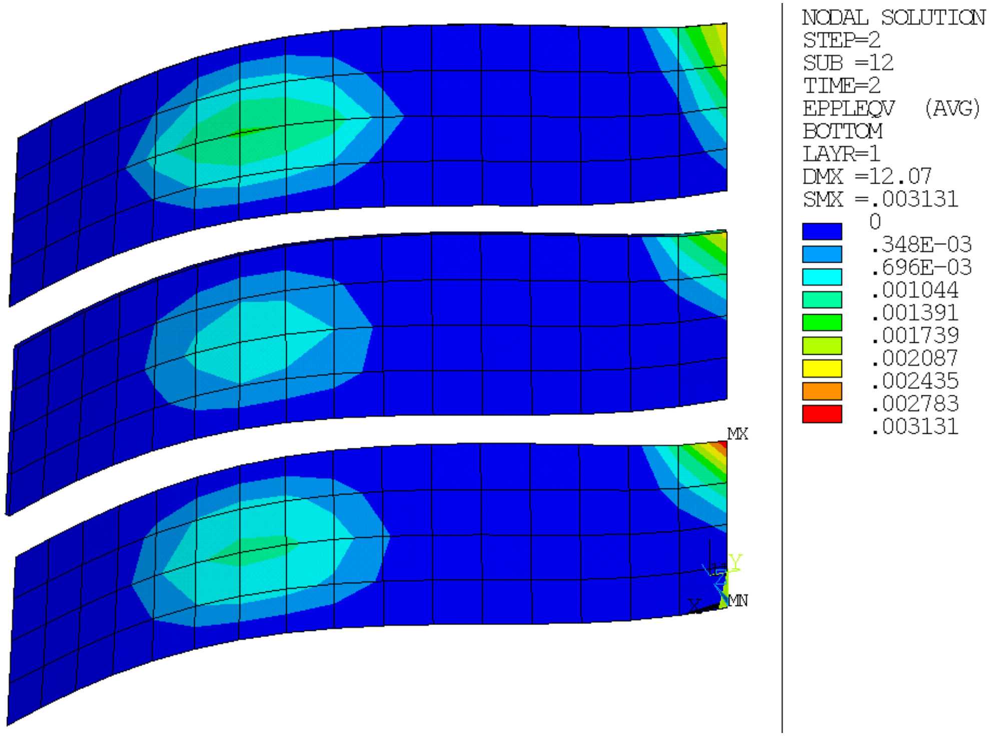
SOLSH190 model

Shell model

Example: thin laminated plate



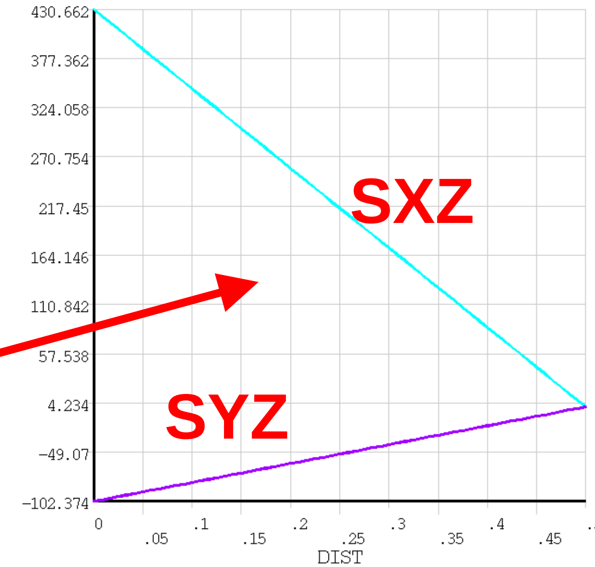
Displacements

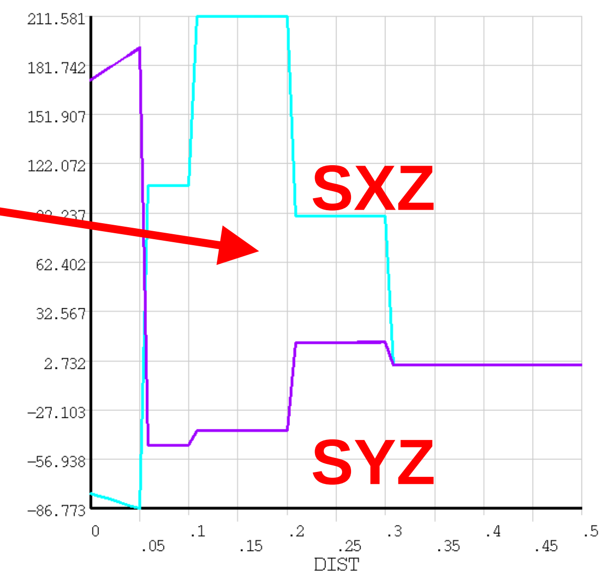


Equivalent Plastic Strain

Example: thin laminated plate

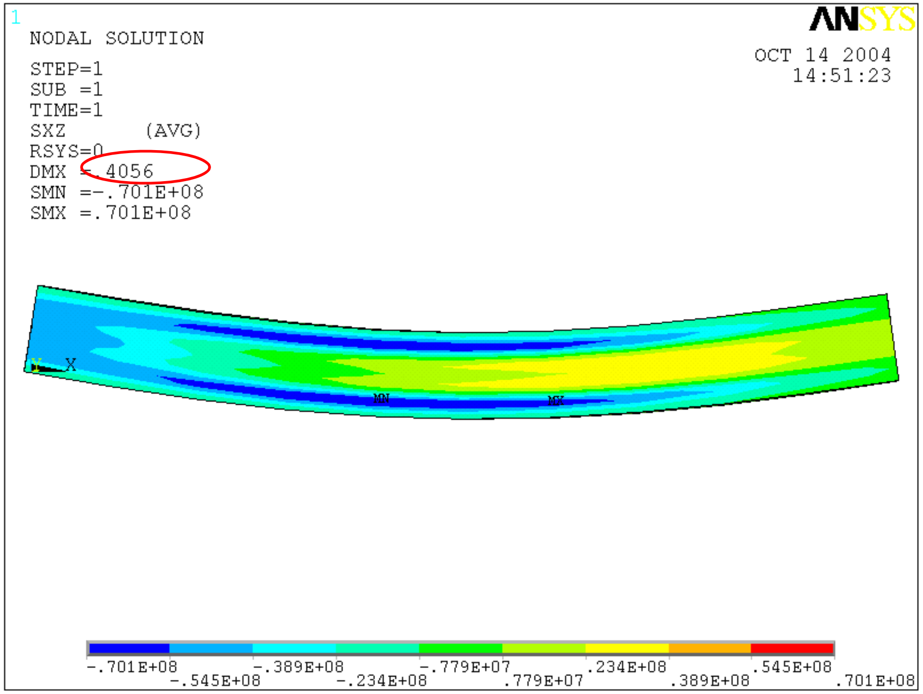
- Interlaminar shear stresses (SXZ,SYZ) are dependent on laminate construction:
  - For the SOLSH190 model consisting of one element through the thickness, the details of the shear stress variation is lost.
  - For the stacked SOLSH190 model consisting of five elements through the thickness, the variation is captured. However, more elements are still needed for a smoother stress distribution.



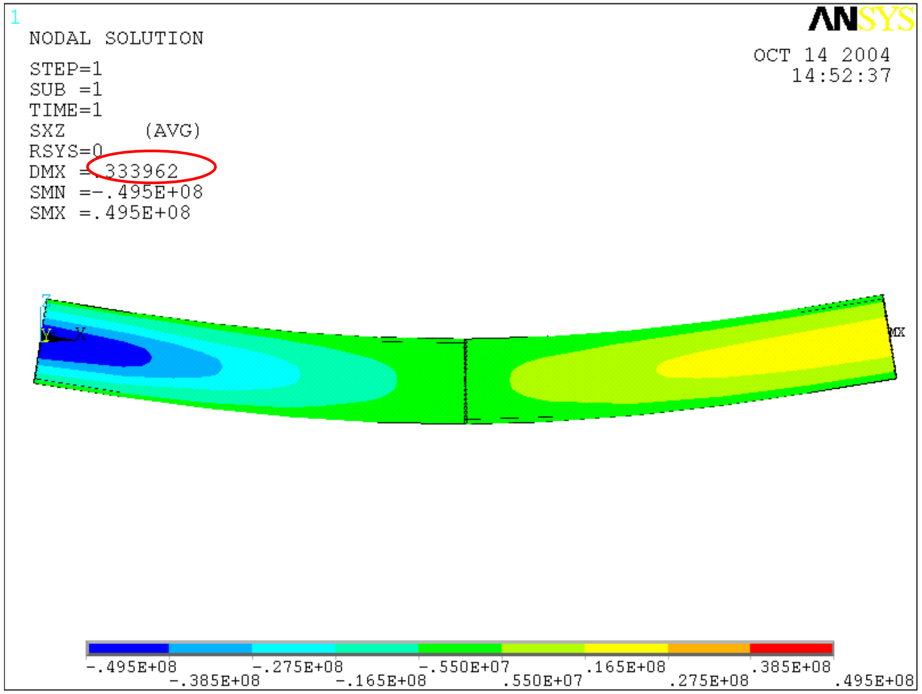


Example: thick laminated plate

- 45/-45 symmetric lay-up, orthotropic materials,20 layers
- Thickness to span ratio: 0.1
- Differences in both global response and stress distributions



SXZ, SOLSH190 (stacked)



SXZ, Shell

Future of Solid-Shell element

- Element formulations
  - Higher order transverse strain formulation with or without layer-wise displacement fields, to improve the accuracy in transverse strains. Suitable for modeling the laminate with single element
  - Hybrid stress formulation that incorporates transverse stresses as independent variables, to ensure the interlaminar transverse stress continuity. Suitable for the stacking option (i.e., one element per layer)
  - Special element shape correction technique for improving robustness and solution consistency in the distorted mesh

Future of Solid-Shell (cont.)

- Multi-physics
  - Thermal, mechanical, and electric potential coupled field solid-shell for modeling smart and piezoelectric structures
- Pre-processing
  - Thin solid meshing algorithm to minimize mesh distortion through the thickness
  - layered composite meshing to create refined mesh in the thickness direction based on the lay-up information
- Membrane option
  - To eliminate numerical ill-conditioning and improve efficiency in extremely thin structure

Conclusion

- With its locking-free property, solid element connectivity, and 3D strain state compatibility, Solid-Shell may represent the future trend in the modeling of shell and laminated composite structures.
- Solid-Shell based simulation can be further enhanced in areas such as meshing, element formulation, and multi-physics capability.